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Gains from trade and efficiency under monopolistic competition: a variable elasticity case*

Kristian Behrens† Yasusada Murata‡

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Abstract

We present a general equilibrium model of monopolistic competition with variable demand elasticities and investigate the impact of free trade on welfare and efficiency. First, contrary to the constant elasticity case, in which all gains from trade are due to increasing product diversity, our model features gains from pro-competitive effects. Second, we prove that the market outcome is not efficient because too many firms operate at an inefficiently small scale. Last, we illustrate that free trade raises efficiency by reducing the gap between the equilibrium utility and the optimal utility.

Keywords: international trade; monopolistic competition; variable elasticity; gains from trade; efficiency
JEL Classification: D43; D51; F12

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†CORE, Université catholique de Louvain, 34 voie du Roman Pays, 1348 Louvain-la-Neuve, Belgium. E-mail: behrens@core.ucl.ac.be

‡Advanced Research Institute for the Sciences and Humanities, Nihon University, 12-5, Gobancho, Chiyoda-ku, Tokyo 102-8251, Japan. E-mail: murata@arish.nihon-u.ac.jp

1 Introduction

Few trade theorists would disagree with the statement that product variety, scale economies, and pro-competitive effects are central to any discussion about gains from trade and efficiency with differentiated goods under imperfect competition.[1] Yet, it is fair to say that, until now, these questions have not been fully explored within a simple and solvable general equilibrium model of monopolistic competition. This is largely due to the fact that the workhorse approach to international trade under monopolistic competition, namely the constant elasticity of substitution (henceforth, CES) framework, displays two peculiar features. First, it does not allow for pro-competitive effects so that *"there is no effect of trade on the scale of production, and the gains from trade come solely through increased product diversity*" (Krugman, 1980, p.953). Second, the equilibrium in the CES model is usually constrained (second-best) optimal, i.e., the market provides the socially desirable number of varieties at an efficient scale (Dixit and Stiglitz, 1977). Consequently, trade is not efficiency enhancing because it does not correct the only market failure, pricing above marginal cost.

In order to fully explore gains from trade and efficiency under monopolistic competition, we must depart from the standard CES model in some respect. In the present paper, we propose a simple and tractable variable elasticity of substitution (henceforth, VES) model, despite the widely-held view that VES models have until now unfortunately "*not proved tractable, and from Dixit and Norman (1980) and Krugman (1980) onwards, most writers have used the CES specification*" (Neary, 2004, p.177).[2,3] Building on previous work by Behrens and Murata (2005), we present a general equilibrium model of international trade that displays the following four features: (i) it has *variable mark-ups* and, therefore, accounts for pro-competitive effects; (ii) it has a *competitive limit* when the mass of firms becomes arbitrarily large; (iii) it exhibits *gains from trade due to both product diversity and pro-competitive effects*; and (iv) it allows for an efficiency analysis.

Previewing our main results, we concisely illustrate the well-known fact that free

[1]Dixit (2004, p.128), e.g., summarizes the gains from trade under monopolistic competition as follows: (i) availability of greater variety; (ii) better exploitation of economies of scale; and (iii) greater degree of competition, driving prices closer to marginal costs.

[2]Interestingly, Lawrence and Spiller (1983, p.66) point out that "the assumption that the elasticity of demand is independent of the number of competitors is not intuitively appealing", and they suggest a specification in which the elasticity of demand depends on the number of varieties while leaving the analysis of its implications for future research.

[3]Another possibility that has been recently explored in the literature is the introduction of heterogeneous firms within the well-established CES framework (Melitz, 2003).

trade leads to an increase in the mass of varieties consumed, and to a decrease in the mass of varieties produced in each country (Feenstra, 2004, Ch.5). We also show that exit under free trade is accompanied by an increase in output per firm, which leads to a better exploitation of firm-level scale economies. Although the per capita consumption of each variety decreases, gains from trade materialize because of product diversity and pro-competitive effects. We illustrate this by providing a welfare decomposition. Finally, we show that free trade *increases efficiency* by moving the market outcome closer to the optimum. The effect is stronger when the trading partners are large, yet even free trade does not lead to a fully efficient outcome in our setting.

Before continuing, one word of caution is in order. Some of the results we present in this paper have been 'in the air' for a quite a while. Indeed, Dixit and Norman (1980), Krugman (1980, 1981), Lawrence and Spiller (1983), Helpman and Krugman (1985), Wong (1995), and Feenstra (2004), among others, analyze certain aspects of gains from trade with differentiated products under monopolistic competition. Yet, these gains have not been systematically explored in a simple and solvable general equilibrium model with VES, as we do in this paper. Our new specification therefore enriches models as in Krugman's (1979) seminal paper, by making them more tractable and by allowing us to investigate additional issues such as whether free trade is efficiency enhancing.

The remainder of the paper is organized as follows. Section 2 develops the model, and Section 3 discusses the autarky case. Section 4 presents the free trade case, analyzes the trade equilibrium (Section 4.1), decomposes the gains from trade (Section 4.2) and shows that free trade moves the economy closer to the optimum and is, therefore, efficiency enhancing (Section 4.3). Section 5 concludes.

2 Model

2.1 Preferences

Consider a world with two countries, labeled r and s. Variables associated with each country will be subscripted accordingly. There is a mass L_r of workers/consumers in country r, and each worker supplies inelastically one unit of labor. Thus, L_r also stands for the total amount of labor available in country r. We assume that labor is internationally immobile and that it is the only factor of production.

There is a single monopolistically competitive industry producing a horizontally differentiated consumption good with a continuum of varieties. Let Ω_r (resp., Ω_s) be the set of varieties produced in country r (resp., s), of measure n_r (resp., n_s). Hence,

$N \equiv n_r + n_s$ stands for the (endogenously determined) mass of available varieties in the global economy. A representative consumer in country r solves the following consumption problem, with 'constant absolute risk aversion' (CARA) sub-utility functions (Behrens and Murata, 2005):

$$\begin{gathered}\max_{q_{rr}(i),\, q_{sr}(j)} \quad U_r \equiv \int_{\Omega_r} \left[1 - \mathrm{e}^{-\alpha q_{rr}(i)}\right] \mathrm{d}i + \int_{\Omega_s} \left[1 - \mathrm{e}^{-\alpha q_{sr}(j)}\right] \mathrm{d}j \\ \text{s.t.} \quad \int_{\Omega_r} p_r(i) q_{rr}(i) \mathrm{d}i + \int_{\Omega_s} p_s(j) q_{sr}(j) \mathrm{d}j = E_r,\end{gathered} \tag{1}$$

where $\alpha > 0$ is a utility parameter; E_r stands for the expenditure; $p_r(i)$ denotes the price of variety i, produced in country r; and $q_{sr}(j)$ stands for the per-capita consumption of variety j, produced in country s and sold in country r.

We show in Appendix A that the demand functions for country-r consumers are given as follows:

$$q_{rr}(i) = \frac{E_r - \dfrac{1}{\alpha}\displaystyle\int_{\Omega_r} \ln\left(\frac{p_r(i)}{p_r(j)}\right) p_r(j)\mathrm{d}j - \dfrac{1}{\alpha}\int_{\Omega_s} \ln\left(\frac{p_r(i)}{p_s(j)}\right) p_s(j)\mathrm{d}j}{\displaystyle\int_{\Omega_r} p_r(j)\mathrm{d}j + \int_{\Omega_s} p_s(j)\mathrm{d}j}, \tag{2}$$

$$q_{sr}(j) = \frac{E_r - \dfrac{1}{\alpha}\displaystyle\int_{\Omega_r} \ln\left(\frac{p_s(j)}{p_r(i)}\right) p_r(i)\mathrm{d}i - \dfrac{1}{\alpha}\int_{\Omega_s} \ln\left(\frac{p_s(j)}{p_s(i)}\right) p_s(i)\mathrm{d}i}{\displaystyle\int_{\Omega_r} p_r(i)\mathrm{d}i + \int_{\Omega_s} p_s(i)\mathrm{d}i}. \tag{3}$$

Mirror expressions hold for country-s consumers. Because of the continuum assumption firms are negligible, so that the own-price derivatives of the demand functions are given as follows:

$$\frac{\partial q_{rr}(i)}{\partial p_r(i)} = -\frac{1}{\alpha p_r(i)} \qquad \frac{\partial q_{sr}(j)}{\partial p_s(j)} = -\frac{1}{\alpha p_s(j)}, \tag{4}$$

which then yields the demand elasticities $\epsilon_{rr}(i) = [\alpha q_{rr}(i)]^{-1}$ and $\epsilon_{sr}(j) = [\alpha q_{sr}(j)]^{-1}$. Mirror expressions hold again for country s.

2.2 Technology

All firms have access to the same increasing returns to scale technology. To produce $Q(i)$ units of any variety requires $l(i) = cQ(i) + F$ units of labor, where F is the fixed and c is the constant marginal labor requirement. We assume that firms can costlessly differentiate their products and that there are no scope economies. Thus, there is a one-to-one correspondence between firms and varieties, so that the mass of varieties N also stands for the mass of firms operating in the global economy. There is free entry and exit in each country, which implies that n_r and n_s are endogenously determined by the zero profit conditions. Consequently, the expenditure E_r equals the wage in country r.

International markets are assumed to be integrated, so that firm $i \in \Omega_r$ sets a unique free-on-board price $p_r(i)$ for consumers in both countries. Its profit is then as follows:

$$\Pi_r(i) = [p_r(i) - cw_r] \, Q_r(i) - Fw_r, \tag{5}$$

where $Q_r(i) \equiv L_r q_{rr}(i) + L_s q_{rs}(i)$ stands for its total output.

2.3 Equilibrium

Country-r (resp., country-s) firms maximize their profit (5) with respect to $p_r(i)$ (resp., $p_s(j)$), taking the vectors (n_r, n_s) and (w_r, w_s) of firm distribution and factor prices as given.[4] This yields the following first-order conditions:

$$\frac{\partial \Pi_r(i)}{\partial p_r(i)} = Q_r(i) + [p_r(i) - cw_r] \left[L_r \frac{\partial q_{rr}(i)}{\partial p_r(i)} + L_s \frac{\partial q_{rs}(i)}{\partial p_r(i)} \right] = 0, \tag{6}$$

$$\frac{\partial \Pi_s(j)}{\partial p_s(j)} = Q_s(j) + [p_s(j) - cw_s] \left[L_s \frac{\partial q_{ss}(j)}{\partial p_s(j)} + L_r \frac{\partial q_{sr}(j)}{\partial p_s(j)} \right] = 0. \tag{7}$$

Conditions (6) and (7) highlight a fundamental property of monopolistic competition models: although each firm is negligible to the market (no 'direct strategic interactions' between firms), it must take into account the aggregate pricing decisions of the other firms since their prices enter the first-order conditions ('weak strategic interactions' via the price aggregates). Formally, our equilibrium concept is that of a Nash equilibrium with a continuum of players.[5,6]

In what follows, let $(\mathbf{p}_r, \mathbf{p}_s)$ stand for a *price equilibrium*, i.e., a distribution of prices satisfying (6) and (7) for all $i \in \Omega_r$ and $j \in \Omega_s$. We will discuss its existence, uniqueness, and some other properties in the following sections.

[4] It is well-known that price and quantity competition yield the same outcome in monopolistic competition models with a continuum of firms (Vives, 1999, p.168).

[5] In the light of this interpretation, one can check that the price equilibrium in the CES model is a dominant strategy Nash equilibrium (Behrens and Murata, 2005).

[6] As shown by Roberts and Sonnenschein (1977), the existence of (price) equilibria is usually problematic in monopolistic competition models, since firms' reaction functions may be badly behaved. Because our model relies on a continuum of firms, which are individually negligible, we do not face these problems in this model. In a similar spirit, Neary (2003) uses a general equilibrium model of oligopolistic competition with a continuum of sectors, in which firms are 'large' in their own markets but 'negligible' in the whole economy. This allows, again, to restore equilibrium since firms cannot directly influence aggregates of the whole economy.

An *equilibrium* is a price equilibrium and vectors (n_r, n_s) and (w_r, w_s) of firm distribution and factor prices such that national factor markets clear, trade is balanced, and firms earn zero profits (in which case $E_r = w_r$ and $E_s = w_s$). More formally, an equilibrium is a solution to the following three conditions:

$$\int_{\Omega_r} \big[cQ_r(i) + F\big]\mathrm{d}i = L_r, \tag{8}$$

$$\int_{\Omega_s} \big[cQ_s(j) + F\big]\mathrm{d}j = L_s, \tag{9}$$

$$L_s \int_{\Omega_r} p_r(i) q_{rs}(i)\mathrm{d}i = L_r \int_{\Omega_s} p_s(j) q_{sr}(j)\mathrm{d}j, \tag{10}$$

where all quantities are evaluated at a price equilibrium. One may set either w_r or w_s as the numeraire. At this stage, however, we do not need to choose a numeraire, since the model is fully determined in real terms.[7] Finally, it is readily verified that firms earn zero profits when conditions (8)–(10) hold.

3 Autarky

Assuming that the two countries cannot trade initially with each other, we first characterize the equilibrium and optimal outcomes in the closed economy. Without loss of generality, we consider country r in what follows.

3.1 Equilibrium

Inserting (2)–(4) into (6), and letting $q_{rs}(i) = \partial q_{rs}(i)/\partial p_r(i) = 0$ and $q_{sr}(j) = \partial q_{sr}(j)/\partial p_s(j) = 0$, Behrens and Murata (2005) have shown that the unique price equilibrium is symmetric and given as follows:

$$p_r^a = \left(c + \frac{\alpha}{n_r^a}\right) w_r^a, \tag{11}$$

where an a-superscript henceforth denotes autarky values.

Given the symmetry of the price equilibrium, the profit of each firm is as follows:

$$\Pi_r^a = L_r q_r^a \left(p_r^a - c w_r^a\right) - F w_r^a.$$

Using the consumer's budget constraint $w_r^a = n_r^a p_r^a q_r^a$, this can be rewritten as

$$\Pi_r^a = p_r^a q_r^a [L_r \left(1 - c n_r^a q_r^a\right) - F n_r^a].$$

[7] Indeed, the choice of the numeraire is immaterial in our monopolistic competition framework. This is an important departure from general equilibrium oligopoly models, where the choice of the numeraire is usually not neutral (Gabszewicz and Vial, 1972).

Zero profits then imply that the quantities must be such that

$$q_r^a = \frac{1}{c}\left[\frac{1}{n_r^a} - \frac{F}{L_r}\right], \tag{12}$$

which are positive because $n_r^a F < L_r$ must hold from the resource constraint when n_r^a firms operate. Utility is then given by

$$U(n_r^a) = n_r^a \left[1 - \mathrm{e}^{-\frac{\alpha}{c}\left(\frac{1}{n_r^a} - \frac{F}{L_r}\right)}\right]. \tag{13}$$

Note that (12) and (13) hold whenever prices are symmetric and firms earn zero profit. This property will prove to be useful when we subsequently compare the equilibrium and the optimum.

Inserting $q_r^a = w_r^a/(n_r^a p_r^a)$ into the labor market clearing condition (8), we get:

$$n_r^a = \frac{L_r}{F}\left(1 - c\frac{w_r^a}{p_r^a}\right). \tag{14}$$

The equilibrium mass of firms can then be found by using (11) and (14), which yields:[8]

$$n_r^a = \frac{\sqrt{4\alpha c F L_r + (\alpha F)^2} - \alpha F}{2cF} > 0. \tag{15}$$

Finally, inserting (15) into (13), the equilibrium utility in autarky is given by

$$U^a = \frac{\sqrt{4\alpha c F L_r + (\alpha F)^2} - \alpha F}{2cF}\left[1 - \mathrm{e}^{-\frac{2\alpha F}{\sqrt{4\alpha c F L_r + (\alpha F)^2} + \alpha F}}\right], \tag{16}$$

which is a strictly increasing and strictly concave function of the population size L_r for all admissible parameter values, i.e., $\alpha > 0$, $c > 0$, $F > 0$, and $L_r > 0$.

3.2 Optimum

We now determine the socially optimal mass of varieties. The planner maximizes the utility, as given in (1), subject to the technology and resource constraints (8). The first-order conditions of this problem with respect to $q(i)$ show that the quantities must be symmetric. This, together with (8), implies that:

$$q_r = \frac{Q_r}{L_r} = \frac{1}{c}\left(\frac{1}{n_r} - \frac{F}{L_r}\right). \tag{17}$$

Hence, the planner maximizes

$$U_r(n_r^o) = n_r^o \left[1 - \mathrm{e}^{-\frac{\alpha}{c}\left(\frac{1}{n_r^o} - \frac{F}{L_r}\right)}\right], \tag{18}$$

[8]Note that the other root is negative and must, therefore, be ruled out.

with respect to n_r^o, where an o-superscript henceforth denotes the optimal values. Note that, as shown in Appendix B, alternative policies such as: (i) marginal cost pricing and lump-sum transfers; or (ii) profit-maximizing prices and non-negative profits, boil down to exactly the same problem. Standard calculations show that

$$\frac{\partial U_r}{\partial n_r^o} = 1 - \left(1 + \frac{\alpha}{c n_r^o}\right) \mathrm{e}^{-\frac{\alpha}{c}\left(\frac{1}{n_r^o} - \frac{F}{L_r}\right)} \tag{19}$$

and

$$\frac{\partial^2 U_r}{\partial (n_r^o)^2} = -\frac{\alpha^2}{c^2 (n_r^o)^3} \mathrm{e}^{-\frac{\alpha}{c}\left(\frac{1}{n_r^o} - \frac{F}{L_r}\right)} < 0,$$

i.e., U_r is a strictly concave function of n_r^o. Equating (19) to zero, utility maximization requires us to solve the first-order condition

$$\frac{c n_r^o}{\alpha + c n_r^o} = \mathrm{e}^{-\frac{\alpha}{c}\left(\frac{1}{n_r^o} - \frac{F}{L_r}\right)}. \tag{20}$$

Using (20) we can show the following result:

Proposition 1 (excess entry) *There is a unique optimal mass of firms n_r^o such that $n_r^a > n_r^o$. Hence, in equilibrium under autarky, there are too many firms operating at an inefficiently small scale.*

Proof. See Appendix C. ■

Note that excess entry arises because of the negative externality each firm imposes on the other firms' through a price decrease (see equation (11)).

Interestingly, this result contrasts starkly with the constant elasticity case, where the equilibrium mass of varieties is also (second-best) optimal.[9] Stated differently, the basic CES model does not account for the tendency that too many firms produce at inefficiently small scale in a closed economy (the so-called 'Eastman-Stykolt hypothesis'; Eastman and Stykolt, 1967), an argument often used to criticize import-substituting industrialization policies (Krugman and Obstfeld, 2003, pp.261-263) or tariff barriers (Horstmann and Markusen, 1986) on efficiency grounds.

[9]This can be seen from Dixit and Stiglitz (1977, p.301), when letting $s = 1$ and $\theta = 0$ in their equations (20) and (21), since there is no homogeneous good in our setting. Note that the two-factor two-sector CES trade model by Lawrence and Spiller (1983, p.68) even displays insufficient entry. This runs against the general tendency of excess entry obtained under "a range of very plausible situations" (Vives, 1999, p.176).

4 Free trade

We now analyze the impacts of free trade on welfare and efficiency in a world with variable mark-ups. Section 4.1 analyzes the equilibrium. Section 4.2 then shows the existence of gains from trade and decomposes these gains into *product diversity* and *pro-competitive effects*. Section 4.3 finally illustrates that trade increases efficiency by moving the equilibrium closer to the optimum.

4.1 Equilibrium

Assume that both countries can trade freely. The profits and the first-order conditions are still given by (5)–(7), respectively. We start by showing that free trade leads to product price equalization, which then also implies factor price equalization.[10]

Proposition 2 (price equalization) *Free trade leads to both product and factor price equalization for all admissible parameter values, i.e.,* $\alpha > 0$, $c > 0$, $F > 0$, $L_r > 0$ *and* $L_s > 0$.

Proof. Conditions (6) and (7) must hold for both country-r and country-s firms at every price equilibrium which, using (2)–(4), yields

$$\frac{\partial \Pi_r(i)}{\partial p_r(i)} - \frac{\partial \Pi_s(j)}{\partial p_s(j)} = 0 \quad \Longleftrightarrow \quad c\left[\frac{w_r}{p_r(i)} - \frac{w_s}{p_s(j)}\right] = \ln\left(\frac{p_r(i)}{p_s(j)}\right). \tag{21}$$

It is also readily verified that

$$Q_r(i) \gtreqqless Q_s(j) \quad \Longleftrightarrow \quad \frac{-(L_r + L_s)}{\alpha} \ln\left(\frac{p_r(i)}{p_s(j)}\right) \gtreqqless 0. \tag{22}$$

Furthermore, an equilibrium is such that firms earn zero profit, i.e.,

$$\begin{aligned} \Pi_r(i) &= w_r\left[\left(\frac{p_r(i)}{w_r} - c\right)Q_r(i) - F\right] = 0 \\ \Pi_s(j) &= w_s\left[\left(\frac{p_s(j)}{w_s} - c\right)Q_s(j) - F\right] = 0. \end{aligned}$$

Assume that there exists $i \in \Omega_r$ and $j \in \Omega_s$ such that $p_r(i) > p_s(j)$. Then condition (21) implies that

$$\frac{w_r}{p_r(i)} > \frac{w_s}{p_s(j)} \qquad \Longrightarrow \qquad \frac{p_r(i)}{w_r} < \frac{p_s(j)}{w_s},$$

[10] Note that there is a priori no reason for product price equalization to hold in our setting, even under free trade. This is because *firms sell differentiated varieties*, factor markets are segmented, and firms are imperfectly competitive. Most studies assume, rather than prove, that product price equalization must hold under free trade in the first place (e.g., Helpman, 1981).

whereas condition (22) implies that $Q_r(i) < Q_s(j)$. Hence, $\Pi_r(i) < \Pi_s(j)$, which is incompatible with an equilibrium. We may hence conclude that $p_r(i) = p_s(j)$ must hold for all $i \in \Omega_r$ and $j \in \Omega_s$, which shows that product prices are equalized. Condition (21) then shows that $w_r = w_s$, i.e., factor prices are equalized whenever product prices are equalized, which proves our claim. ■

From Proposition 2, we know that $p_r = p_s = p$ and $w_r = w_s = w$, which allows us to rewrite (2)–(4) as follows:

$$q_{rr} = q_{sr} = q_{ss} = q_{rs} = \frac{w}{Np} \tag{23}$$

and

$$\frac{\partial q_{rr}}{\partial p_r} = \frac{\partial q_{sr}}{\partial p_s} = \frac{\partial q_{ss}}{\partial p_s} = \frac{\partial q_{rs}}{\partial p_r} = -\frac{1}{\alpha p}. \tag{24}$$

Inserting (23) and (24) into the first order condition (6), we obtain the price equilibrium:

$$p = \left(c + \frac{\alpha}{N}\right) w, \tag{25}$$

which is an extension of the autarky case (11).

Since prices and wages are equalized, all firms sell the same total quantity $Q = (L_r + L_s)q$. Labor market clearing then implies that $n_r/n_s = L_r/L_s$, which yields

$$n_r = \frac{L_r}{F}\left(1 - c\frac{w}{p}\right). \tag{26}$$

Plugging (25) into (26) and the analogous expression for country s, we obtain two equations with two unknowns n_r and n_s. Solving for the equilibrium masses of firms, we get

$$n_r = \frac{L_r}{L_r + L_s} \frac{\sqrt{4\alpha cF(L_r + L_s) + (\alpha F)^2} - \alpha F}{2cF}$$

and

$$n_s = \frac{L_s}{L_r + L_s} \frac{\sqrt{4\alpha cF(L_r + L_s) + (\alpha F)^2} - \alpha F}{2cF}.$$

Thus, the equilibrium mass of firms in the global economy is given by

$$N = n_r + n_s = \frac{\sqrt{4\alpha cF(L_r + L_s) + (\alpha F)^2} - \alpha F}{2cF}, \tag{27}$$

which is an extension of the autarky expression (15). This immediately shows that $N > \max\{n_r^a, n_s^a\}$, thus implying from (11) and (25) that $p/w < \min\{p_r^a/w_r^a, p_s^a/w_s^a\}$. Finally, from expressions (14) and (26), we obtain $n_r < n_r^a$ and $n_s < n_s^a$. Hence, the relationship between trade and product diversity can be summarized as follows:

Proposition 3 (trade and product diversity) *When compared with autarky, the mass of varieties produced in each country decreases under free trade, whereas the mass of varieties consumed in each country increases.*

Proposition 3 illustrates exit of firms due to the *pro-competitive effects of international trade.* Once trade occurs, the price-cost margins in *both* countries decrease, thus driving some firms out of each national market (Feenstra, 2004).[11] Factor market clearing then makes sure that firm-level and total production expands, as labor is reallocated from the (unproductive) fixed requirements of closing firms to the (productive) marginal requirements of surviving firms. This is an important departure from the CES model, in which such an effect does not arise. Note also that Proposition 2 holds regardless of country size. In autarky, a smaller country tends to have a smaller mass of firms, which implies a higher price-wage ratio. Therefore, the price-wage ratio in a small country decreases more than that in a large country under free trade, i.e., we observe convergence in price-wage ratios across countries.[12]

Note, finally, that although there is a growing literature on firm heterogeneity and exit in international trade (e.g., Melitz, 2003), the price-cost margin for each firm is usually assumed to be constant in these models because of the CES specification.[13] By contrast, our model captures the 'old idea' that international trade in the presence of imperfect competition leads to decreasing mark-up rates and hence to exit of firms even without heterogeneity (Dixit and Norman, 1980).

4.2 Welfare decomposition and gains from trade

We now discuss gains from trade by decomposing welfare as in Krugman (1981). Since varieties are symmetric under both free trade and autarky, the utility difference can be expressed as follows:

$$U_r - U_r^a = N\left(1 - \mathrm{e}^{-\frac{\alpha w}{Np}}\right) - n_r^a\left(1 - \mathrm{e}^{-\frac{\alpha w_r^a}{n_r^a p_r^a}}\right).$$

[11]In the model of Lawrence and Spiller (1983, Proposition 7), international trade simply leads to a redistribution of existing firms between the two countries while the total mass of firms remains unchanged. This result is driven by changes in relative factor prices under free trade and, as pointed out by the authors, need not hold under variable mark-ups.

[12]In the two-sector two-factor model of Lawrence and Spiller (1983, Proposition 5), the price of the monopolistically competitive good falls in one country and rises in the other due to changes in relative factor prices. Yet, the price-cost margins remain constant because of the CES assumption.

[13]One notable exception is the work by Melitz and Ottaviano (2005) who recently proposed a model that explains trade-induced exit by combining pro-competitive effects and firm heterogeneity in a quasi-linear framework.

Adding and subtracting $n_r^a \mathrm{e}^{-\frac{\alpha w}{n_r^a p}}$, and rearranging the resulting terms, we obtain the following decomposition:

$$U_r - U_r^a = \underbrace{N\Big(1 - \mathrm{e}^{-\frac{\alpha w}{Np}}\Big) - n_r^a\Big(1 - \mathrm{e}^{-\frac{\alpha w}{n_r^a p}}\Big)}_{\text{Product diversity}} + \underbrace{n_r^a\Big(\mathrm{e}^{-\frac{\alpha w_r^a}{n_r^a p_r^a}} - \mathrm{e}^{-\frac{\alpha w}{n_r^a p}}\Big)}_{\text{Pro-competitive effects}}, \tag{28}$$

which isolates the two channels, namely product diversity and pro-competitive effects, through which gains from trade materialize.

We now examine the role and the sign of each component in expression (28) in more details, both from a theoretical and an empirical point of view.

Product diversity: As shown in Proposition 3, free trade expands consumers' choice set in our model, as it does in the CES case, despite the exit of some national producers. Ceteris paribus this raises utility in this model via 'love-of-variety'. This can be seen as follows. Given the wage-price ratio under free trade, w/p, we have

$$U_r = N\left(1 - \mathrm{e}^{-\frac{\alpha w}{Np}}\right), \qquad \frac{\partial U_r}{\partial N} = 1 - \mathrm{e}^{-\frac{\alpha w}{Np}}\left(1 + \frac{\alpha w}{Np}\right) > 0 \qquad \forall N.$$

To obtain the last inequality, let $z \equiv (\alpha w)/(Np)$ and $h(z) \equiv 1 - e^{-z}(1+z)$. Clearly, $h(0) = 0$ and $h'(z) > 0$ for all $z > 0$, which shows that for any given wage-price ratio w/p, utility increases with the range of varieties consumed.

Despite its central theoretical role in new trade theory, little is known until now about the empirical importance of gains from product diversity (Feenstra, 1995). Using extremely disaggregated data, Broda and Weinstein (2005) document that the number of product varieties in US imports rose by 251% between 1972 and 2001, and according to their estimates this maps into US welfare gains of about 2.8% of GDP. These findings suggest that gains from product diversity may be an important real-world aspect of international trade.

Pro-competitive effects: The second term in (28) captures the beneficial effects of increased price competition in the product market, driving prices closer to marginal costs. This can be seen by comparing expressions (11) and (25). Note that $w_r^a/p_r^a = w/p$ would hold in the constant elasticity case, i.e., there would be no gains from trade due to pro-competitive effects.

It is well-known from various industrial organization studies that prices in many imperfectly competitive industries are increasing functions of producer concentration (see Schmalensee, 1989, pp.987-988, for a survey). In our symmetric equilibrium, the Herfindahl-index of concentration, defined as the sum of squared market shares, reduces to $H = N(1/N)^2 = 1/N$. Hence, by increasing N trade decreases concentration, which in turn maps into lower consumer prices. Several case studies in international trade confirm this 'imports-as-market-discipline hypothesis', i.e. import competition, by increasing the number of competitors in each market, decreases concentration and reduces prices (e.g., Levinsohn, 1993; Harrison, 1994).

Let us summarize our results as follows:

Proposition 4 (gains from trade) *Free trade has a positive effect on welfare both by expanding consumers' choice sets ('product diversity') and by driving prices closer to marginal costs ('pro-competitive effects').*

Proof. To prove our claim, it is sufficient to discuss the sign of the two components in expression (28). As shown above, they are both positive, which ensures gains from trade. ■

4.3 Optimum and efficiency

In what follows, we focus on the case in which a global planner maximizes world welfare under a global resource constraint ('centralized' first-best). As shown in Appendix D, doing so is equivalent to letting each national government maximize its own welfare under its domestic resource constraint ('decentralized' first-best).

Since in our model free trade amounts to increasing the population size, the result on excess entry established in Section 3.2 continues to hold, even under free trade. Stated differently, there is again a unique optimal mass of firms N^o, satisfying the first-order condition (20), under free trade such that $N > N^o$. Hence, there are too many firms operating at an inefficiently small scale and the market outcome is not efficient. Yet, we know that the model has a competitive limit, which yields the following result:

Proposition 5 (trade and efficiency) *When the population gets arbitrarily large, the equilibrium utility approaches the optimal utility, i.e.,*

$$\lim_{L\to\infty} U(N(L), L) = \lim_{L\to\infty} U(N^o(L), L) = \frac{\alpha}{c}.$$

Proof. See Appendix E. ■

The limit result established in Proposition 5 may be extended to a finite economy by investigating whether

$$\frac{U(n_r^a)}{U(n_r^o)} < \frac{U(N)}{U(N^o)} < 1, \tag{29}$$

where the last inequality comes from the definition of the optimum N^o. To do this, we check whether the ratio of equilibrium to optimal utility $\Gamma \equiv U/U^o$ monotonically increases in L. Note that this is a non-trivial question. Indeed, we know from expression (16) that there are gains from free trade or from an exogenous increase in population, i.e., $\mathrm{d}U/\mathrm{d}L > 0$. Yet, the optimal utility also rises due to free trade or to an increase in population. To see this, apply the implicit function theorem to (20), which yields

$$\frac{\mathrm{d}N^o}{\mathrm{d}L} = \frac{F(\alpha + cN^o)\,(N^o)^2}{\alpha L^2} > 0$$

i.e., the optimal mass of firms increases in the population size L. Plugging (20) into (18), the optimal utility satisfies $U^o = \alpha N^o/(\alpha + cN^o)$, which shows that

$$\frac{\mathrm{d}U^o}{\mathrm{d}L} = \left(\frac{\alpha}{\alpha + cN^o}\right)^2 \frac{\mathrm{d}N^o}{\mathrm{d}L} > 0.$$

Given that both the equilibrium and the optimal utility increase in L, it is a priori unclear whether free trade (or an exogenous increase in L) leads the economy closer to the optimum.

[**Insert Figure 1 around here**]

Figure 1 illustrates a typical example of Γ as a function of L.[14] Three remarks are in order. First, as can be seen from the fact that $\Gamma < 1$, the market outcome remains inefficient for finite population sizes. Second, since Γ increases monotonically with L, free trade between large countries yields higher efficiency than free trade between small countries (see also Lawrence and Spiller, 1983, Proposition 10). Third, when the population gets arbitrarily large ($L \to \infty$), the economy converges to the efficient outcome, because our model has a competitive limit (in the sense that prices converge to marginal costs, as can be seen from expressions (11) and (25)). Last, the maximal utility that can be achieved in the limit is determined solely by tastes and technology (i.e., α/c).

5 Conclusions

We have developed a simple monopolistic competition model that allows us to concisely highlight the *gains from trade due to product diversity and pro-competitive effects*. We have shown that, as in the CES model, trade expands consumers' choice

[14]The parameter values are as follows: $\alpha = 2$, $F = 1$, and $c = 0.5$. Other admissible parameter values yield qualitatively similar figures, thus suggesting that the underlying property is general.

sets which, ceteris paribus, makes them better off due to 'love-of-variety'. In addition, and contrary to the CES model, trade also drives prices closer to marginal costs via pro-competitive effects, which leads to a reduction of the mass of varieties produced in each country due to exit of firms. Finally, we also illustrated that free trade is likely to enhance efficiency by moving the economy closer to the optimum. When the population size gets arbitrarily large, our model has a competitive limit and the market outcome is efficient.

A final word of caution is in order. The model itself may be simply viewed as an analytically solvable example of Krugman (1979). Indeed, Proposition 3 is not original since the result has been around in the literature (e.g., Feenstra, 2004). Yet, due to its tractability, our specification allows us to go beyond Krugman's original contribution in three respects: first, it allows us to concisely analyze the optimum; second, we can decompose gains from trade into those due to product diversity and those due to pro-competitive effects; and third, we can analyze the issue of efficiency in a general equilibrium model of monopolistic competition with pro-competitive effects and a competitive limit.

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Appendix A: Derivation of the demand functions

A representative consumer in country r solves problem (1). Letting λ stand for the Lagrange multiplier, the first order conditions for an interior solution are given by:

$$\alpha \mathrm{e}^{-\alpha q_{rr}(i)} = \lambda p_r(i), \qquad \forall i \in \Omega_r \tag{30}$$

$$\alpha \mathrm{e}^{-\alpha q_{sr}(j)} = \lambda p_s(j), \qquad \forall j \in \Omega_s \tag{31}$$

and the budget constraint

$$\int_{\Omega_r} p_r(k) q_{rr}(k) \mathrm{d}k + \int_{\Omega_s} p_s(k) q_{sr}(k) \mathrm{d}k = E_r. \tag{32}$$

Taking the ratio of (30) with respect to i and j, we obtain

$$\mathrm{e}^{-\alpha[q_{rr}(i) - q_{rr}(j)]} = \frac{p_r(i)}{p_r(j)} \quad \Longrightarrow \quad q_{rr}(i) = q_{rr}(j) + \frac{1}{\alpha} \ln \left[\frac{p_r(j)}{p_r(i)} \right] \quad \forall i, j \in \Omega_r.$$

Multiplying the last expression by $p_r(j)$ and integrating with respect to $j \in \Omega_r$ we obtain:

$$q_{rr}(i) \int_{\Omega_r} p_r(j) \mathrm{d}j = \int_{\Omega_r} p_r(j) q_{rr}(j) \mathrm{d}j + \frac{1}{\alpha} \int_{\Omega_r} \ln \left[\frac{p_r(j)}{p_r(i)} \right] p_r(j) \mathrm{d}j. \tag{33}$$

Analogously, taking the ratio of (30) and (31) with respect to i and j, we get:

$$\mathrm{e}^{-\alpha[q_{rr}(i)-q_{sr}(j)]} = \frac{p_r(i)}{p_s(j)} \quad \Longrightarrow \quad q_{rr}(i) = q_{sr}(j) + \frac{1}{\alpha}\ln\left[\frac{p_s(j)}{p_r(i)}\right] \quad \forall i \in \Omega_r, \forall j \in \Omega_s.$$

Multiplying the last expression by $p_s(j)$ and integrating with respect to $j \in \Omega_s$ we obtain:

$$q_{rr}(i)\int_{\Omega_s} p_s(j)\mathrm{d}j = \int_{\Omega_s} p_s(j)q_{sr}(j)\mathrm{d}j + \frac{1}{\alpha}\int_{\Omega_s} \ln\left[\frac{p_s(j)}{p_r(i)}\right] p_s(j)\mathrm{d}j. \tag{34}$$

Summing (33) and (34), and using the budget constraint (32), we finally obtain the demands (2). The derivations of the demands (3) are analogous.

Appendix B: Alternative policies

(i) Marginal cost pricing and lump-sum transfers:

Denote by a c-superscript the values when the planner can use lump-sum transfers to compensate firms for the losses they make under marginal cost pricing, $p_r^c = cw_r$. This implies that firms' profit is given by $\Pi_r = -Fw_r < 0$. Hence, when there is a mass of firms n_r^c operating, a lump-sum tax $(Fw_r n_r^c)/L_r$ is levied on each consumer's income in country r. The income net of this tax is then given by

$$E_r = w_r\left(1 - \frac{Fn_r^c}{L_r}\right), \tag{35}$$

which is positive since $Fn_r^c < L_r$ must hold from the resource constraint. Using (35), $p_r^c = cw_r$ and $q_r^c = E_r/(n_r^c p_r^c)$, we get

$$q_r^c = \frac{w_r}{p_r^c}\left(\frac{1}{n_r^c} - \frac{F}{L_r}\right) = \frac{1}{c}\left(\frac{1}{n_r^c} - \frac{F}{L_r}\right). \tag{36}$$

which is the same as expression (17) and, therefore, yields the same utility function to maximize.

(ii) Profit-maximizing prices and non-negative profits:

Denote by a s-superscript the values under profit-maximizing prices and non-negative profits. To begin with, it is worth noting that we cannot a priori rule out that firms make non-negative profits since the planner controls entry and may opt for a mass of firms incompatible with zero profits. Let $\pi_r \equiv n_r^s \Pi_r / L_r$ stand for per-capita profits in the economy, so that consumers' budget constraint is given by $n_r^s p_r^s q_r^s = w_r + \pi_r$.

Behrens and Murata (2005) have shown that the profit-maximizing price is symmetric and unique even when wages and expenditures are different, and given by:[15]

$$p_r^s = cw_r + \frac{\alpha(w_r + \pi_r)}{n_r^s}. \tag{37}$$

Substituting (37) and the quantities $q_r^s = (w_r + \pi_r)/n_r^s p_r^s$ into (5) under autarky, the per-capita profits associated with a mass of firms n_r^s are given by

$$\pi_r = \frac{w_r \left[L_r \alpha - n_r^s F(cn_r^s + \alpha)\right]}{L_r(cn_r^s - \alpha) + Fn_r^s \alpha}. \tag{38}$$

Finally, plugging (37) and (38) into q_r^s yields

$$q_r^s = \frac{w_r + \pi_r}{n_r^s p_r^s} = \frac{1}{c}\left(\frac{1}{n_r^c} - \frac{F}{L_r}\right), \tag{39}$$

which is the same as expression (17) and, therefore, yields the same utility function to maximize. Hence, (13) or (18) holds regardless of whether firms earn zero profits or not.

Appendix C: Proof of Proposition 1

We need to compare the market outcome n_r^a and the optimum n_r^o defined as the solution of the equation:

$$f(n_r) = g(n_r), \quad \text{where} \quad f(n_r) \equiv \frac{cn_r}{\alpha + cn_r} \quad \text{and} \quad g(n_r) \equiv \mathrm{e}^{-\frac{\alpha}{c}\left(\frac{1}{n_r} - \frac{F}{L_r}\right)}. \tag{40}$$

Note first that f is strictly increasing in n_r, taking values from 0 to 1, and that g is also strictly increasing, taking values from 0 to $\mathrm{e}^{\alpha F/(cL_r)} > 1$. Some standard calculations show that there is a unique intersection since: (i) both functions are continuous; (ii) f is concave, whereas g is convex for n_r sufficiently small; (iii) the slope of f is strictly greater than that of g for n_r sufficiently small;[16] and (iv) g admits a single value for which its second order derivative is equal to zero.

[15] Note that $n_r^s \Pi_r / L_r = \pi_r \Rightarrow (n_r^s/L_r)\left[L_r q_r^s (p_r^s - cw_r) - Fw_r\right] = n_r^s p_r^s q_r^s - w_r$, which reduces to $n_r^s[cL_r q_r^s + F] = L_r$ and thus the resource constraint is satisfied.

[16] To check this, note that $\lim_{n_r \to 0} f'(n_r) = c/\alpha > \lim_{n_r \to 0} g'(n_r) = 0$. The last equality is obtained as follows. Noting that

$$\ln g'(n_r) = -\frac{2}{n_r}\left[\frac{\ln(n_r)}{1/n_r} + \frac{\alpha}{2c}\right] + \ln\left(\frac{\alpha}{c}\right) + \frac{\alpha F}{cL},$$

and that $\lim_{n_r \to 0} \ln n_r/(1/n_r) = 0$ by l'Hospital's rule, we have

$$\lim_{n_r \to 0} \ln g'(n_r) = -\lim_{n_r \to 0} \frac{2}{n_r} \times \lim_{n_r \to 0}\left[\frac{\ln(n_r)}{1/n_r} + \frac{\alpha}{2c}\right] + \ln\left(\frac{\alpha}{c}\right) + \frac{\alpha F}{cL} = -\infty,$$

which, by continuity of the logarithmic function, implies $\lim_{n_r \to 0} g'(n_r) = 0$.

We next show that $n_r^a > n_r^o$. To prove our claim, we use a convexity argument. The equilibrium mass of varieties is given by (15), whereas the optimal mass of varieties is the unique solution to (40). First, evaluate f at n_r^a, which yields

$$f(n_r^a) = \frac{cn_r^a}{\alpha + cn_r^a} = \frac{-\alpha F + \sqrt{\alpha F(4cL_r + \alpha F)}}{\alpha F + \sqrt{\alpha F(4cL_r + \alpha F)}} = \frac{-2\alpha F + X_r}{X_r}, \tag{41}$$

where $X_r \equiv \alpha F + \sqrt{\alpha F(4cL_r + \alpha F)}$. Second, evaluate g at n_r^a to get

$$g(n_r^a) = \mathrm{e}^{-\frac{\alpha}{c}\left(\frac{1}{n_r^a} - \frac{F}{L_r}\right)} = \mathrm{e}^{-\frac{2\alpha F}{X_r}}. \tag{42}$$

Let $Y_r \equiv (2\alpha F)/X_r < 1$ and $g(Y_r) = \mathrm{e}^{-Y_r}$. Note that (41) can then be expressed as $f(Y_r) = 1 - Y_r$, which is tangent to (42) at $Y_r = 0$:

$$1 - Y_r = g(0) + g'(0)(Y_r - 0).$$

Since (42) is strictly convex, it lies strictly above its tangent. Put differently, $f(Y_r) = 1 - Y_r < \mathrm{e}^{-Y_r} = g(Y_r)$ holds for all $Y_r > 0$ (see Figure 2). Hence, the right-hand side of (40) exceeds the left-hand side of (40) at the equilibrium mass of firms n_r^a. By uniqueness of the optimal mass of firms, and since the right-hand side of (40) exceeds the left-hand side if and only if $n_r > n_r^o$, we may conclude that $n_r^a > n_r^o$, which proves our claim (see Figure 3).

[**Insert Figures 2 and 3 around here**]

Appendix D: Equivalence of the 'centralized' and 'decentralized' first-best

We prove that the 'centralized' first-best is equivalent to the 'decentralized' first-best in the following sense: *If N^o is a centralized optimum, then $n_r^o = (L_r/L)N^o$ and $n_s^o = (L_s/L)N^o$ is a decentralized optimum. Conversely, if n_r^o and n_s^o is a decentralized optimum, then $N^o = n_r^o + n_s^o$ is a centralized optimum. The optimal utilities under both regimes are the same.*

Because of symmetry across varieties and from the labor market clearing conditions in each country, we get:

$$q_r = \frac{1}{c}\frac{L_r}{L}\left(\frac{1}{n_r} - \frac{F}{L_r}\right) \qquad \text{and} \qquad q_s = \frac{1}{c}\frac{L_s}{L}\left(\frac{1}{n_s} - \frac{F}{L_s}\right), \tag{43}$$

where q_r (resp., q_s) is the per capita quantity of a variety produced in r (resp., s) available for consumption in the integrated economy under free trade. Utility in each country is then given as follows:

$$U_r = U_s = n_r[1 - \mathrm{e}^{-\alpha q_r}] + n_s[1 - \mathrm{e}^{-\alpha q_s}]. \tag{44}$$

(i) 'Decentralized' solution $\Rightarrow$ 'centralized' solution: Let n_r^o and n_s^o denote the decentralized optimum. Differentiating (44) with respect to n_r and n_s, we obtain the first-order conditions

$$\mathrm{e}^{-\frac{\widetilde{\alpha}_r}{c}\left(\frac{1}{n_r^o}-\frac{F}{L_r}\right)} = \frac{cn_r^o}{\widetilde{\alpha}_r + cn_r^o} \qquad \text{and} \qquad \mathrm{e}^{-\frac{\widetilde{\alpha}_s}{c}\left(\frac{1}{n_s^o}-\frac{F}{L_s}\right)} = \frac{cn_s^o}{\widetilde{\alpha}_r + cn_s^o}, \tag{45}$$

where $\widetilde{\alpha}_r = \alpha(L_r/L) < \alpha$. From the labor market clearing condition in each country, we know that $n_r^o = L_r/\kappa_r$ and $n_s^o = L_s/\kappa_s$ for some $\kappa_r, \kappa_s > 0$. Hence, (45) reduce to

$$\mathrm{e}^{\frac{\alpha(F-\kappa_r)}{cL}} = \frac{cL}{\alpha\kappa_r + cL} \qquad \text{and} \qquad \mathrm{e}^{\frac{\alpha(F-\kappa_s)}{cL}} = \frac{cL}{\alpha\kappa_s + cL}, \tag{46}$$

thus showing that $\kappa_r = \kappa_s$. Denote by κ^o the common optimal solution to these conditions, so that $n_r^o = L_r/\kappa^o$ and $n_s^o = L_s/\kappa^o$. Let $N^o \equiv n_r^o + n_s^o$, which implies $N^o = L/\kappa^o$. It is readily verified that N^o satisfies the centralized first-order condition as follows:

$$\frac{cN^o}{\alpha + cN^o} = \frac{cL}{\alpha\kappa^o + cL} = \mathrm{e}^{\frac{\alpha(F-\kappa^o)}{cL}} = \mathrm{e}^{-\frac{\alpha}{c}\left(\frac{1}{N^o}-\frac{F}{L}\right)},$$

thus showing that N^o is the centralized optimum.

(ii) 'Centralized' solution $\Rightarrow$ 'decentralized' solution: Conversely, assume that N^o solves the centralized first-order condition:

$$\mathrm{e}^{-\frac{\alpha}{c}\left(\frac{1}{N^o}-\frac{F}{L}\right)} = \frac{cN^o}{\alpha + cN^o}. \tag{47}$$

Let $n_r^o = (L_r/L)N^o$ and $n_s^o = (L_s/L)N^o$, which implies $N^o = (L/L_r)n_r^o = (L/L_s)n_s^o$. Inserting these expressions into (47), we obtain (45), thus showing that n_r^o and n_s^o are the decentralized optimum.

Finally, inserting (45) into (44) one can check that

$$U(n_r^o, n_s^o) = \frac{\widetilde{\alpha}_r}{\frac{\widetilde{\alpha}_r}{n_r^o} + c} + \frac{\widetilde{\alpha}_s}{\frac{\widetilde{\alpha}_s}{n_s^o} + c} = \frac{\alpha(L_r/L)}{\frac{\alpha(L_r/L)}{(L_r/\kappa^o)} + c} + \frac{\alpha(L_s/L)}{\frac{\alpha(L_s/L)}{(L_s/\kappa^o)} + c} = \frac{\alpha}{\frac{\alpha}{N^o} + c} = U(N^o),$$

i.e., utilities under both regimes are the same.

Appendix E: Proof of Proposition 5

For any given value of L, the planner maximizes

$$U(N) = N\left[1 - \mathrm{e}^{-\frac{\alpha}{c}\left(\frac{1}{N}-\frac{F}{L}\right)}\right], \tag{48}$$

with respect to N to find the optimal mass of varieties N^o. To see that the equilibrium converges to the optimum as L increases, we may proceed as follows. First, using (48) and (27), it is readily verified that

$$\frac{\partial U}{\partial N}(N) = 1 - \mathrm{e}^{-\frac{2\alpha F}{X}} \frac{X}{X - 2\alpha F} = 1 - \mathrm{e}^{-Y} \frac{1}{1 - Y}, \qquad (49)$$

where $Y < 1$ and X are defined as in Appendix C. Expression (49) is always negative because of the tangency property as in Appendix C (which implies excess entry). Hence, by definition we have

$$\frac{\partial U}{\partial N}(N^o) = 0 > \frac{\partial U}{\partial N}(N),$$

i.e., the derivative at the equilibrium is bounded from above by the derivative at the optimum (see Figure 4). Re-deriving (49) with respect to L and evaluating the resulting expression at (27) some tedious, but standard, calculations shows that

$$\frac{\partial^2 U}{\partial N \partial L}(N(L), L) = \frac{4(\alpha F)^3 \left(2cL + \alpha F - D\right) e^{-\frac{2\alpha F}{\alpha F + D}}}{LD \left(D - \alpha F\right)^3} > 0, \qquad (50)$$

where we set $D = \sqrt{\alpha F(4cL + \alpha F)}$. Expression (50) monotonically converges to 0 as $L \to \infty$. We may therefore conclude that

$$\lim_{L \to \infty} \frac{\partial U}{\partial N}(N(L), L) = \frac{\partial U}{\partial N}(N^o) = 0,$$

which, by strict concavity of U (uniqueness of the optimum), then implies that

$$\lim_{L \to \infty} \left[U(N^o(L), L) - U(N(L), L)\right] = 0, \qquad (51)$$

hence establishing the result. The value $\lim_{L \to \infty} U(N^o(L), L) = \lim_{L \to \infty} U(N(L), L) = \alpha/c$ can then be obtained by noting that $U(N^o) = \alpha N^o/(\alpha + cN^o)$ and that N^o is strictly increasing in L, as established before.

[**Insert Figure 4 around here**]

Figure 1: Γ as a function of L

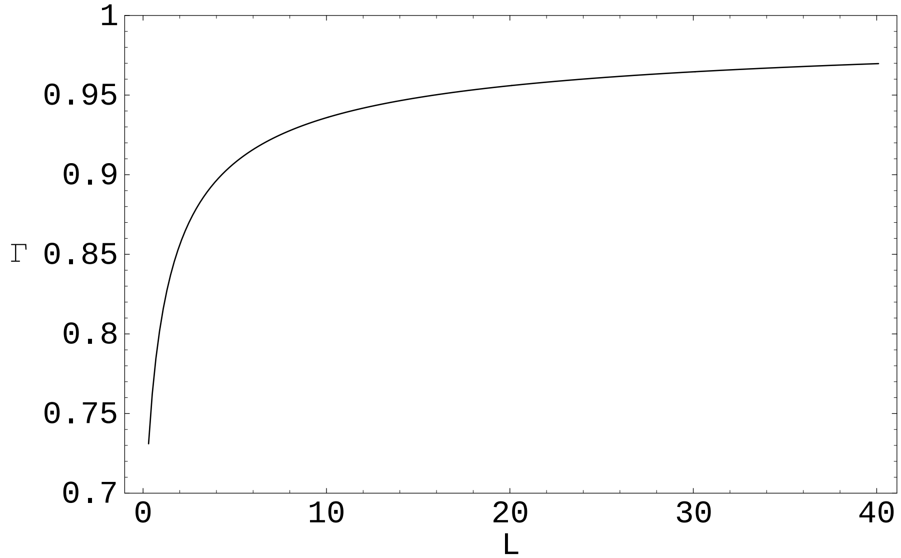

Figure 2: f and g as a function of Y_r.

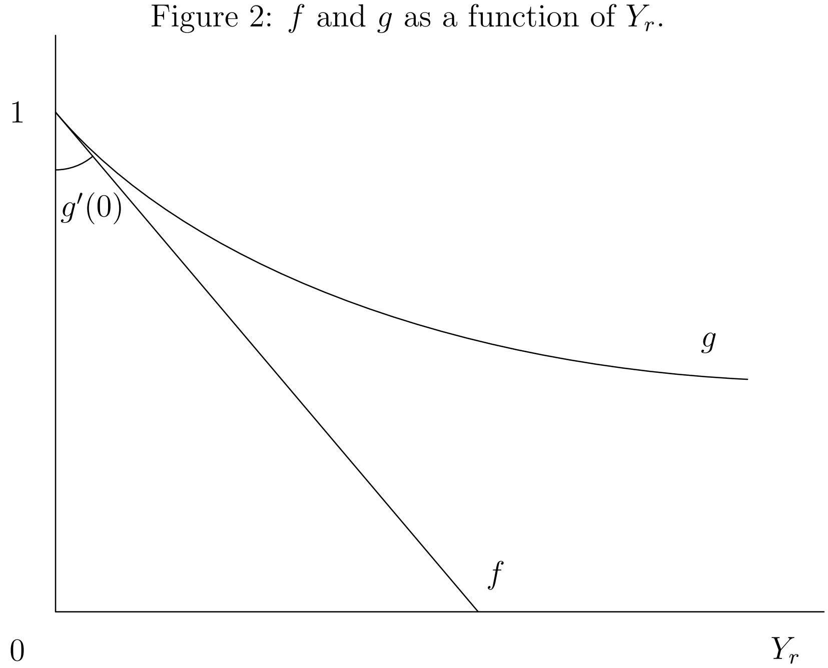

Figure 3: Excess entry.

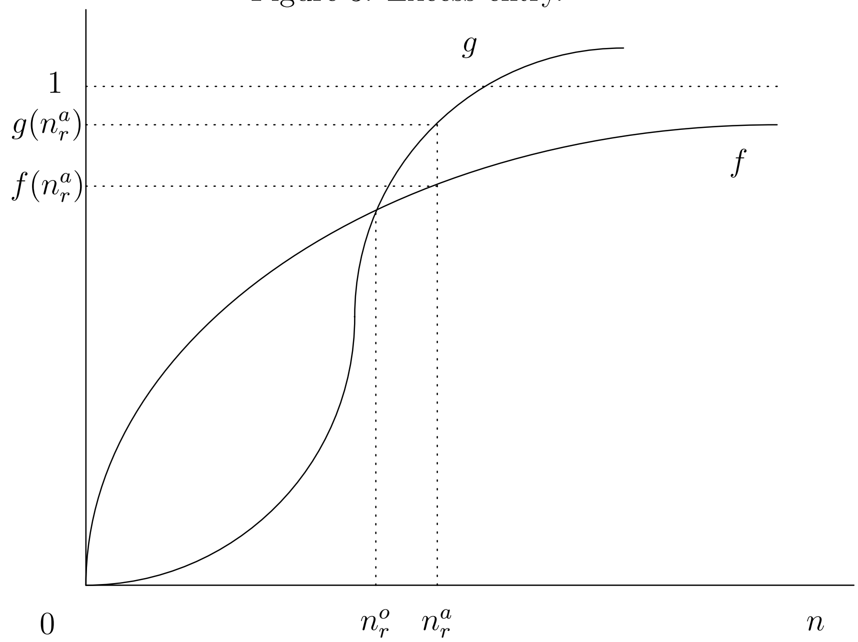

Figure 4: Equilibrium and optimal utility.