Family altruism with a renewable resource and population growth

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Abstract

We develop an overlapping-generation model *à la* Diamond with a non-constant population growth in which households privately own a natural renewable resource and have a family-altruism resource bequest motive. The natural resource can be either extracted and sold to the producing firms as a production factor, or bequeathed to the offspring to increase his adult disposable income. With a numerical application, we analyze how family altruism interplays with population growth to shape the dynamics of the whole economy. We also highlight the role of altruism in the case of a temporary negative demographic shock. The simulations we present show that a fall in the size of families increases the family natural resource stock but reduces resource extraction on the transition, through a reinforcement of family altruistic links. Hence, family altruism plays a key role in the recovery of the economy after the shock.

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1 Introduction

In the literature on sustainability with overlapping-generations models, most of the time the population size is assumed to be constant and households selfish. We consider in this paper that the population has its own dynamics and the households have a taste to bequeath a renewable resource to their heirs out of family altruism. Our purpose is to study how these features influence the pressure on the stock of a renewable natural resource and how they shape the dynamics of the whole economy,both on the transition path and at the steady state. We also analyze the contribution of family altruism in response to a negative demographic shock.

In the literature on intergenerational transfers (e.g. Barro (1974) and Becker (1991)), parents feel concerned about their children through altruistic links. These links are operative when parents make positive transfers of the *numeraire* good to their offspring in order to iron out shocks in their relative well-being. Barro (1974) has formulated a version of the altruistic hypothesis in which altruistic households solve a problem formally equivalent to the infinitely-lived representative agent. When applied to the study of the effects of public debt this so-called *dynastic* altruistic model concludes that households are able to offset any policy aiming at redistributing income between generations through public debt. Barro's (1974) paper was a revival of the Ricardian equivalence argument and gave rise to a large debate on the intergenerational effect of public policies.

One among the many criticisms addressed to the dynastic model is that it assumes that the sequence of individuals of one dynasty not only are willing to behave as one decision unit but also have the capacity to foresee the indefinite future, *i.e.* the whole future paths of prices and incomes. If this model is used to study how a decentralized competitive economy manages a renewable resource, it is no surprise that the sustainability issue is entirely solved. Households fully take into account the consequences of their decisions on the whole future and no threat is put on the sustainability of the growth process.

Our motivation is to study the interplay between population growth and the use of a renewable resource with an alternative approach of altruism: the *family altruism* hypothesis. The idea of family altruism has been developed recently in two papers. Lambrecht *et al.* (2005) use it to study how pay-as-you-go pensions can foster growth in a model with human and physical capital. Lambrecht *et al.* (2006) discuss the implication of the family altruism hypothesis in the public debt policy debate. The main assumption of the family model, as opposed to the dynastic model, is that the decision unit in which intergenerational links are operative is the family, as opposed to the dynasty. In our model, the children's adult disposable income enters the utility function of each family head and the natural resource stock, privately owned, constitutes the means of the bequest.

The paper is organized as follows. The next section presents the model, *i.e.* the dynamics of the population and renewable resource, the family-altruism bequest motive and the individuals' and firms' behavior. A special attention will be paid on the family altruism bequest motive and the way it interferes with the population dynamics. The competitive temporary and intertemporal equilibrium are defined and characterized in Section 3. Section 4 presents a numerical application of the model. A reference scenario will help us to highlight the main dynamic properties of the model, and we shall also study the impacts of a temporary negative demographic shock. Lastly, Section 5 concludes.

2 The model

We model an overlapping-generation (OLG) economy *à la* Diamond (1965). We extend this basic model in two respects. First we assume that individuals leave bequests of natural resource to their offspring out of the family altruism bequest motive (Lambrecht *et al.* (2005), Lambrecht *et al.* (2006)) and second we assume that the size of each generation has its own dynamics along which the growth rate is changing over time.

2.1 The family altruism bequest motive and the fertility factor

We first present the dynamics of the size of generations and then the concept of family altruism applied to the bequests of natural resource.

Let N_t denote the number of young individuals at time t. The generations' size dynamics are governed by the following equation:

$$N_{t+1} = BN_t^{\nu}, \tag{1}$$

with $\nu \in (0,1)$, $B > 0$ a scale factor and N_{-1} the exogenously-given number of old individuals in the initial period. Whatever the initial N_{-1}, the steady state size of each generations is given by: $N = B^{1/(1-\nu)}$. Consider any time period t on the transition. N_t is the number of young individuals. The number of children who will be young adults at time $t+1$ is denoted by N_{t+1}. We label the ratio N_{t+1}/N_t as the fertility factor. On the transition of the generations' size dynamics, the fertility factor changes and converges to unity. This is at odd with the standard OLG model *à la* Diamond (1965) in which the ratio between the size of a young and an old generation, N_{t+1}/N_t, remains constant. It is often denoted by the factor $1+n$. In line with this notation we will denote our fertility factor by $1 + n_{t+1}$. It is easy to write this fertility factor as a function of the size of the old time t generation. This yields:

$$1 + n_{t+1} \equiv \frac{N_{t+1}}{N_t} = BN_t^{\nu-1}. \tag{2}$$

Let the initial generation's size N_{-1} be less (resp. greater) than the steady state size N. The convergence towards N is monotonically increasing (resp. decreasing). As far as the fertility factor $1 + n_{t+1}$ is concerned, it follows a decreasing (resp. increasing) path toward unity as the generation's size increases (resp.decreases). Since all the households are homogenous, the factor $1 + n_{t+1}$ is also the number of children in each household.

Let us now describe how we apply the concept of family altruism to our model with bequests of resource. We define the family as a decision unit which survives for two periods. It is composed of a family head, namely an individual over his life cycle, and his $1 + n_{t+1}$ children during the first period of their life cycle (adulthood). As a result, each individual is a member of the family started by his parent one period before and starts his own family when he is young. His own family lives for two periods. Altruism is assumed to be descendant, i.e. parents care about their children but not the reverse.

The difference between a typical *household* of the Diamond's (1965) model and a *family* in Lambrecht *et al.*'s (2006) model is the following. Families are actually equivalent to Diamond's (1965) households plus the next households during their adulthood[1].

The preferences of a family head are defined over his life cycle consumption, c_t and d_{t+1}, and over their $1 + n_{t+1}$ children's adult disposable income ω_{t+1}[2]. With such preferences, the sequence of altruistic descendants of the same time $t = 0$ founding father, does not behave as a single dynasty and there is no need to foresee the indefinite future.

In this paper we use the family altruism model. But we need to extend it to our framework. First of all, the fertility factor $1 + n_{t+1}$ and, hence the size of families $2 + n_{t+1}$, changes over time. The generation's size dynamics is increasing and concave which has a very simple implication. If the size of generations increases toward the steady state, the family size decreases[3] and, as times goes by and generations follow each other, the family heads care about the adult disposable incomes of less and less children[4]. The population dynamics thus introduces a trend in the utility function. This feature will explain some of our results.

We assume the utility function to be additively separable:

$$U_t = (1 - \beta)u_1(c_t) + \beta u_2(d^e_{t+1}) + (1 + n_{t+1})\gamma u_3(\omega^e_{t+1}), \tag{3}$$

where d^e_{t+1} and ω^e_{t+1} are respectively the expected second-period consumption and the expected adult disposable income of each of the $1 + n_{t+1}$ children.

The other extension to the standard family altruism model concerns the expectations formed by the family head on the adult disposable incomes ω^e_{t+1}. To understand this, we need to present the sources of revenues of a young individual. Each young individual works and extracts a renewable resource in the first period of his life. More precisely, he supplies to firms (i) one unit of labor inelastically on the labor market, for a real wage w_t, and (ii) the

[1]During childhood, individuals make no decision.

[2]In the dynastic model, preferences are defined over consumptions and the children's utility, which is formally equivalent to the infinite sum of utilities defined over the whole sequence of consumptions of all generations.

[3]In the standard family altruism model with constant population growth, the family size remains constant like the fertility factor.

[4]The reverse is true for a decreasing population.

quantity e_t of extracted resource on the resource market, for a real price q_t. Each time t family head has to form expectations about the real wage and resource price at time $t+1$, namely he has to try to decide about the value of, respectively, w^e_{t+1} and q^e_{t+1}. Moreover he has to form expectation about the extraction behavior if his offspring, e^e_{t+1}. As a result the expected adult disposable income of a young individual, as anticipated by a time t family head, is the following:

$$w^e_{t+1} + q^e_{t+1} e^e_{t+1} = \omega^e_{t+1}. \tag{4}$$

2.2 The renewable resource dynamics

We assume that there exists a renewable resource in private property. At any time t, each family head inherits a share z_{t-1} of the family resource stock. This individual stock has its own natural return, which yields Cz^{ζ}_{t-1}, with $C > 0$ a scale factor, to each family heads. In the absence of extraction this stock Cz^{ζ}_{t-1} is shared among the $1 + n_{t+1}$ children. Thus the dynamics of the families' resource stock without extraction writes as follows:

$$z_t = \frac{Cz^{\zeta}_{t-1}}{1 + n_{t+1}}, \tag{5}$$

with $\zeta \in (0, 1)$. Without extraction, the family head's resource stock converges to a steady state equal to[5] $z = C^{1/(1-\zeta)}$.

2.3 The individuals' problem

We now characterize the behavior of the family heads. We make the assumption that their utility function is of the log-linear type:

$$U_t = (1 - \beta) \log c_t + \beta \log d^e_{t+1} + (1 + n_{t+1})\gamma \log \omega^e_{t+1}. \tag{6}$$

As we already explained, the first period income of a young family head is $\omega_t = w_t + q_t e_t$. This first-period income is shared between consumption c_t and saving s_t. This is summarized by the first-period budget constraint:

$$w_t + q_t e_t = c_t + s_t. \tag{7}$$

[5]Indeed $1 + n_{t+1}$ tends to unity.

The amount of resource which has not been extracted, i.e. $Cz_{t-1}^{\zeta} - e_t$, is bequeathed equally to the $1 + n_{t+1}$ children by the family head. This means that the dynamics of the families' resource stock with extraction is given by:

$$z_t = \frac{Cz_{t-1}^{\zeta} - e_t}{1 + n_{t+1}}. \tag{8}$$

When old, individuals hold the firms' capital stock through their savings and earn a capital income which they entirely consume. As anticipated from period t, this summarized by :

$$R_{t+1}^e s_t = d_{t+1}^e, \tag{9}$$

where R_{t+1}^e is the expected interest factor on saving s_t, i.e. one plus the expected interest factor r_{t+1}^e, and d_{t+1}^e is the expected old-age consumption.

As explained before young family heads form expectations to evaluate the adult disposable income of their offspring. This is given by equation (4). They can sustain their offspring's adult disposable income by increasing their resource bequests[6]. Indeed equation (4) can be re-written as follows :

$$w_{t+1}^e + q_{t+1}^e \left[\left(\frac{Cz_{t-1}^{\zeta} - e_t}{1 + n_{t+1}} \right)^{\zeta} - (1 + n_{t+2}) z_{t+1}^e \right] = \omega_{t+1}^e. \tag{10}$$

Family heads maximize their utility (6) under the constraints (7), (9) and (10) and taking prices and expectations as given. The solution to this problem can be characterized by studying the saving and the resource extraction decisions, i.e. by studying the following problem obtained after substitution:

$$\begin{aligned} &\max_{s_t, e_t} (1-\beta) \log(w_t + q_t e_t - s_t) + \beta \log(R_{t+1}^e s_t) \\ &+ (1 + n_{t+1}) \gamma \log \left(w_{t+1}^e + q_{t+1}^e \left[\left(\frac{Cz_{t-1}^{\zeta} - e_t}{1 + n_{t+1}} \right)^{\zeta} - (1 + n_{t+2}) z_{t+1}^e \right] \right). \end{aligned} \tag{11}$$

The first-order conditions are:

$$\frac{1-\beta}{w_t + q_t e_t - s_t} = \frac{\beta}{s_t}, \tag{12}$$

$$\frac{(1-\beta) q_t}{w_t + q_t e_t - s_t} \leq \frac{\gamma q_{t+1}^e \zeta \left(\frac{Cz_{t-1}^{\zeta} - e_t}{1 + n_{t+1}} \right)^{\zeta - 1}}{w_{t+1}^e + q_{t+1}^e \left[\left(\frac{Cz_{t-1}^{\zeta} - e_t}{1 + n_{t+1}} \right)^{\zeta} - (1 + n_{t+2}) z_{t+1}^e \right]}. \tag{13}$$

[6] In this paper we rule out bequest of the *numeraire* like in the most of altruistic models

The last condition holds with equality if extraction is positive. Instead it holds with inequality when optimal extraction is zero, i.e. when, at zero extraction, the marginal benefit from extraction in terms of consumption c_t is larger then the marginal loss in terms of the offspring's expected adult disposable income ω^e_{t+1}. In the sequel we focus on the case of optimal positive extraction, i.e. the case when, at zero extraction, the marginal benefit of extraction is smaller than the marginal gain. Savings can be written as a function of extraction e_t:

$$s_t = \beta(w_t + q_t e_t). \tag{14}$$

and the second condition with equality can be re-written as:

$$(1-\beta)q_t\left[w^e_{t+1} + q^e_{t+1}\left[\left(\frac{Cz^{\zeta}_{t-1} - e_t}{1+n_{t+1}}\right)^{\zeta} - (1+n_{t+2})z^e_{t+1}\right]\right] \\ - \gamma q^e_{t+1}\zeta\left(\frac{Cz^{\zeta}_{t-1} - e_t}{1+n_{t+1}}\right)^{\zeta-1}(w_t + q_t e_t - s_t) = 0. \tag{15}$$

Thus we have a system of two equations in the variables s_t and e_t

It easy to shed light on the family head decision problem by building the family income and the family intertemporal budget constraint. In the Diamons's (1965)model, the life cycle income and budget constraint are built by adding up the incomes and expenditures of the whole life cycle in present value. In the family model, we simply add up all the incomes and expenditures of the life cycle plus the definition of the adult disposable incomes of the $1 + n_{t+1}$ children, in present value, i.e. we add the present value of equations (7), (9) and (4) times $(1 + n_{t+1})$. Denote the family intertemporal income by Ω_t, we have that:

$$\Omega_t \equiv w_t + q_t e_t + \frac{1+n_{t+1}}{R^e_{t+1}}(w^e_{t+1} + q^e_{t+1}e^e_{t+1}) = c_t + \frac{d^e_{t+1}}{R^e_{t+1}} + \frac{1+n_{t+1}}{R^e_{t+1}}\omega^e_{t+1}. \tag{16}$$

This family budget displays in the RHS the three utility elements over which preferences are defined. Those are the three items of expenditures of the family head. Any increase in the family income Ω_t is spent over these three items. The buffer used by the family heads to transfer incomes from the c_t to d^e_{t+1} is saving s_t and the one used to transfer income from c_t to ω^e_{t+1} is resource bequest z_t. We shall see later in the simulation how the family income evolves across time and which results can be deduced from this feature.

2.4 The firms' problem

The representative firm produces the output Y_t by combining three production factors capital K_t, labor L_t and extracted resource E_t with a Cobb-Douglas technology:

$$Y_t = AK_t^{\alpha_K} L_t^{\alpha_L} E_t^{\alpha_E}. \tag{17}$$

Considering factor prices as given, namely the real interest factor R_t, the real wage w_t and the real resource price q_t, the representative firm maximizes its profit in real terms π_t by choosing its demands of capital, labor and resource. We define the real profit as follows:

$$\pi_t = AK_t^{\alpha_K} L_t^{\alpha_L} E_t^{\alpha_E} - R_t K_t - w_t L_t - q_t E_t. \tag{18}$$

The first-order conditions are given by:

$$q_t = \alpha_E A \left(\frac{K_t}{L_t}\right)^{\alpha_K} \left(\frac{E_t}{L_t}\right)^{\alpha_E - 1}; \tag{19}$$

$$R_t = \alpha_K A \left(\frac{K_t}{L_t}\right)^{\alpha_K - 1} \left(\frac{E_t}{L_t}\right)^{\alpha_E}; \tag{20}$$

$$w_t = \alpha_L A \left(\frac{K_t}{L_t}\right)^{\alpha_K} \left(\frac{E_t}{L_t}\right)^{\alpha_E}. \tag{21}$$

The firm hires the services of capital, labor and resource up to the point where their respective marginal productivities equal their respective price.

3 The competitive equilibrium

We first analyze the temporary equilibrium of period t and then the intertemporal equilibrium.

3.1 The time t temporary equilibrium

We now turn to the definition and characterization of the time t temporary equilibrium. At any time period t, we consider the following variables as given:

- the aggregate capital stock K_t, which depends on past saving decisions ($K_t = N_{t-1}s_{t-1}$);

- the family inherited resource stock z_{t-1}, which depends on past extraction decision and past family resource bequest ($z_{t-1} = (Cz_{t-2}^{\zeta} - e_{t-1})/(1+n_{t+1})$);
- the young generation size N_t, which is follows from the population dynamics ($N_t = BN_{t-1}^{\nu}$);
- the expectations on the next period:
  - real wage w_{t+1}^e,
  - resource prices q_{t+1}^e,
  - bequest of resource z_{t+1}^e.

For all t, we define the temporary time t equilibrium as:

- a vector of prices R_t, w_t, q_t,
- individual quantities c_t, s_t, e_t, z_t, d_t,
- aggregate quantities $Y_t, K_t, L_t, E_t, N_{t+1}$,

such that

- all agents, families and firm, maximize their objective function subject to their constraints,
- all markets, i.e. output, capital, labor and resource, clear.

We characterize the time t equilibrium values of the above endogenous variables as a function of the variables considered as given, $\{K_t, z_{t-1}, N_t, w_{t+1}^e, q_{t+1}^e, z_{t+1}^e\}$. First, the conditions of equality between supply and demand of, respectively, labor, capital and resource are given by:

- $N_t = L_t$ (exogenous labor supply);
- $K_{t+1} = N_t s_t$;
- $N_t e_t = E_t$.

This implies that the equilibrium prices are given by:

- $R_t = R(\frac{K_t}{N_t}, \frac{E_t}{N_t}) \equiv \alpha_K A \left(\frac{K_t}{N_t}\right)^{\alpha_K - 1} \left(\frac{E_t}{N_t}\right)^{\alpha_E}$;
- $q_t = q(\frac{K_t}{N_t}, \frac{E_t}{N_t}) \equiv \alpha_E A \left(\frac{K_t}{N_t}\right)^{\alpha_K} \left(\frac{E_t}{N_t}\right)^{\alpha_E - 1}$;

- $w_t = w(\frac{K_t}{N_t}, \frac{E_t}{N_t}) \equiv \alpha_L A \left(\frac{K_t}{N_t}\right)^{\alpha_K} \left(\frac{E_t}{N_t}\right)^{\alpha_E}$.

Let $k_t = K_t/N_t$ and $e_t = E_t/N_t$. In equilibrium, we thus write the system of two equations in saving s_t and extraction e_t by replacing prices w_t and q_t by their equilibrium expressions $w(k_t, e_t)$ and $q(k_t, e_t)$. The solutions of this system of equations can be written as functions of $\{K_t, z_{t-1}, N_t, w^e_{t+1}, q^e_{t+1}, z^e_{t+1}\}$. The other individual variables in equilibrium are thus easily obtained by using the families constraints. As far as aggregate variables are concerned, we can also write them as functions of $\{K_t, z_{t-1}, N_t, w^e_{t+1}, q^e_{t+1}, z^e_{t+1}\}$.

3.2 The competitive intertemporal equilibrium

We now turn to the characterization of the competitive intertemporal equilibrium. We define the competitive intertemporal equilibrium as a sequence of temporary equilibria, given the initial conditions $\{K_0,\ N_{-1},\ z_{-1}\}$ and a rule for the formation of expectations on w^e_{t+1}, q^e_{t+1} and z^e_{t+1}.

At this stage it is important to stress the following point. Under the assumption of perfect foresight, family heads would be considered as able to foresee the entire sequence of prices. Indeed, perfect foresight would imply $w^e_{t+1} = w(k_{t+1}, e_{t+1})$ and $q^e_{t+1} = q(k_{t+1}, e_{t+1})$. In other words, family heads at time t would have to compute the next period extraction behavior of their offspring. *In se* this is could be fairly well hypothesized. Indeed, his offspring extraction decision is contemporaneous of his second-period of life.

The problem is that the offspring extraction decision in turn will depends on the offspring's expectations about their own children's decision, and so on. To be consistent with our hypothesis that family heads organize their resource bequests decision in a finite entity, i.e.the family unit, we make the assumption that family heads expect their offspring to extract the same amount as themselves. Hence:

$$w^e_{t+1} = \alpha_L A k_t^{\alpha_K} e_t^{\alpha_E}; \tag{22}$$

$$q^e_{t+1} = \alpha_E A k_t^{\alpha_K} e_t^{\alpha_E - 1}; \tag{23}$$

$$z^e_{t+1} = z_t. \tag{24}$$

This assumption is known as the *myopic* expectations hypothesis.

Given the initial condition K_0, N_{-1}, z_{-1} and our rule of expectations, the competitive intertemporal equilibrium with myopic foresight is characterized a sequence $\{k_{t+1}, e_t, z_t\}_{t=0}^{+\infty}$ which verifies the following system of equations:

$$(1 + n_{t+1})k_{t+1} - \beta(1 - \alpha_K)Ak_t^{\alpha_K} e_t^{\alpha_E} = 0, \tag{25}$$

$$(1-\beta)\alpha_E Ak_t^{\alpha_K} e_t^{\alpha_E - 1}\left(\alpha_L Ak_t^{\alpha_K} e_t^{\alpha_E} + \alpha_E Ak_t^{\alpha_K} e_t^{\alpha_E - 1}\left[\left(\frac{Cz_{t-1}^{\zeta} - e_t}{1 + n_{t+1}}\right)^{\zeta} - (1+n_{t+2})z_t\right]\right)$$
$$-\gamma\alpha_E Ak_t^{\alpha_K} e_t^{\alpha_E - 1}\zeta\left(\frac{Cz_{t-1}^{\zeta} - e_t}{1 + n_{t+1}}\right)^{\zeta - 1}[(1-\alpha_K)Ak_t^{\alpha_K} e_t^{\alpha_E} - (1+n_{t+1})k_{t+1}] = 0 \tag{26}$$

$$z_t = \frac{Cz_{t-1}^{\zeta} - e_t}{1 + n_{t+1}} \tag{27}$$

4 Numerical application

This section gathers the computational application of this model and applications. We first present the parameters values and computation issues. Then we analyze a reference scenario and the impacts of a negative demographic shock.

4.1 Parameters value and computation

The computational version of the model depicts the temporary dynamic equilibria of the economy. It consists of :

- two pre-determined variables: labor supply (N_t) and demand (L_t),
- six simultaneous equations: resource extraction (e_t), bequeathed resource stock (z_t), savings (s_t) and real resource price (q_t), interest factor (R_t) and wage rate (w_t),
- and a set of post-determined variables and identities giving, *e.g.*, the utility level, aggregates variables, etc.
- three expected variables of the next period: wage (w_{t+1}^e), resource price (q_{t+1}^e) and bequeathed resource stock (z_{t+1}^e).

Initial conditions are K_0, N_{-1} and z_{-1}. Since, in addition to childhood, individuals live for two periods, one period of time represents roughly 25 years. The model runs over a 15-period time span. The implicit equation giving the level of individual extraction is solved with the Newton-Raphson algorithm and the whole model is solved with the Gauss-Seidel algorithm[7].

4.2 A reference scenario

The table 1 displays the parameters value of the reference scenario. Most of these values come from conventional practice and do not require further comments. We set up the parameters ν and ζ of the population and resource dynamics such that (i) the population steady state is reached within 15 periods of time, (ii) the family resource stock without extraction increases over time. Scale parameters have been used for the population and resource own dynamics such that their level at the steady state is higher than one. The time profile of the main variables is displayed in the set of Figures 1.a to 1.f (thick lines).

Table 1 Parameters value

α_k	share of capital in output	0.30
α_l	share of labour in output	0.60
α_e	share of natural resource in output	0.10
β	weight of old-age consumption in utility function	0.25
γ	degree of family altruism	1.50
ν	population own's dynamics	0.65
ζ	natural resource own's dynamics	0.65

< insert figure 1 >

The model converges to a steady state. Given the initial conditions and parameters value, capital per head increases and so does the family income Ω_t of equation (16), along the transition path. Population, aggregate natural resource and aggregate capital stocks also increase over time. Yet, some special features deserve more attention.

[7] The model runs under the integrated software IODE developed by the Belgian Federal Planning Bureau and publicly available at www.plan.be

Firstly, one can observe (Fig. 1.b) that the family natural resource stock is decreasing on the transition path. This simply results from the fact that, considering the initial conditions and parameters values, the demographic (population growth) and economic (extraction at equilibrium) pressures temporarily dominate the natural resource own's dynamics.

Secondly, Fig.1.e shows that the utility level is decreasing on the transition path. This originates from the combination of four effects, two positive and two negative. The two positive effects regard consumption when young and adult disposable income of the offspring (ω_{t+1}). They increase over time. These two positive effect are more than offset by the negative effects. The first one concerns consumption when old (d_{t+1}). It decreases, which can be easily understood considering that,

$$d_{t+1} = R_{t+1}s_t = [1 + n_{t+1}] \left[\alpha_k A k_{t+1}^{\alpha_k} e_{t+1}^{\alpha_e}\right]$$

where the first term in brackets is decreasing over time while the second term increases at a decreasing rate. Of course, consumption when old could grow for another set of parameters value or initial conditions[8]. The second negative effect concerns the fertility factor and, consequently the utility weight of the offspring adult disposable income, $(1 + n_{t+1})\gamma$. It is decreasing over time, as explained in section 2.1 and shown is Fig.1.f.

Thirdly, and finally, the role of family altruism. Altruism is the only motive for households not to extract and sell the whole resource stock. So it is straightforward that, if the degree of altruism is too low, then the natural resource may collapse, entailing the collapse of the whole economy. In a slightly different setting[9], Bréchet and Lambrecht (2006) formally demonstrate the possibility of such a result. In this paper with family altruism, numerically, given the parameters value, the lowest value of γ compatible with a positive resource stock is 1.3.

Furthermore, the higher the value of γ, the higher both the resource stock and extraction at the steady state. And so is capital intensity. As a consequence, family income also increases with γ, and so does the utility level at

[8]Let us note that, consumptions over the life cycle, $c_t + d_{t+1}/(R_{t+1})$, is nevertheless strictly increasing over time in this simulation.

[9]An OLG model with a *joy-of-giving* bequest motive.

the steady state. This is not in contradiction with the fact that the ratio of extraction over bequest, e/z, is decreasing with γ.

4.3 The effects of a negative transitory demographic shock

We now focus on the impacts of a negative transitory demographic shock. Let x_t and $\widetilde{x}_t$ denote the level of a variable without and with the shock, respectively. The shock consists in a one-third drop in N_3, the size of the young generation at time $t = 3$. The shock is unexpected and does not affect the low of motion of the size of generations. Put differently, the population own's dynamics remains untouched but undergoes the following one-period exogenous shock:

$$\widetilde{N}_3 < N_3 \tag{28}$$

In period 4, we have:

$$\widetilde{N}_3 < \widetilde{N}_4 < N_4 \tag{29}$$

Since the fertility factor of the time $t = 3$ generation is given by $\widetilde{N}_3^{\nu-1}$, the shock leads to a temporary jump

$$1 + \widetilde{n}_4 > 1 + n_4 \tag{30}$$

Afterward the fertility factor goes back gradually to its steady state value.

The impacts are shown in Fig.1.a to 1.f. (thin lines).

Because the number of children dropped with the shock, each child of generation young in $t = 3$ inherits a higher resource stock. What will be the arbitrage of these young family heads at time $t = 3$ between their current consumption ($\widetilde{c}_3$), their consumption when old ($\widetilde{d}_4$) and the income of its heirs ($\widetilde{\omega}_4$)? Given that all these are normal goods, young individuals will increase all three. They do so by increasing savings $\widetilde{s}_3$ and the resource bequest $\widetilde{z}_3$, with respect to their level without shock. The size of the families at time $t = 3$ becomes $(1 + \widetilde{n}_4) > (1 + n_4)$, so the total family bequest is higher than it would have been without shock, $(1 + \widetilde{n}_4)\, \widetilde{z}_3 > (1 + n_4)\, z_3$.

As for extraction ($\widetilde{e}_3$) is concerned, the theoretical impact is ambiguous. There are two opposite effects. The first one is that a higher inherited stock allows a higher extraction and that the induced higher real wage and resource price foster equilibrium extraction. The second one is more complex. The argument runs as follows. The negative demographic shock leads to a higher fertility rate $\widetilde{n}_4$. As a result, the RHS of equation (13) (with equality) increases (through higher n_{t+1}), which means that the marginal cost of extraction increases. Family heads react to this shock by a lower extraction $\widetilde{e}_3$ ($< e_3$). Indeed decreasing extraction e_t reduces the RHS of (13) and increases its LHS. For the chosen parameter values this negative effect dominates the two positive effects. All in all, the shock temporarily rises the utility level.

A standard comparative statics exercise in models with altruism is to look at the effect of a rise in the degree γ of altruism. So let us now compare the effects of the negative demographic shock with a higher value of gamma. The Fig.2 shows the impact of the shock on utility expressed in % with respect to the reference simulation[10]. It clearly appears that the higher γ, the higher the positive impact of the shock on the utility level.

< insert figure 2 >

4.4 Conclusion

In this paper we develop a OLG model in which the size of generations evolves across time and converges to a steady state. A private natural renewable resource is both extracted and bequeathed out the family altruism bequest motive.

We study how these two features influence the pressure on the renewable resource and on the equilibrium path of the economy. We highlight the role of altruism in the case of a temporary negative demographic shock. The simulation we present shows that a fall in the size of families increases the family natural resource stock but reduces resource extraction on the transition, through a reinforcement of family altruistic links. Hence, family altruism plays a key role in the recovery of the economy after the shock.

[10]Naturally, each value of γ is associated with a specific reference path.

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