Networks of Manufacturers and Retailers: The Appendix †

(only for referee use)

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1 Appendix A : The benchmark case

1.1 The equilibrium outcomes

We give the outputs, transfer prices and payoffs of the symmetric distribution networks between n manufacturers and n retailers and other networks necessary to check for stability of networks C and ED.

(a) We start with the complete network C where the n manufacturers are linked with the n retailers, for all i, j, $n_{b_j} = n_{r_i} = r = m = n$. As indicated in the text $p_i = a - Q_i - d\sum_{l\neq i} Q_l$ $i, l = 1, 2,..., n$. Where $Q_i = \sum_{j=1}^{n} q_{ij}$, for all $i = 1, 2,..., n$. Retailers maximize the following payoffs choosing n different quantities. Each retailer j, $j = 1, 2, ..., n$ maximizes

$$\max_{\{q_{1i}, q_{2i}, \ldots, q_{ni}\}} \Pi_{R_j}(\cdot) = \sum_{i=1}^{n} (p_i - w_i) q_{ij} - nk,$$

Note that symbol "$\cdot$" in the above expression means that retailers payoffs are functions of all the q's. The n first order conditions (FOC) for retailer j are:

$$-q_{ij} + p_i - w_i - d\sum_{k\neq i} q_{kj} = 0,\ i = 1, 2, ..., n.$$

Substituting in each FOC for p_i and adding for all retailers' FOC for each product i we get,

$$-(n+1)Q_i + n(a - w_i) - d(n+1)\sum_{k\neq i} Q_k = 0,\ i = 1, 2, ..., n.$$

Next, adding for all the above expressions and solving for $\sum_{i=1}^{n} Q_i$, we obtain:

$$\sum_{i=1}^{n} Q_i = \frac{n\sum_{i=1}^{n} aw_i}{(n+1)(1 + d(n-1))}$$

Where aw_i is a shortcut for $(a - w_i)$ and it is convenient for the analysis to express the derived demands for manufacturers in terms of expressions like aw_i. Then, the derived demand functions for manufacturers and retailers' margins easily follow, $i = 1, 2,..., n$.

$$\begin{aligned} Q_i(w_1, w_2, ..., w_n) &= \frac{n((1 + d(n-2)aw_i - d\sum_{k\neq i} aw_k)}{(1-d)(n+1)(1 + d(n-1))}, \\ p_i - w_i &= \frac{aw_i}{n+1}. \end{aligned}$$

Manufacturers decide on transfer prices, thus each manufacturer i maximizes the following:

$$\max_{\{w_i\}} \Pi_{M_i}(w_1, w_2, ..., w_n) = (w_i - c)Q_i(w_1, w_2, ..., w_n) - nk$$

Then, manufacturer $i's$ FOC is $Q_i + \frac{\partial Q_i}{\partial w_i}(w_i - c) = 0$, which can be rewritten as,

$$Q_i + \frac{\partial Q_i}{\partial w_i} ac - \frac{\partial Q_i}{\partial w_i} aw_i = 0,$$

where ac stands for $(a - c)$. In this way we can substitute above for Q_i, apply manufacturers' symmetry ($aw_i = aw_k$ for all i, k) and solve for awi. We obtain that at equilibrium,

$$awi(C) = \frac{(1 + d(n-2))ac}{(2 + d(n-3))},\ i = 1, 2, ..., n.$$

The equilibrium expressions are now easily obtained:

$$\begin{aligned} w_ic(C) &= ac - aw_i(C) = \frac{(1-d)ac}{(2 + d(n-3))},\ \forall i \\ Q_i(C) &= \frac{n(1 + d(n-2))ac}{(n+1)(1 + d(n-1))(2 + d(n-3))},\ \forall i \\ \Pi_{M_i}(C) &= \frac{n(1-d)(1 + d(n-2))ac^2}{(n+1)(1 + d(n-1))(2 + d(n-3))^2} - nk, \forall i \\ \Pi_{R_j}(C) &= \frac{n(1 + d(n-2))^2 ac^2}{(n+1)^2(1 + d(n-1))(2 + d(n-3))^2} - nk, \forall j \end{aligned}$$

(b) Next, we present the equilibrium expressions for the exclusive distribution and dealing network, ED. The one where each manufacturer is linked with one and different retailer, that is for all i, j, $n_{b_j} = n_{r_i} = 1$, and $r = m = n$. Each agent bears only k regarding link costs. Now it follows that $Q_i = q_{ii}$, for all $i = 1, 2, ..., n$. So, only one output is chosen by each retailer. Manipulating the FOC for each retailer we reach the following expressions for the derived demand functions for manufacturers and retailers margins, for $i = 1, 2, ...n$,

$$\begin{aligned} Q_i(w_1, w_2, ..., w_n) &= \frac{(2 + d(n-2))aw_i - d\sum_{k \neq i} aw_k}{(2-d)(2 + d(n-1))}, \\ p_i - w_i &= Q_i(w_1, w_2, ..., w_n). \end{aligned}$$

We are ready to obtain the FOC for each manufacturer and apply symmetry to these FOC's obtaining the equilibrium transfer prices,

$$aw_i(ED) = \frac{(2 + d(n-2))ac}{(4 + d(n-3))},\ i = 1, 2, ..., n$$

The other relevant equilibrium expressions follow:

$$\begin{aligned} w_ic(ED) &= \frac{(2-d)ac}{(4 + d(n-3))},\ \forall i \\ Q_i(ED) &= \frac{(2 + d(n-2))ac}{(2 + d(n-1))(4 + d(n-3))},\ \forall i \\ \Pi_{M_i}(ED) &= \frac{(2-d)(2 + d(n-2))ac^2}{(2 + d(n-1))(4 + d(n-3))^2} - k,\ \forall i \\ \Pi_{R_j}(ED) &= \frac{(2 + d(n-2))^2 ac^2}{(2 + d(n-1))^2(4 + d(n-3))^2} - k,\ \forall j. \end{aligned}$$

(c) Consider *homogeneous oligopoly network*, H_r, where only one manufacturer is linked with r retailers, $r \leq n$. Only one product is sold, say product 1, with the following inverse demand function, $p_1 = a - Q_1$, with $Q_1 = \sum_{j=1}^{r} q_{1j}$. Retailers profits are $\Pi_{R_j}(\cdot) = (p_1 - w_1)q_{1j} - k$, $j = 1, 2, ...r$. Each maximizes profits choosing its level of output. Noting that at equilibrium $p_1 - w_1 = q_{1j}$ for all j and manipulating the r FOC's we obtain the derived demand for manufacturer 1: $Q_1(w_1) = \frac{raw_1}{r+1}$, which is increasing in aw_1 and in r. Next, M_1 maximizes profits choosing w_1. It happens that the equilibrium transfer price is defined by $aw_1(H_r) = \frac{ac}{2}$ $\forall r \leq n$, then the equilibrium expressions are, for all $r \leq n$,

$$
\begin{aligned}
w_1c(H_r) &= \frac{ac}{2} \\
Q_1(H_r) &= \frac{rac}{2(r+1)}, \\
\Pi_{M_1}(H_r) &= \frac{r(ac)^2}{4(r+1)} - rk, \\
\Pi_{R_j}(H_r) &= \frac{ac^2}{4(r+1)^2} - k, \ \forall j.
\end{aligned}
$$

(d) We analyze now the *multiproduct monopolist retailer network*, MR_m, where only one retailer, say R_1, is linked with m manufacturers, $m \leq n$. In this case since there is only one channel to distribute manufacturers brands, then $Q_i = q_{i1}$ for all i. Retailer 1 profits are $\Pi_{R_1}(\cdot) = \sum_{i=1}^{m}(p_i - w_i)q_{i1} - mk$. This retailer chooses m outputs to maximize profits. We can easily compute the m FOCs as in the case of network C. Next, we add the m FOCs to obtain the sum of retailer 1's equilibrium outputs. Finally, take one of the FOC and plug it into the latter expression. The result is the derived demand for each manufacturer i and R_1's margin in brand i:

$$
\begin{aligned}
Q_i(w_1, w_2, ..., w_n) &= \frac{(1 + d(m-2))aw_i - d\sum_{k \neq i} aw_k}{2(1-d)(1 + d(m-1))}, \\
p_i - w_i &= \frac{aw_i}{2}, \forall i.
\end{aligned}
$$

Once we have the derived demands for manufacturers, we are ready to compute the FOC for each manufacturer, add the m FOCs and apply symmetry to obtain:

$$
awi(MR_m) = \frac{(1 + d(m-2))ac}{(2 + d(m-3))}, \ i = 1, 2, ..., m.
$$

The remaining relevant expressions at equilibrium follow:

$$\begin{aligned} w_i c(MR_m) &= \frac{(1-d)ac}{(2+d(m-3))}, \ \forall i \\ Q_i(MR_m) &= \frac{(1+d(m-2))ac}{2(1+d(m-1))(2+d(m-3))}, \ \forall i \\ \Pi_{M_i}(MR_m) &= \frac{(1-d)(1+d(m-2))ac^2}{2(1+d(m-1))(2+d(m-3))^2} - k, \ \forall i \\ \Pi_{R_1}(MR_m) &= \frac{m(1+d(m-2))^2ac^2}{4(1+d(m-1))(2+d(m-3))^2} - nk. \end{aligned}$$

(e) To analyze network C stability, we need to provide the analytical solutions for the network consisting of the complete network with one link less, for instance, the link between manufacturer 1 and retailer 1. Denote this network by $C_{-(1,1)}$. We need to obtain the incremental payoffs for manufacturer 1's and for retailer 1's when the $C_{-(1,1)}$ is formed starting from C. If the increment in payoffs for both retailer 1 and manufacturer 1 is negative then C is pairwise stable. The following expressions have been obtained by a similar reasoning as in the networks above but taking now into account the corresponding asymmetries. Derived demand for manufacturer one is now.[1]

$$Q_1(w_1, w_2, ..., w_n) = \frac{(n-1)[(1+d(n-2))aw_i - d\sum_{k\neq i} aw_k]}{n(1-d)(1+d(n-1))},$$

and the equilibrium transfer price for product 1 is:

$$\begin{aligned} w_1 c(C_{-(1,1)}) &= \frac{ac(1-d)W_1}{(1+d(n-2))W_2} \\ W_1 &= 2n^2 + n^2(4n-7)d + (2n^4 - 7n^3 + 6n^2 - 1)d^2 \\ W_2 &= 4n^2 + 6n^2(n-2)d + (2n^3 - 7n^2 + 2n + 1)d^2 \end{aligned}$$

The equilibrium transfer prices for the other symmetric manufacturers and retailers margins for products $i = 2, 3, ..., n$ are:

$$\begin{aligned} w_i c(C_{-(1,1)}) &= \frac{ac(1-d)(2n^2 + (n-1)^2(2n+1)d)}{W_2}, \\ p_i - w_i(C_{-(1,1)}) &= \frac{ac(2n^2 + (n^2(4n-7) - 1)d + n(n-1)(n(2n-5)+1)d^2)}{(n+1)W_2}. \end{aligned}$$

Finally, the payoffs for the relevant manufacturer and retailer are:

$$\begin{aligned} \Pi_{M_1}(C_{-(1,1)}) &= \frac{(n-1)(1+d(n-2))(w_1 c(C_{-\{11\}}))^2}{n(1-d)(1+d(n-1))} - (n-1)k \\ \Pi_{R_1}(C_{-(1,1)}) &= \frac{(n-1)(p_j - w_j(C_{-\{11\}}))^2}{(1+d(n-2))} - (n-1)k \end{aligned}$$

[1] The derived demands for the other manufacturers are not presented here to save space, but can be obtained from the authors upon request.

Network C is pairwise stable if and only if $k \leq \min\{\pi_{M_1}(C) - \pi_{M_1}(C_{-(1,1)}), \pi_{R_1}(C) - \pi_{R_1}(C_{-(1,1)})\}$.

(f) To analyze ED stability, we need to provide the analytical solutions for the network consisting of the ED network plus one more link, i.e. the link between manufacturer 1 and retailer 2. Denote this network by $ED_{+(1,2)}$. In this way we can compute the incremental payoffs for both retailer 2 and manufacturer 1 and if both are positive then ED is not pairwise stable. Now it follows that $Q_i = q_{ii}$, for all $i = 2,...,n$. and $Q_1 = q_{11} + q_{12}$. So, only one output is chosen by each retailer different from retailer two while this one chooses outputs q_{12} and q_{22}. In fact, we have that, at equilibrium Q_1 and Q_2 are different each other and different from the symmetric $Q_i,\ \ i = 3, 4, ..., n$. Similarly, for aw_1, aw_2 and aw_i $i = 3, 4, ..., n$. By the use of the retailers FOC's we obtain the following derived demands for manufacturers 1 and 2:

$$
\begin{aligned}
Q_1(\cdot) &= \frac{A_1 aw_1 - dRaw_2 - d(1-d)(4+d)\sum_{k\neq 1,2} aw_k}{(2-d)(1-d)(6+3nd+(n-2)d^2)} \\
Q_2(\cdot) &= \frac{A_2 aw_2 - dRaw_1 - 3d(1-d)\sum_{k\neq 1,2} aw_k]}{(2-d)(1-d)(6+3nd+(n-2)d^2)} \\
A_1 &= 8 + 4(n-3)d - 2(n-1)d^2 + (n-3)d^3 \\
A_2 &= 6 + 3(n-3)d - 2(n-2)d^2 \\
R &= 6 + (n-5)d
\end{aligned}
$$

Next, using the retailer 2's FOCs we can write down her equilibrium payoffs as $\Pi_{R_2}(ED_{+(1,2)}) = q_{12}^2 + Q_2^2 + 2dq_{12}Q_2 - 2k$. Then, in order to compute payoffs we need the expression for q_{12} which is

$$
q_{12} = \frac{(2+(n-2)d+d^2)aw_1 - d(3+d(n-2))aw_2 - d(1-d)\sum_{k\neq 1,2} aw_k}{(1-d)(6+3nd+(n-2)d^2)}
$$

Once Q_1,Q_2 and Q_j are found we can compute the FOC for manufacturers and obtain the relevant equilibrium transfer prices:

$$
\begin{aligned}
aw_1 &= \frac{ac\,(A_2(2A_1+dR)(2A_i-dG) - dF[9A_1 - A_2(4+d)(2A_i+3d) - 3R])}{(4A_1A_2 - d^2R^2)(2A_i-dG) - dF[18A_1 + d(4+d)(2A_2(4+d)+3R)]}, \\
aw_2 &= \frac{ac\,(A_1(2A_2+dR)(2A_i-dG) + F[6A_1A_i + d(4+d)(3A_1 - d(4+d)A_2 + dRA_i)])}{(4A_1A_2 - d^2R^2)(2A_i-dG) - dF[18A_1 + d(4+d)(2A_2(4+d)+3R)]}, \\
aw_i &= \frac{ac\left(A_i(4A_1A_2 - d^2R^2) + A_2 d[A_1(2(7+d)+3dR) + dR(4+d)]\right)}{(4A_1A_2 - d^2R^2)(2A_i-dG) - dF[18A_1 + d(4+d)(2A_2(4+d)+3R)]},\ \ i = 3, 4, ...n. \\
A_i &= 6 + 3(n-1)d + (n-3)d^2 \\
G &= (3+d)(n-3) \\
F &= (1-d)(n-2)
\end{aligned}
$$

The expressions for $\Pi_{M_1}(ED_{+(1,2)}) = (ac - aw_1)Q_1 - 2k$ and $\Pi_{R_2}(ED_{+(1,2)}) = q_{12}^2 + Q_2^2 + 2dq_{12}Q_2 - 2k$ are too "awful" to be quoted here, although Mathematica software is able to give an analytical expression for each of them. Note that ED pairwise stability requires that $k > \min\{\pi_{M_1}(ED_{+(1,2)}) - \pi_{M_1}(ED), \pi_{R_2}(ED_{+(1,2)}) - \pi_{R_2}(ED)\}$.

1.2 The expressions in Figure 2

$$\begin{aligned}\Delta_M^{ED}(d) &= \pi_{M_1}(X) - \pi_{M_1}(ED) \\ &= \frac{(2-d)(1024 - 1088d^2 - 1120d^3 - 728d^4 - 52d^5 + 45d^6 - 25d^7)(a-c)^2}{6(1+d)(2+d)(4-d)^2(16-7d^2)^2}.\end{aligned}$$

$$\begin{aligned}\Delta_R^{ED}(d) &= \pi_{R_1}(X) - \pi_{R_1}(ED) \\ &= \frac{(1024 + 1152d + 832d^2 + 176d^3 - 8d^4 + 68d^5 - 3d^6 - d^7)(a-c)^2}{36(1+d)(16-7d^2)(2+d)^2(4-d)^2}.\end{aligned}$$

$$\begin{aligned}\Delta_M^{C}(d) &= \pi_{M_2}(C) - \pi_{M_2}(X) \\ &= \frac{(1-d)(256 - 320d^2 - 96d^3 + 88d^4 + 36d^5 - 3d^6)(a-c)^2}{6(1+d)(2+d)^2(16-7d^2)^2}.\end{aligned}$$

$$\begin{aligned}\Delta_R^{C}(d) &= \pi_{R_2}(C) - \pi_{R_2}(X) \\ &= \frac{(1-d)(256 + 64d - 100d^2 + 28d^3 - 7d^4 - 20d^5 + 4d^6)(a-c)^2}{9(1+d)(2+d)^2(16-7d^2)}.\end{aligned}$$

2 Appendix B : The two-part tariff case

2.1 The equilibrium outcomes

- **Network** H_1:

$$Q_1^t(H_1) = \frac{a-c}{2},\ p_1^t(H_1) = Q_1^t(H_1) + w_1^t(H_1) = \frac{a+c}{2},\ w_1^t(H_1) - c = 0,$$

$$F_1^t(H_1) = \frac{(a-c)^2}{4} - k,\ \Pi_{M_1}^t(H_1) = \frac{(a-c)^2}{4} - 2k,\ \Pi_{M_2}^t(H_1) = 0,$$

$$\Pi_{R_1}^t(H_1) = \Pi_{R_2}^t(H_1) = 0,\ CS^t(H_1) = \frac{(a-c)^2}{8},\ W^t(H_1) = \frac{3(a-c)^2}{8} - 2k.$$

- **Network** H :

$$Q_1^t(H) = \frac{a-c}{2},\ p_1^t(H) = \frac{a+c}{2},\ w_1^t(H) - c = \frac{a-c}{4},\ F_{1j}^t(H) = \frac{(a-c)^2}{16} - k,\ \ j = 1,2,$$

$$\Pi_{M_1}^t(H) = \frac{(a-c)^2}{4} - 4k,\ \Pi_{M_2}^{tp}(H) = 0,\ \Pi_{R_1}^t(H) = \Pi_{R_2}^t(H) = 0,$$

$$CS^t(H) = \frac{(a-c)^2}{8}, W^t(H) = \frac{3(a-c)^2}{8} - 4k.$$

- **Network** ED :

$$Q_i^t(ED) = \frac{2(a-c)}{4+2d-d^2},\ p_i^t(ED) = \frac{(2-d^2)a + 2(1+d)c}{4+2d-d^2},\ \ i = 1,2,$$

$$w_i^t(ED) - c = \frac{-d^2(a-c)}{4+2d-d^2},\ F_i^t(ED) = \frac{4(a-c)^2}{(4+2d-d^2)^2} - k,\ \ i = 1,2,$$

$$\Pi_{M_i}^t(ED) = \frac{2(2-d^2)(a-c)^2}{(4+2d-d^2)^2} - 2k,\ \ i = 1,2,\ \Pi_{R_1}^t(ED) = \Pi_{R_2}^t(ED) = 0,$$

$$CS^t(ED) = \frac{4(a-c)^2}{(4+2d-d^2)^2}, W^t(ED) = \frac{4(3-d^2)(a-c)^2}{(4+2d-d^2)^2} - 4k.$$

- **Network** MR :

$$Q_1^t(MR) = Q_2^t(MR) = \frac{a-c}{2(1+d)},\ p_1^t(MR) = p_2^t(MR) = \frac{a+c}{2},$$

$$w_i^t(MR) - c = 0,\ F_i^t(MR) = \frac{(1-d)(a-c)^2}{4(1+d)},\ \ i = 1,2,$$

$$\Pi_{M_1}^t(MR) = \Pi_{M_2}^t(MR) = \frac{(1-d)(a-c)^2}{4(1+d)} - k,\ \Pi_{R_1}^t(MR) = \frac{d(a-c)^2}{2(1+d)} - 2k,$$

$$\Pi_{R_2}^t(MR) = 0, CS^t(MR) = \frac{(a-c)^2}{4(1+d)^2}, W^t(MR) = \frac{(3+2d)(a-c)^2}{4(1+d)^2} - 4k.$$

- **Network** X**, where** M_1 **and** R_1 **have two links while** M_2 **and** R_2 **only one** :

$$
\begin{aligned}
q_{11}^t(X) &= \frac{(2-d)(2+d)(32-20d-20d^2+12d^3+6d^4-d^5)(a-c)}{2(1+d)(256-204d^2+36d^4-7d^6)} \\
q_{12}^t(X) &= \frac{(64+24d-48d^2-24d^3+12d^4-d^6)(a-c)}{(256-204d^2+36d^4-7d^6)} \\
p_1^t(X) &= \frac{a(16-2d-10d^2+5d^3)(8-2d-2d^2-d^3)}{256-204d^2+36d^4-7d^6} \\
&\quad + \frac{2c(64+24d-48d^2-24d^3+12d^4-d^6)}{256-204d^2+36d^4-7d^6} \\
Q_2^t(X) &= q_{12}^t(X) = \frac{(256+64d-212d^2-68d^3+40d^4+4d^5-3d^6)(a-c)}{2(1+d)(256-204d^2+36d^4-7d^6)} \\
p_2^t(X) &= \frac{a(256-64d-228d^2+48d^3+60d^4-12d^5-7d^6+d^7)}{2(256-204d^2+36d^4-7d^6)} \\
&\quad + \frac{c(256+64d-180d^2-48d^3+12d^4+12d^5-7d^6-d^7)}{2(256-204d^2+36d^4-7d^6)}
\end{aligned}
$$

$$
\begin{aligned}
w_1^t(X)-c &= \frac{4(2-d)(1-d)^2(1+d)(8+3d+d^2)(a-c)}{(256-204d^2+36d^4-7d^6)} \\
w_2^t(X)-c &= 0 \\
F_{11}^t(X) &= \frac{(1-d)(32-20d-20d^2+12d^3+6d^4-d^5)}{4(1+d)(256-204d^2+36d^4-7d^6)^2} \cdot \\
&\quad (512+192d-224d^2-212d^3-20d^4+52d^5+2d^6-5d^7)(a-c)^2 \\
F_{12}^t(X) &= \frac{(64+24d-48d^2-24d^3+12d^4-d^6)^2(a-c)^2}{4(1+d)(256-204d^2+36d^4-7d^6)^2} - k \\
F_2^t(X) &= \frac{(1-d)(256+64d-212d^2-68d^3+40d^4+4d^5-3d^6)^2(a-c)^2}{4(1+d)(256-204d^2+36d^4-7d^6)^2}
\end{aligned}
$$

$$
\begin{aligned}
\Pi_{M_1}^t(X) &= \frac{(256-64d-156d^2+12d^3+32d^4+8d^5-15d^6-d^7)(a-c)^2}{2(1+d)(256-204d^2+36d^4-7d^6)} - 3k \\
\Pi_{M_2}^t(X) &= \frac{(1-d)(256+64d-212d^2-68d^3+40d^4+4d^5-3d^6)^2(a-c)^2}{4(1+d)(256-204d^2+36d^4-7d^6)^2} - k \\
\Pi_{R_1}^t(X) &= \frac{d\rho_1(d)(a-c)^2}{4(1+d)(256-204d^2+36d^4-7d^6)^2} - 2k \\
\Pi_{R_2}^t(X) &= 0 \\
CS^t(X) &= \frac{\rho_3(d)(a-c)^2}{8(1+d)^2(256-204d^2+36d^4-7d^6)^2} \\
W^t(X) &= \frac{\rho_4(d)(a-c)^2}{8(1+d)^2(256-204d^2+36d^4-7d^6)^2} - 4k
\end{aligned}
$$

where

$$\rho_1(d) = \begin{cases} 49152 + 7168d - 65024d^2 - 24208d^3 + 31472d^4 + 17616d^5 \\ -7440d^6 - 4840d^7 + 1472d^8 + 652d^9 - 164d^{10} - 33d^{11} + 9d^{12} \end{cases}$$

$$\rho_2(d) = \begin{cases} 131072 - 222208d^2 - 6400d^3 + 143696d^4 + 8048d^5 - 49520d^6 - 2608d^7 \\ +10312d^8 + 280d^9 - 1064d^{10} - 56d^{11} + 105d^{12} + 7d^{13} \end{cases}$$

$$\rho_3(d) = \begin{cases} 131072 + 81920d - 177152d^2 - 1315884d^3 + 77968d^4 + 68256d^5 - 16544d^6 \\ -15168d^7 + 3880d^8 + 2584d^9 - 328d^{10} - 192d^{11} + 57d^{12} + 16d^{13} + d^{14} \end{cases}$$

$$\rho_4(d) = \begin{cases} 393216 + 344064d - 621568d^2 - 588800d^3 + 352560d^4 + 371744d^5 - 99488d^6 \\ -119424d^7 + 19288d^8 + 23768d^9 - 1896d^{10} - 2432d^{11} + 155d^{12} + 240d^{13} + 15d^{14} \end{cases}$$

- **Network** C :

$$Q_i^t(C) = \frac{2(a-c)}{(4-d)(1+d)},\ p_i^t(C) = \frac{(4-d)a+2c}{4-d},\ i=1,2,\ w_i^t(C) - c = \frac{(1-d)(a-c)}{(4-d)},$$

$$F_i^t(C) = \frac{(1-d)(a-c)^2}{(1+d)(4-d)^2},\ i=1,2,\ \Pi_{M_i}^t(C) = \frac{4(1-d)(a-c)^2}{(1+d)(4-d)^2} - 2k,\ i=1,2,$$

$$\Pi_{R_j}^t(C) = \frac{2d(a-c)^2}{(1+d)(4-d)^2} - 2k,\ j=1,2,$$

$$CS^t(C) = \frac{4(a-c)^2}{(1+d)^2(4-d)^2}, W^t(C) = \frac{4(3+d-d^2)(a-c)^2}{(1+d)^2(4-d)^2} - 8k.$$

2.2 The expressions in Figure 3

$$\Delta t_M^{ED}(d) = \pi_{M_1}^t(X) - \pi_{M_1}^t(ED) =$$
$$\frac{d(1024 - 768d - 2240d^2 + 368d^3 + 1056d^4 + 44d^5 + 4d^6 + 12d^7 - 16d^8 + 11d^9 + d^{10})(a-c)^2}{4(1+d)(256 - 204d^2 + 36d^4 - 7d^6)(4 - 2d + d^2)^2}$$

$$\begin{aligned} \Delta t_M^C(d) &= \pi_{M_2}^t(C) - \pi_{M_2}^t(X) \\ &= \frac{3(1-d)d^2(32 + 20d - 28d^2 + 8d^3 - 4d^4 - d^5)}{4(1+d)(256 - 204d^2 + 36d^4 - 7d^6)^2(4-d)^2} \cdot \\ &\quad (2048 - 1728d^2 - 60d^3 + 372d^4 - 24d^5 - 44d^6 + 3d^7)(a-c)^2 \end{aligned}$$

3 Appendix C : No bargaining power upstream

3.1 The profit expressions

$$\Pi^r_{R_j}(MR_1) = \frac{(a-c)^2}{4} - 2k,\ \Pi^r_{R_j}(MR) = \frac{(a-c)^2}{2(1+d)} - 4k,\ \Pi^r_{R_j}(H) = \frac{(a-c)^2}{9} - 2k, j = 1, 2,$$

$$\Pi^r_{R_j}(ED) = \frac{(a-c)^2}{(2+d)^2} - 2k, j = 1, 2,\ \Pi^r_{R_2}(X) = \frac{(a-c)^2}{9} - 2k,\ R_2 \text{ with one link in } X,$$

$$\Pi^r_{R_j}(C) = \frac{2(a-c)^2}{9(1+d)} - 4k, j = 1, 2, \Pi^r_{R_1}(X) = \frac{(13-5d)(a-c)^2}{36(1+d)} - 4k, R_1 \text{ with two links in } X.$$

4 Appendix D : Differentiated retailers and price competition

4.1 The equilibrium outcomes

- **Network** H_1, where M_1 and R_1 the only linked agents:

$$\Pi^p_{R_1}(H_1) = (q_{11}(H_1))^2,\ \Pi^p_{M_1}(H_1) = 2(q_{11}(H_1))^2,\ p_{11}(H_1) = q_{11}(H_1) + w_1(H_1),$$

$$q_{11}(H_1) = \frac{(a-c)}{4},\ w_1(H_1) - c = \frac{(a-c)}{2}.$$

- **Network** H, where M_1 the only linked manufacturer:

$$\Pi^p_{R_j}(H) = (1-s^2)(q_{11}(H))^2,\ \ j = 1, 2,\ \Pi^p_{M_1}(H) = 2(2-s)(1+s)(q_{11}(H))^2,$$

$$p_{1j}(H) = (1-s^2)q_{1j}(H) + w_1(H),\ \ j = 1, 2,\ q_{1j}(H) = \frac{(a-c)}{2(2-s)(1+s)},$$

$$w_1(H) - c = \frac{(a-c)}{2}.$$

- **Network** MR, where R_1 the only linked retailer:

$$\Pi^p_{R_1}(MR) = 2(1+d)(q_{11}(MR))^2,\ \Pi^p_{M_i}(MR) = 2(1-d^2)(q_{11}(MR))^2\ i = 1,2,$$

$$p_{i1}(MR) = (1+d)q_{i1}(MR) + w_i(MR),\ i = 1,2,\ q_{i1}(MR) = \frac{(a-c)}{2(2-d)(1+d)},$$

$$w_i(MR) - c = \frac{(1-d)(a-c)}{2-d},\ i = 1,2.$$

- **Network** ED:

$$\Pi^p_{R_1}(ED) = \Pi^p_{R_2}(ED) = (1-s^2d^2)(q_{11}(ED))^2,$$

$$\Pi^p_{M_1}(ED) = \Pi^p_{M_2}(ED) = \frac{(4-s^2d^2)(1-s^2d^2)}{2-s^2d^2}(q_{11}(ED))^2,$$

$$p_{ii}(ED) = (1-s^2d^2)q_{ii}(ED) + w_i(ED),\ \ i = 1,2,$$

$$q_{ii}(ED) = \frac{(2-s^2d^2)(a-c)}{(1+sd)(2-sd)(4-sd-2s^2d^2)},\ w_i(ED) - c = \frac{(1-sd)(2+sd)(a-c)}{(4-sd-2s^2d^2)},\ i = 1,2.$$

- **Network** C:

$$\Pi^p_{R_j}(C) = 2(1+d)(1-s^2)(q_{11}(C))^2,\ j = 1,2,$$

$$\Pi^p_{M_i}(C) = 2(2-s)(1+s)(1-d^2)(q_{11}(C))^2,\ i = 1,2,$$

$$p_{ij}(C) - w_i(C) = (1+d)(1-s^2)q_{ij}(C),\ i,j = 1,2,$$

$$q_{ij}(C) = \frac{(a-c)}{(2-d)(1+d)(2-s)(1+s)},\ w_i(C) - c = \frac{(1-d)(a-c)}{2-d},\ i,j = 1,2.$$

- **Network** X, where M_1 and R_1 have two links while M_2 and R_2 only one:

$$\Pi^p_{R_1}(X) = (1-s^2)(q_{12}(X))^2 + (1-d^2)(q_{21}(X))^2,\ \Pi^p_{R_2}(X) = (1-s^2)(q_{12}(X))^2,$$

$$\Pi^p_{M_1}(X) = \frac{(2(2-s)(1+s)(1-d^2))}{(4-(2-s)(1+s)d^2)}(q_{11}(X)+q_{12}(X))^2,\ \Pi^p_{M_2}(X) = 2(1-d)(q_{21}(X))^2,$$

$$p_{11}(X) - w_1(X) = p_{12}(X) - w_1(X) = (1-s^2)q_{12}(X),$$

$$p_{21}(X) - w_2(X) = d(1-s^2)q_{12}(X) + (1-d^2)q_{21}(X),$$

$$q_{11}(X) = \frac{(16+2(2-3s+3s^2)d - 2(6-s+s^2)d^2 - (2-s+s^2)^2d^3)(a-c)}{2(2-s)(1+s)(1+d)(16-(10-3s+3s^2)d^2)},$$

$$q_{12}(X) = \frac{(8+(2-s)(1+s)d - 2(2-s+s^2)d^2)(a-c)}{(2-s)(1+s)(16-(10-3s+3s^2)d^2)},$$

$$q_{21}(X) = \frac{(8+4d-(2-s+s^2)d^2)(a-c)}{2(1+d)(16-(10-3s+3s^2)d^2)},$$

$$w_1(X) - c = \frac{(1-d)(8+(6-s+s^2)d)(a-c)}{(16-(10-3s+3s^2)d^2)},$$

$$w_2(X) - c = \frac{(1-d)(8+4d-(2-s+s^2)d^2)(a-c)}{(16-(10-3s+3s^2)d^2)}.$$

5 Appendix E : Social welfare

5.1 The benchmark case and the expressions in Figure 5

Thresholds on k that are used to find the social welfare maximizer network:

(i) $W(X) - W(ED) > 0$ if $k < \Omega_{X-ED} =$

$$\frac{(69632+26624d+25344d^2+130048d^3+18460d^7+2967d^8+68d^9+335d^{10})(a-c)^2}{144(4-d)^2(2+d)^2(1+d)^2(16-7d^2)^2}$$
$$-\frac{\left(29984d^4+141504d^5+30548d^6\right)(a-c)^2}{144(4-d)^2(2+d)^2(1+d)^2(16-7d^2)^2}.$$

(ii) $W(C) - W(ED) > 0$ if $k < \Omega_{C-ED} =$

$$(34 + 3d - 30d^2 - d^3 + 3d^4)\frac{(2 - d + d^2)(a - c)^2}{9(4 - d)^2(2 - d)^2(2 + d)^2(1 + d)^2}.$$

(iii) $W(C) - W(X) > 0$ if $k < \Omega_{C-X} =$

$$\frac{(4352 - 3072d - 5632d^2 + 3040d^3 + 1096d^4 - 476d^5 + 1081d^6 - 68d^7 - 335d^8)(a - c)^2}{144(2 - d)^2(1 + d)^2(16 - 7d^2)^2}.$$

(iv) $W(X) - W(MR) > 0$ if $k < \Omega_{X-MR}$

$$\frac{(4352 - 2304d - 5824d^2 + 3680d^3 + 2270d^4 - 1990d^5 - 267d^6 + 335d^7)(a - c)^2}{144(2 - d)^2(1 + d)(16 - 7d^2)^2}.$$

(v) $W(C) - W(MR) > 0$ if $k < \Omega_{C-MR} =$

$$\frac{(17 - 2d + 12d^2)(a - c)^2}{144(2 - d)^2(1 + d)^2}.$$

5.2 The two-part tariff case and the expressions in Figure 6

Thresholds on k that are used to find the social welfare maximizer network:

(i) $W^t(ED) - W^t(H_1) > 0$ if $k < \Omega t_{ED-H_1} =$

$$\frac{48 - 48d - 20d^2 + 12d^3 - 3d^4}{16(4 + 2d - d^2)^2}.$$

(ii) $W^t(X) - W^t(H_1) > 0$ if $k < \Omega t_{X-H_1} =$

$$\frac{\lambda(d)(a-c)^2}{16(1+d)^2(256-204d^2+36d^4-7d^6)^2},$$

where

$$\lambda(d) = 98304 - 24576d - 252416d^2 + 18944d^3 + 242880d^4 + 5728d^5 - 112408d^6 - 4896d^7 + 30824d^8 - 572d^9 - 6420d^{10} + 296d^{11} + 760d^{12} - 27d^{13} - 66d^{14}.$$

(iii) $W^t(X) - W^t(ED) > 0$ if $k < \Omega t_{X-ED} =$

$$\frac{d\eta(d)(a-c)^2}{16(1+d)^2(4+2d-d^2)^2(256-204d^2+36d^4-7d^6)^2},$$

where

$$\eta(d) = -786432 - 180224d + 1033312d^2 + 740608d^3 - 1439488d^4 - 751808d^5 + 362688d^6 + 353712d^7 - 30304d^8 - 151040d^9 + 1600d^{10} + 55784d^{11} + 5736d^{12} - 7972d^{13} - 636d^{14} + 703d^{15} + 180d^{16} + 15d^{17}.$$

6 Appendix F : The proofs

6.1 Proof of Result 1

Consider the $H_{(n-1)}$ distribution network where M_1 is linked will all the retailers except one. To add a new link between M_1 and the remaining retailer both firms must agree. It is apparent (follows from the definition of $\bar{k}_n$) that the retailer not already linked will always agree since there is no competition effect and the output expansion effect outweights the cost effect ($\pi_{R_j}(H) \geq \bar{k}_n$). Thus, any isolated firm is always willing to form a link since the output expansion effect always offsets the cost effect. However, for M_1, we need to compute and compare the three effects. The output expansion effect is the amount of profits that comes from the new distribution channel, which is

$$\frac{(a-c)^2}{4(n+1)}.$$

The competition effect is computed as the variation (reduction) in profits that accrues from the $(n-1)$ distribution channels by the effect of the new link, which is

$$\frac{(n-1)(a-c)^2}{4(n+1)} - \frac{(n-1)(a-c)^2}{4n} = -\frac{(n-1)(a-c)^2}{4n(n+1)}.$$

Thus both the output and the competition effects are decreasing in n. The difference between effects is

$$\frac{(a-c)^2}{4n(n+1)}.$$

Finally, the link is created if the latter difference outweights the cost effect, i.e.

$$\frac{(a-c)^2}{4n(n+1)} > k;$$

which is satisfied since

$$\frac{(a-c)^2}{4n(n+1)} > \frac{(a-c)^2}{4(n+1)^2} = \pi_{R_j}(H) \geq \bar{k}_n.$$

6.2 Proof of Result 3

Take network MR_{n-1}, and take the multiproduct monopolist retailer, say R_1. The output expansion effect of forming the last link is

$$\frac{(1+d(n-2))^2(a-c)^2}{4(1+d(n-1))(2+d(n-3))^2}.$$

The competition effect is

$$\frac{(n-1)(a-c)^2}{4}[\frac{(1+d(n-2))^2}{(1+d(n-1))(2+d(n-3))^2} - \frac{(1+d(n-3))^2}{(1+d(n-2))(2+d(n-4))^2}].$$

Adding both effects we get

$$\frac{(a-c)^2}{4}[\frac{n(1+d(n-2))^2}{(1+d(n-1))(2+d(n-3))^2} - \frac{(n-1)(1+d(n-3))^2}{(1+d(n-2))(2+d(n-4))^2}],$$

which must be greater than k the cost effect of the new link in order to be profitable for R_1. Take

$$\frac{\pi_{M_i}(C)}{n} = \frac{(1-d)(1+d(n-2))(a-c)^2}{(n+1)^2(1+d(n-1))(2+d(n-3))^2}.$$

Since we know that $\Pi_{M_i}(C) \geq 0$, then $\frac{\pi_{M_i}(C)}{n} \geq \bar{k}_n$ and consequently, if

$$\frac{(a-c)^2}{4}[\frac{n(1+d(n-2))^2}{(1+d(n-1))(2+d(n-3))^2} - \frac{(n-1)(1+d(n-3))^2}{(1+d(n-2))(2+d(n-4))^2}]$$

is greater than $\frac{\pi_{M_i}(C)}{n}$, then the new link is profitable for R_1. The expression

$$\frac{(a-c)^2}{4}[\frac{n(1+d(n-2))^2}{(1+d(n-1))(2+d(n-3))^2} - \frac{(n-1)(1+d(n-3))^2}{(1+d(n-2))(2+d(n-4))^2}] - \frac{(1-d)(1+d(n-2))(a-c)^2}{(n+1)^2(1+d(n-1))(2+d(n-3))^2}$$

is positive iff $n(n+1)^2(A+d)^3(A+1-d)^2 - (n-1)(n+1)^2(A)^2(A+2d)(A+1)^2 - (1-d)(A+d)^2(A+1-d)^2 > 0$, where $A = 1+d(n-3)$, which can be written as

$(n(n+1)^2A-4+d(n(n+1)^2+4)(A+d)^2(A+1-d)^2-(n-1)(n+1)^2(A)^2(A+2d)(A+1)^2 > 0$, and is positive since $(A+d)^2(A+1-d)^2 > (A)^2(A+1)^2$ for all $d \in (0,1)$ and $n > 0$; and $(n(n+1)^2A-4+d(n(n+1)^2+4) > (n-1)(n+1)^2(A+2d)$ for all $n > 1$. This completes the proof of Result 3.

Regarding the competition effect, it is positive iff

$$[\frac{(1+d(n-2))^2}{(1+d(n-1))(2+d(n-3))^2} - \frac{(1+d(n-3))^2}{(1+d(n-2))(2+d(n-4))^2}] > 0.$$

In terms of expression $A = 1+d(n-3)$, iff $\Phi(A,d) = -A^4 - 2dA^3 + (1-2d^2)A^2 + d(3-d)(1-d)A + d^2(1-d)^2 > 0$. Note that A is increasing in n, and $\Phi(A,d)$ is decreasing in A, for $n > 2$. Take $n = 2$, then $A = 1-d$ and Φ simplifies to $(3-d)(1-d)^2d^2$, positive for all d. Take now $n = 3$, then $A = 1$ and Φ is $(1-5d+d^2(1-d))d^2$ which is positive for $d \in (0, 0.193937)$ and negative otherwise. Finally, if $n = 4$, $A = 1+d$ and Φ is $-(1+13d+19d^2+3d^3)d^2$, which is obviously negative. Thus for $n > 4$, the competition effect is negative too since $\Phi(A,d)$ is decreasing in A for $n > 2$.

6.3 Proof that the upper bound $\overline{k}_2 = \min\{\pi_{R_j}(H), \pi_{M_i}(C)/2\}$

Consider networks H_1 and H. From direct inspection of the agents' profits, the most binding constraint for k is $k < \pi_{R_1}(H) = \frac{(a-c)^2}{36}$. Similarly: i) for network ED the most binding constraint for k is $k < \pi_{R_1}(ED) = \frac{4(a-c)^2}{(4-d)^2(2+d)^2}$. ii) For network MR, $k < \frac{\pi_{R_1}(MR)}{2} = \frac{(a-c)^2}{4(2-d)^2(1+d)}$ is the most binding constraint. It is now easy to show $\pi_{R_1}(H)$ is lower than $\pi_{R_1}(ED)$ and $\pi_{R_1}(MR)$.

Consider now networks X and C, it follows that $k < \pi_{R_1}(H) = \frac{(a-c)^2}{36}$ is a more binding condition for k than those imposed by $\Pi_{R_1}(X) > 0$, $\Pi_{R_2}(X) > 0$ and $\Pi_{R_1}(C) > 0$, for any $d \in (0,1)$. Similarly, the condition on k that ensures $\Pi_{M_1}(C) > 0$, that is $k < \frac{\pi_{M_1}(C)}{2} = \frac{(1-d)(a-c)^2}{3(2-d)^2(1+d)}$, is more binding that those ensuring $\Pi_{M_1}(X) > 0$ and $\Pi_{M_2}(X) > 0$. Therefore, by comparing expressions $\frac{(a-c)^2}{36}$ and $\frac{(1-d)(a-c)^2}{3(2-d)^2(1+d)}$ we have the upper bound on k as a function of d.

6.4 Proof of Proposition 1

This proposition is about stability in case of $n = 2$. Remind that Results 1, 2 and 3 apply.

i) By the use of Result 1 network H_1 is not stable.

ii) Network H is not stable because: a) Result 1 applies then the manufacturer already linked does not break any link and b) retailer 1 is always willing to form a link with the unlinked manufacturer:

$$\pi_{R_1}(X) - \pi_{R_1}(H) > k$$

and

$$\pi_{R_1}(X) - \pi_{R_1}(H) = \frac{(6+2d+d^3)(a-c)^2}{6(1+d)(16-7d^2)} > \frac{(1-d)(a-c)^2}{3(1+d)(2-d)^2} \geq \bar{k}_2 > k.$$

iii) Network MR is not stable because: a) Result 3 applies and the retailer already linked does not break any link and b) manufacturer 1 is always willing to form a link with the unlinked retailer:

$$\pi_{M_1}(X) - \pi_{M_1}(MR) > k$$

and

$$\pi_{M_1}(X) - \pi_{M_1}(MR) = \frac{(1-d)(256-208d^2+64d^3+45d^4-25d^5)(a-c)^2}{6(2-d)^2(16-7d^2)^2} \geq \bar{k}_2 > k.$$

iv) The remaining three networks, ED, X and C can be pairwise stable depending on the particular values of k and d. Since the relevant parameter region in the space (k, d) is bounded, it is enough to provide graphical proofs. The cost effect bound and the expressions Δ's (the combined effect of the output expansion and the competition effect) are all functions of d than can be plotted in the same graph and properly compared. The graph of the proof is Figure 2 in the text where the important expressions are the following and $(a-c)^2$ is set to one without loss of generality:

$$\bar{k}_2(d) = \min\{\pi_{R_1}(H), \frac{\pi_{M_1}(C)}{2}\} = \min\{\frac{(a-c)^2}{36}, \frac{(1-d)(a-c)^2}{3(1+d)(2-d)^2}\}.$$

$$\begin{aligned} \Delta_M^{ED}(d) &= \pi_{M_1}(X) - \pi_{M_1}(ED) \\ &= \frac{(2-d)(1024-1088d^2-1120d^3-728d^4-52d^5+45d^6-25d^7)(a-c)^2}{6(1+d)(2+d)(4-d)^2(16-7d^2)^2}. \end{aligned}$$

$$\begin{aligned} \Delta_R^{ED}(d) &= \pi_{R_1}(X) - \pi_{R_1}(ED) \\ &= \frac{(1024+1152d+832d^2+176d^3-8d^4+68d^5-3d^6-d^7)(a-c)^2}{36(1+d)(16-7d^2)(2+d)^2(4-d)^2}. \end{aligned}$$

$$\Delta_M^{C}(d) = \pi_{M_2}(C) - \pi_{M_2}(X) = \frac{(1-d)(256-320d^2-96d^3+88d^4+36d^5-3d^6)(a-c)^2}{6(1+d)(2+d)^2(16-7d^2)^2}.$$

$$\Delta_R^{C}(d) = \pi_{R_2}(C) - \pi_{R_2}(X) = \frac{(1-d)(256+64d-100d^2+28d^3-7d^4-20d^5+4d^6)(a-c)^2}{9(1+d)(2+d)^2(16-7d^2)}.$$

Noting that network C is pairwise stable iff $k \leq \min\{\Delta_M^C, \Delta_R^C\}$. Network ED is pairwise stable iff $k > \min\{\Delta_M^{ED}, \Delta_R^{ED}\}$. Finally, since network MR is not stable, network X is pairwise stable iff $\min\{\Delta_M^C, \Delta_R^C\} < k < \min\{\Delta_M^{ED}, \Delta_R^{ED}\}$.

6.5 Stability of networks ED and C in the benchmark case for $n > 2$

- Bounds on k :

For simplicity, we consider that the upper bound on k for the $(n \times n)$ case is the counterpart to the one we prove for the $n = 2$ case. That is,

$$k < \bar{k}_n = \min\{\pi_{R_j}(H), \frac{\pi_{M_i}(C)}{n}\}.$$

Where

$$\pi_{R_j}(H) = \frac{(a-c)^2}{4(n+1)^2}$$

and

$$\frac{\pi_{M_i}(C)}{n} = \frac{(1-d)(1+d(n-2))(a-c)^2}{(n+1)(1+d(n-1))(2+d(n-3))^2}.$$

- Stability of network C :

To check for the stability of network C we evaluate $\Delta_M^C = \pi_{M_1}(C) - \pi_{M_1}(C_{-(1,1)})$ and $\Delta_R^C = \pi_{R_1}(C) - \pi_{R_1}(C_{-(1,1)})$ by using the command Manipulate in the Mathematica 6.0 software. We plot in the same graph each of the two differences and the k upper bounds for $d \in [0,1]$ and for all the values of n from 2 to 50. Finally we plot together Δ_M^C, Δ_R^C, $\frac{\pi_{M_i}(C)}{n}$, $\pi_{R_1}(H)$. We find that both Δ_M^C and Δ_R^C are decreasing in d, being $\Delta_R^C = \pi_{R_1}(H) < \Delta_M^C$ if $d = 0$ and both $\Delta_R^C = \Delta_M^C = 0$ if $d = 1$. We further find that the $\min\{\Delta_M^C, \Delta_R^C\}$ is Δ_R^C for small d, while is Δ_M^C for large d. In any case $\min\{\Delta_M^C, \Delta_R^C\}$ is smaller than $\bar{k}_n$ for all $d < 1$. Therefore we can conclude that the network C is stable for small k for all possible d. The required level of k to obtain that C is stable is decreasing in d. Thus, the result about stability for C in the case of two agents is robust to the case of n agents.

- Stability of network ED :

To check for stability of network ED we evaluate $\Delta_M^{ED} = \pi_{M_1}(ED_{+(1,2)}) - \pi_{M_1}(ED)$ and $\Delta_R^{ED} = \pi_{R_2}(ED_{+(1,2)}) - \pi_{R_2}(ED)$ as above and plot in the same graph each of them with the k upper bounds. We find that retailer 2 is always willing to form the new link with M_1 ($\Delta_R^{ED} > \bar{k}_n$) for $n > 2$ and $d \in (0,1)$. For M_1 the new link does not pay if d and k are sufficiently large. In particular, we identify two thresholds for d: d_1 and d_2.

If $d < d_1$, network ED is never stable. If $d_1 < d < d_2$ the network is stable for k large enough and the required k for stability decreasing in d. Finally, for $d > d_2$ the network is stable for all k. Once again we conclude that the result about stability for ED in the case of two agents is robust to the case of n agents. The ED network is stable for large d and k.

6.6 Proof that the upper bound $\overline{k}_2^t = \min\{\pi^t_{R_j}(C)/2, \pi^t_{M_j}(C)/2\}$

Consider networks H_1 and H, from direct inspection of the agents' profits, the most binding constraint for k is $k < \pi^t_{M_1}(H) = \frac{(a-c)^2}{16}$. However, the latter is not the upper bound for k since $\pi^t_{M_1}(H) > \frac{\pi^t_{R_1}(C)}{2}$. Similarly for network MR, it happens that $\pi^t_{R_1}(MR) > \frac{\pi^t_{R_1}(C)}{2}$ and $\pi^t_{M_1}(MR) > \frac{\pi^t_{M_1}(C)}{2}$. Finally, for network X it is shown that $\pi^t_{M_2}(X) > \frac{\pi^t_{M_1}(C)}{2}$, $\pi^t_{R_1}(X) > \frac{\pi^t_{R_1}(C)}{2}$ and $\pi^t_{M_1}(X) > \min\{\frac{\pi^t_{R_j}(C)}{2}, \frac{\pi^t_{M_j}(C)}{2}\}$. Then we conclude that the most binding constraint for k is $0 < k < \bar{k}_2^t \equiv \min\{\frac{\pi^t_{R_j}(C)}{2}, \frac{\pi^t_{M_j}(C)}{2}\}$. Notice that $\bar{k}_2^t = \frac{\pi^t_{R_j}(C)}{2} = \frac{d(a-c)^2}{(1+d)(4-d)^2}$ if $0 < d \leq \frac{2}{3}$ and $\bar{k}_2^t = \frac{\pi^t_{M_j}(C)}{2} = \frac{2(1-d)(a-c)^2}{(1+d)(4-d)^2}$ if $\frac{2}{3} < d < 1$.

6.7 Proof of Proposition 2

Network H is not stable because M_1 has always incentives to sever a link to reach H_1 getting the same gross profits and saving $2k$. However, network H_1 is not stable either since the not linked pair manufacturer-retailer has always incentives to form a link reaching network ED.

Next, network MR is not stable because there is always a manufacturer, say M_1, that is willing to introduce intrabrand rivalry in the market:

$$\begin{aligned}
\frac{\pi^t_{M_1}(X) - \pi^t_{M_1}(MR)}{2} &= \frac{d(48 - 36d - 12d^2 + 11d^3 - 2d^4)\,(a-c)^2}{512 - 08d^2 + 72d^4 - 14d^6} \\
&\geq \frac{d\,(a-c)^2}{(1+d)\,(4-d)^2} \geq \bar{k}_2^t.
\end{aligned}$$

Therefore we are left with three networks ED, X and C which are stable for some particular values of k and d. As occurs in the benchmark case, the relevant parameter region in the space (k, d) is bounded, and then it is enough to provide a graphical proof. The expressions Δ's (the combined effect of the output expansion and the competition effect) and $\bar{k}_2^t$ are all functions of d than can be plotted in the same graph and properly compared. The graph of the proof is Figure 3 in the text where the important expressions are the following and $(a-c)^2$ is set to one without loss of generality:

$$\begin{aligned}\Delta t_M^{ED}(d) &= \pi_{M_1}^t(X) - \pi_{M_1}^t(ED)\\ &= \frac{d(a-c)^2}{4(1+d)(256-204d^2+36d^4-7d^6)(4-2d+d^2)^2}\cdot(1024-768d-2240d^2\\ &\quad +368d^3+1056d^4+44d^5+4d^6+12d^7-16d^8+11d^9+d^{10})\end{aligned}$$

$$\begin{aligned}\Delta t_M^{C}(d) &= \pi_{M_2}^t(C) - \pi_{M_2}^t(X)\\ &= \frac{3(1-d)d^2(32+20d-28d^2+8d^3-4d^4-d^5)}{4(1+d)(256-204d^2+36d^4-7d^6)^2(4-d)^2}\cdot\\ &\quad (2048-1728d^2-60d^3+372d^4-24d^5-44d^6+3d^7)(a-c)^2\end{aligned}$$

Note that since retailers are always better off being multiproduct sellers, pairwise stability is conducted by the manufacturers incentives. Thus network C is pairwise stable iff $k < \Delta t_M^C$. Network ED is pairwise stable iff $k > \Delta t_M^{ED}$. Finally, since network MR is not stable, network X is pairwise stable if $\Delta t_M^C < k < \Delta t_M^{ED}$.

6.8 Proof of Proposition 3

By Results 1 and 2 neither H_1 nor MR_1 are pairwise stable. Also MR is not pairwise stable since the isolated retailer has always incentives to form a link with one manufacturer. The stability of network C requires

$$k < \Delta r_R^C \equiv \frac{\pi_{R_j}^r(C) - \pi_{R_j}^r(X)}{2} = \frac{(1-d)(a-c)^2}{18(1+d)},$$

being R_j the retailer that deletes a link. The stability of network ED requires

$$k > \Delta r_R^{ED} \equiv \frac{\pi_{R_j}^r(X) - \pi_{R_j}^r(ED)}{2} = \frac{(16+12d+5d^2)(1-d)(a-c)^2}{72(1+d)(2+d)^2},$$

and the stability of network H requires

$$k > \Delta r_R^H \equiv \frac{\pi_{R_j}^r(X) - \pi_{R_j}^r(H)}{2} = \frac{(1-d)(a-c)^2}{8(1+d)},$$

being R_j the retailer that forms the new link. The link costs are bounded to $0 \leq k \leq \bar{k}_2^r \equiv \frac{(a-c)^2}{18(1+d)}$. We find four different regions by plotting together $\bar{k}_2^r$, Δr_R^C, Δr_R^{ED} and Δr_R^H. See Figure 4, where we find that $\Delta r_R^{ED} < \Delta r_R^C < \bar{k}_2^r < \Delta r_R^H$ if $d < \frac{5}{9}$, while $\Delta r_R^{ED} < \Delta r_R^C < \Delta r_R^H < \bar{k}_2^r$ if $d > \frac{5}{9}$. When the combinations of d and k are below Δr_R^{ED}, then network C is pairwise stable. When those combinations are between Δr_R^{ED}

and Δr_R^C, both networks C and ED are pairwise stable. When they are between Δr_R^C and $\min\{\bar{k}_2^r, \Delta r_R^H\} = \bar{k}_2^r$ only network ED is stable. Finally, when they are between Δr_R^H and $\bar{k}_2^r$ with $\Delta r_R^H < \bar{k}_2^r$, both networks H and ED are pairwise stable.

6.9 Proofs for Result 4 and Proposition 4 when retailers are differentiated and compete in prices

(a) Network H_1 is not stable against network ED since there is always a pair manufacturer-retailer that gets positive profits be forming a link.

(b) Network H is not stable against network X, since the isolated manufacturer is always willing to link with one of the retailers and the retailer agrees. We only need to prove that $\Pi^p_{R_1}(H) = (1-s^2)(q_{11}(H))^2 < \Pi^p_{R_1}(X) = (1-s^2)(q_{11}(X))^2 + (1-d^2)(q_{21}(X))^2$. However it suffices to check that $q_{11}(H) \leq q_{12}(X)$, which is true for all d and $s \in (0,1)$.

(c) Network MR is not stable against network X, since the isolated retailer is always willing to link with one of the manufacturers and one manufacturer agrees. To prove that M_1 agrees we prove the following: $\Pi^p_{M_1}(MR) = (w_1(MR) - c)(q_{11}(MR)) < \Pi^p_{M_1}(X) = (w_1(X)-c)(q_{11}(X)+q_{12}(X))$. Note first that $(w_1(MR)-c) < (w_1(X)-c)$ iff

$$\frac{(1-d)(a-c)}{2-d} < \frac{(1-d)(8+(6-s+s^2)d)(a-c)}{(16-(10-3s+3s^2)d^2)}$$

which is true for all d and $s \in (0,1)$. Next $q_{11}(MR) < q_{11}(X) + q_{12}(X)$ iff

$$\frac{(a-c)}{2(2-d)(1+d)} < \frac{(4-(2-s+s^2)d^2)(8+(6-s+s^2)d)(a-c)}{2(2-s)(1+s)(1+d)(16-(10-3s+3s^2)d^2)}.$$

The latter expression is equivalent to: $(2-s)(1+s)(16-(10-3s+3s^2)d^2) < (2-d)(4-(2-s+s^2)d^2)(8+(6-s+s^2)d)$ and now is easy to prove that it is satisfied since: i) $(4-(2-s+s^2)d^2) > (2-s)(1+s)$, and ii) $(2-d)(8+(6-s+s^2)d) > (16-(10-3s+3s^2)d^2)$, for all d and $s \in (0,1)$.

(d) Retailer R_2 in network X always wants to form a link with M_2. We prove that $\Pi^p_{R_j}(C) = 2(1+d)(1-s^2)(q_{11}(C))^2 > \Pi^p_{R_2}(X) = (1-s^2)(q_{12}(X))^2$. In doing so is enough to prove that $(1+d)q_{11}(C) > q_{12}(X)$. The latter inequality holds if the following one does: $(16-(10-3s+3s^2)d^2) > (2-d)(8+(2-s)(1+s)d-2(2-s+s^2)d^2)$; which simplifies to $2d(1-d^2)(2-s+s^2) > 0$, that holds for all d and $s \in (0,1)$.

(e) Retailer R_j in network ED always wants to form a link. We compute the difference between $\Pi^p_{R_1}(ED) = (1-s^2d^2)(q_{11}(ED))^2$ and $\Pi^p_{R_1}(X) = (1-s^2)(q_{12}(X))^2 + (1-$

$d^2)(q_{21}(X))^2$ and we check numerically that this difference is positive for all $s, d \in (0,1)$.

(f) Manufacturer M_i in network ED wants to form a link if

$$\begin{aligned}\Pi^p_{M_1}(X) &= \frac{(2(2-s)(1+s)(1-d^2))}{(4-(2-s)(1+s)d^2)}(q_{11}(X)+q_{12}(X))^2 > \\ \Pi^p_{M_1}(ED) &= \frac{(4-s^2d^2)(1-s^2d^2)}{2-s^2d^2}(q_{11}(ED))^2.\end{aligned}$$

We check numerically for all s and $d \in (0,1)$ using Mathematica and find that: (i) for $d > 0.971$ and for all s then $\Pi^p_{M_1}(ED) > \Pi^p_{M_1}(X)$; (ii) for $d < 0.800$ and for all s then $\Pi^p_{M_1}(ED) < \Pi^p_{M_1}(X)$; (iii) for $0.800 < d < 0.971$, then $\Pi^p_{M_1}(ED) > \Pi^p_{M_1}(X)$ for some s.

(g) Manufacturer M_i in network C wants to break a link if $\Pi^p_{M_2}(C) = 2(2-s)(1+s)(1-d^2)(q_{11}(C))^2 < \Pi^p_{M_2}(X) = 2(1-d)(q_{21}(X))^2$. We check numerically for all s and $d \in (0,1)$ using Mathematica and find that: (a) for $d < 0.9109$ and for all s then $\Pi^p_{M_2}(C) > \Pi^p_{M_2}(X)$; (b) for $d > 0.9121$ and for all s then $\Pi^p_{M_2}(C) < \Pi^p_{M_2}(X)$; (c) for $0.9109 < d < 0.9121$, then $\Pi^p_{M_2}(C) > \Pi^p_{M_2}(X)$ for some s.

Summarizing:

Proposition

(a) Network C is stable

(a.1) for $d < 0.9109$ and regardless of s,

(a.2) a.2) for $0.9109 < d < 0.9121$ and for some levels of s.

(b) Network ED is stable

(b.1) for $d > 0.971$ and regardless of s,

(b.2) for $0.800 < d < 0.971$ and for some levels of s.

Finally, note that network X can also be stable if network C is unstable for all s (that is item b) above), and simultaneously network ED is also unstable (that is item iii) above). Then, network X is stable if $d \in [0.9121, 0, 971]$ and s sufficiently large. For example, for $d = 0.913$ and $s > 0.970$, or for $d = 0.970$ and $s > 0.999$.

6.10 Social welfare for the benchmark case

First note that H_1 and H are not welfare maximizers as

$$W(H) - W(H_1) = \frac{17(a-c)^2}{288} - 2k > 0 \ \forall\ k \in [0, \overline{k}_2)$$

and

$$W(ED) - W(H) = \frac{(184 - 160d - 12d^2 + 20d^3 - 5d^4)(a-c)^2}{18(4-d)^2(2+d)^2} > 0 \ \forall\ d \in (0,1).$$

Notice that

$$W(ED) - W(MR) = \frac{d(2-3d)(48 + 10d - 39d^2 - 2d^3 - 4d^4)(a-c)^2}{4(4-d)^2(2-d)^2(2+d)^2(1+d)^2}$$

is positive if and only if $0 < d < \frac{2}{3}$. Finally, we define the thresholds on k that are used to find the social welfare maximizer network.

(i) $W(X) - W(ED) > 0$ if $k < \Omega_{X-ED} =$

$$\frac{(69632 + 26624d + 25344d^2 + 130048d^3 + 18460d^7 + 2967d^8 + 68d^9 + 335d^{10})(a-c)^2}{144(4-d)^2(2+d)^2(1+d)^2(16-7d^2)^2}$$
$$-\frac{(29984d^4 + 141504d^5 + 30548d^6)(a-c)^2}{144(4-d)^2(2+d)^2(1+d)^2(16-7d^2)^2}$$

(ii) $W(C) - W(ED) > 0$ if $k < \Omega_{C-ED} =$

$$(34 + 3d - 30d^2 - d^3 + 3d^4)\frac{(2 - d + d^2)(a-c)^2}{9(4-d)^2(2-d)^2(2+d)^2(1+d)^2}.$$

(iii) $W(C) - W(X) > 0$ if $k < \Omega_{C-X} =$

$$\frac{(4352 - 3072d - 5632d^2 + 3040d^3 + 1096d^4 - 476d^5 + 1081d^6 - 68d^7 - 335d^8)(a-c)^2}{144(2-d)^2(1+d)^2(16-7d^2)^2}.$$

(iv) $W(X) - W(MR) > 0$ if $k < \Omega_{X-MR} =$

$$\frac{(4352 - 2304d - 5824d^2 + 3680d^3 + 2270d^4 - 1990d^5 - 267d^6 + 335d^7)(a-c)^2}{144(2-d)^2(1+d)(16-7d^2)^2}.$$

(v) $W(C) - W(MR) > 0$ if $k < \Omega_{C-MR} =$

$$\frac{(17 - 2d + 12d^2)(a-c)^2}{144(2-d)^2(1+d)^2}.$$

Therefore plotting all the above $\Omega's$ in the same graph, noting that $W(ED) > W(MR)$ if $0 < d < \frac{2}{3}$, and comparing them with $\bar{k}_2$ we can easily obtain the areas in the parameter space (k, d) where each network C, X, ED and MR maximizes social welfare. See Figure 5.

6.11 Social welfare for the two-part tariff case

First we eliminate networks H, MR and C as social welfare maximizers. Note that $W^t(H_1) > W^t(H)$ since with network H_1 the same market outcome (integrated monopolist) is attained as with network H, but only forming one link. Next,

$$W^t(ED) - W^t(MR) = \frac{d(2+d)(8 + 2d - 7d^2 - 2d^3)(a-c)^2}{4(4 + 6d + d^2 - d^3)}$$

which is positive for all $d \in (0, 1)$.

Finally,

$$W^t(X) - W^t(C) = \frac{\alpha_1(d)\alpha_2(d)d(a-c)^2}{8(4-d)^2(1+d)^2(256 - 204d^2 + 36d^4 - 7d^6)^2} + 2k > 0$$

for all d and k, where $\alpha_1(d) = (32 + 20d - 28d^2 + 8d^3 - 4d^4 - d^5)$ and $\alpha_2(d) = (8192 - 10752d - 9920d^2 + 14208d^3 + 5436d^4 - 6372d^5 - 1340d^6 + 1032d^7 + 77d^8 - 60d^9 - 15d^{10})$.

Now the thresholds on k that are used to rank ED, H_1 and X are:

(i) $W^t(ED) - W^t(H_1) > 0$ if $k < \Omega t_{ED-H_1} =$

$$\frac{(48 - 48d - 20d^2 + 12d^3 - 3d^4)(a-c)^2}{16(4 + 2d - d^2)^2}.$$

(ii) $W^t(X) - W^t(H_1) > 0$ if $k < \Omega t_{X-H_1} =$

$$\frac{\lambda(d)(a-c)^2}{16(1+d)^2(256-204d^2+36d^4-7d^6)^2}$$

where

$$\begin{aligned}\lambda(d) \quad = \quad & 98304 - 24576d - 252416d^2 + 18944d^3 + 242880d^4 + 5728d^5 - 112408d^6 \\ & -4896d^7 + 30824d^8 - 572d^9 - 6420d^{10} + 296d^{11} + 760d^{12} - 27d^{13} - 66d^{14}\end{aligned}$$

(iii) $W^t(X) - W^t(ED) > 0$ if $k < \Omega t_{X-ED} =$

$$\frac{d\eta(d)(a-c)^2}{16(1+d)^2(4+2d-d^2)^2(256-204d^2+36d^4-7d^6)^2}$$

where

$$\begin{aligned}\eta(d) \quad = \quad & -786432 - 180224d + 1033312d^2 + 740608d^3 - 1439488d^4 - 751808d^5 \\ & +362688d^6 + 353712d^7 - 30304d^8 - 151040d^9 + 1600d^{10} + 55784d^{11} \\ & +5736d^{12} - 7972d^{13} - 636d^{14} + 703d^{15} + 180d^{16} + 15d^{17}\end{aligned}$$

Therefore plotting all the above Ωt's in the same graph and comparing them with $\bar{k}_2^t$ we can easily obtain the areas in the parameter space (k, d) where each network X, ED and H_1 are maximizing social welfare. See Figure 6.