Puzzling over sustainability: an equilibrium analysis

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Abstract

In this paper we model an overlapping generations economy in which individuals are endowed with a renewable resource. This resource can be exploited at no cost by the young households and provided to production. A joy-of-giving bequest motive motivates the transfer of the unexploited resource to the heirs. The study of intertemporal equilibrium reveals three puzzles neglected by the literature on sustainability. First, the existence of a bequest motive does not automatically guarantee a sustainable future. Second, human exploitation may preserve the resource in equilibrium but at a sub-optimal rate; in this case, both those who exploit too much and those who do not exploit enough should run a capital stock lower than the golden rule level. Third, there exist fluctuating transitions to a sustainable future in which some generations are worse off than both their ascendants and their descendants.

Keywords: overlapping generations, sustainability, altruism, natural resource

JEL codes: D91; Q20; D64

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1 Introduction

Nowadays, beyond the buzzword, sustainability does constitute the cornerstone for most policy issues. While being a subject of ideological or political debates, sustainability has been largely disregarded by economic theory, as Dasgupta and Mäler (1995) point out: "most writings on sustainable development start from scratch and some proceeds to get things hopelessly wrong. It would be difficult to find another field of research endeavor in the social sciences that has displayed such intellectual regress". Recently, however, an increasing number of theoretical economists have paid attention to this question [1]. The motivation of this paper was inspired by Solow (1991) who stressed out that defining "sustainability as an obligation to leave the world as we found it in detail is glib but essentially unfeasible" (p. 132). Our own perspective is that leaving the world untouched is unfeasible but, above all, glib as an economic concept. The definition from the Brundtland report is probably the best illustration of this issue: "sustainable development is a development that meets the needs of the present without compromising the ability of future generations to meet their own needs" (WCED, 1987, p. 43). Unambiguously, room is left for many kinds of development paths, *i.e.* there does not exist one and only one sustainability path and any generation, in particular ours, should be authorized to choose its preferred path to the sustainable future. The problem is not to leave our world untouched but to let the forthcoming generations the opportunity to satisfy their own needs, whatever these needs may be. So, a *necessary condition* for sustainability would be to let the opportunity for future generations to make their own choices.

We model an overlapping generations (OLG) economy in which individuals are privately-endowed with a renewable resource. This resource can be extracted at no cost by the young households and provided to production as a source of revenue. However, a joy-of-giving bequest motive motivates the transfer of the unexploited resource to the heirs so as to let them the opportunity to raise their own revenues from the resource. The exploited resource is combined with man-made capital and labor to produce a consumption/investment good. The issues of substitution between the forms

[1] See for example Pezzey (2004), Asheim *et al.* (2003), Arrow *et al.* (2003), Stavins *et al.* (2003), Pezzey and Withagen (1998), Howarth and Norgaard (1992) or Solow (1991).

of capital is addressed, as well as the issue of selfishness *versus* altruism and their implications on the opportunity sets left to future generations. This paper contributes both to the literature on sustainability and to the literature on OLG models with renewable resource.

On the one hand, the contribution to the latter is threefold. First, we consider a production function encompassing labor, natural capital (a renewable resource) and man-made capital, the three markets being fully characterized. In the literature, generally, the production side of the economy is simplified or even left outside (Amacher *et al.* (1999)). To our knowledge there is no paper where the renewable resource is required within the production process along with capital. Generally, only one stock of capital is considered, either the man-made capital or the natural capital (Koskela *et al.* (2002), Gerlagh and Keyser (2001)). Second, we play attention both to the dynamic paths and steady states of the economy whereas most of the papers in the literature only focus on one of them. Most papers devoted to renewable resources in overlapping economies just look at dynamic efficiency of the steady state (*e.g.* Mourmouras (1993), Koskela *et al.* (2002) or Gerlagh and Keyser (2003)). The occurrence of cycles in the presence of wealth effects was first considered by Kurz (1968) and then by Levhari *et al.* (1981) in the context of renewable resources. Smeegmuller and Verchere (2004) point out the implications of such endogenous oscillations in terms of intergenerational equity. We show that the convergence towards the steady state is not necessarily monotonic and that limit cycle may even occur. Third, despite the importance of bequests there are few studies that incorporate them into analysis. Löfgren (1991) pointed out that bequests allow to reconciliate economists and forestry experts. Hultkrantz (1992) examines the implications of bequests in an OLG economy with forest and timber bequest occurring in the form of unharvested forest stock. Amacher *et al.* (1999) introduce a more complex forest management with harvesting and silviculture investment. The representative landowner derives utility both from consuming and leaving a timber bequest for the next generation, the indirect utility of the next generation entering its own utility function. In all these papers (see also Jouvet *et al.*, 2000), however, bequest motives always rely on the assumption of altruism à la Barro (1974). As Becker (1993) himself admits, this form of intergenerational concern requires human capacities that are beyond the capacities of the most prescient. In our paper we considered an alternative form of altruism inside the family based on Andreoni's (1989) warm glow.

Concerning the contribution of the paper to the literature on sustainability, we intentionally exclude any policy device aiming at internalizing the intergenerational externalities, for example trust funds as in Howarth (1997) or Gerlagh and Keyser (2001), and we focus on the preferences and technology conditions for sustainability in a decentralized economy. More precisely we shed light on a set of three puzzling outcomes which are generally neglected by the literature. First altruistic households do not automatically succeed in implementing a sustainable path though their bequest motive is operative on the whole way. Second, the mere preservation of natural capital does not guarantee that it is exploited at the best rate. Third, the way to a preserved future may follow oscillating trajectories with sacrificed generations.

The paper is organized as follows. Section 2 defines a sustainability target for an economy with a renewable resource. In Section 3 we describe a decentralized economy with overlapping generations, a renewable resource and a man-made capital stock, in which households have a bequest motive. Section 4 presents the three puzzles and Section 5 concludes.

2 Natural resource and sustainability

2.1 The natural resource dynamics

We consider a renewable natural resource. Its stock is shared equally between the N first young individuals: $z_{-1} = Z_{-1}/N$. Let us first, in this section, describe the resource own dynamics, i.e without human exploitation (the extraction decision will be studied in the next section). The equation which governs the evolution of each individual endowment in the renewable resource, with zero extraction, is given by

$$z_t = z_{t-1} + h\left(Z_{t-1}\right) z_{t-1} \tag{1}$$

where Z_{t-1} is the aggregate resource stock inherited from time $t-1$, z_{t-1} is the individual stock inherited from time $t-1$ by each of the N time t

young individuals and where the function $h(.)$ is the resource natural return. Since the function h is assumed linear, we can also write this equation as $z_t = z_{t-1} + Nh(z_{t-1}) z_{t-1}$.

We make the following hypotheses on the function h: $h(0) = 0$, $\exists z > 0 : h(z) = -1/N$ and $h''(z) < 0$, $\forall z$. Given our notation $H(.)$ for the available resource, we denote the total individual available resource at time t by $H(z_{t-1}) z_{t-1} = (1 + Nh(z_{t-1})) z_{t-1}$. Hence these dynamics of the individual resource stock in the absence of extraction are given by

$$z_t = H(z_{t-1}) z_{t-1} \tag{2}$$

Example 1 ***The quadratic specification*** - *Let us consider a quadratic specification for the total natural return which is added to the existing stock each period*

$$z_t = z_{t-1} + N(\mu - \nu z_{t-1}) z_{t-1} \tag{3}$$

These dynamics can be represented by a bell-shape curve. Their properties are the following. The stock z_{t-1} must belong to the interval $(0, z_{\max})$, where $z_{\max}$ is the threshold value of z_{t-1} such that the total natural return is negative and annihilates all the existing stock: $z_{\max} + N(\mu - \nu z_{\max}) z_{\max} = 0$, i.e. $z_{\max} = (N\nu)^{-1}(1 + N\mu)$. The maximum of these dynamics is reached when $z_{t-1} = \bar{z} = (2N\nu)^{-1}(1 + N\mu)$. The steady state is given by $z^{ne} = \mu/\nu$, where the upper-script "ne" stands for "no extraction" of the resource. At the steady state z^{ne}, the total natural return is equal to zero. The steady state may be on any side of the bell-shape dynamics. If $z^{ne} \leq \bar{z}$ then the slope of the dynamics at the steady state is positive. If $z^{ne} > \bar{z}$ then the slope at the steady state is negative. The resource own dynamics are not explosive only when the slope at the steady state is smaller than -1. This occurs under the following condition $1 + N\mu - 2N\nu z^{ne} < -1 \Leftrightarrow \mu > 2/N$.

< Figure 1. Phase diagram for the natural resource >

2.2 The long run regime

In the literature on sustainability, when economists think about preserving the future, they are worried about not "(...) compromising the ability of future generations to meet their own needs", as stated in the so-called

Brundtland report (WCED, 1987). We formalize this idea by looking at stationary paths which maximize consumption per head in a given economy.

Assume a constant population N and that the economy is at a steady state. Let the technology be neo-classical and combine three factors: capital K, labor N and extraction of a renewable resource E. We can express production per head as a function of capital intensity k and extracted resource intensity e: $f(k,e)$. At each period the available resource stock per head is composed of the existing stock z plus the bell-shaped natural return on this stock, $h(z)z$. Thus at any period the steady resource stock is the difference between the available resource and extraction $e : z = H(z)z - e$.

Consumption per head $\tilde{c}$ is simply defined as the difference between production per head and investment in capital per head, *i.e.* $\tilde{c} = f[k, H(z)z - z] - k$. We want to maximize consumption per head by choosing the pair (k,z) which solves the following problem:

$$\max_{\{k,z\}} \tilde{c} = f[k, H(z)z - z] - k \tag{4}$$

Under suitable conditions on the limit properties of capital marginal productivity f'_k (*i.e.* $\lim_{k\to+\infty} f'_k(k,.) = 0 < 1 < \lim_{k\to 0} f'_k(k,.) = +\infty$), there exists an interior solution to the consumption maximization problem. The first-order conditions for an interior maximum are the following:

$$f'_k(k^*, H(z^*)z^* - z^*) = 1 \tag{5}$$

$$H'(z^*)z^* + H(z^*) = 1 \tag{6}$$

The first equation in k and z is the equivalent of the standard condition defining the Golden rule capital stock. The choice of the maximizing capital stock is determined by the usual trade-off between the marginal productivity of capital and the population growth rate (here 1). The second equation only depends on z and always has an interior solution. At z^*, the steady exploitation e is maximized. The trade-off for the extracted resource is similar to the one for capital. The marginal natural return $(H'(z^*)z^* + H(z^*))$ must equal the marginal effort to leave the resource stock unchanged next period (*i.e.* 1). We illustrate these properties with a simple example.

Example 2 ***The Cobb-Douglas-quadratic example*** **-** *Assume a Cobb-Douglas production function, $y = Ak^{\alpha_K} e^{\alpha_E}$, with $A > 0$ the multi-factor productivity index and α_K and α_E the elasticities of capital and extracted resource intensities; assume further that the resource evolves according to the quadratic function $z = [1 + N(\mu - \nu z)] z - e$, with $\mu > 0$ and $\nu > 0$. In this case, we have*

$$z^* = \frac{\mu}{2\nu} \tag{7}$$

$$e^* = \frac{N\mu^2}{4\nu} \tag{8}$$

$$k^* = \left(\alpha_K A \left(e^*\right)^{\alpha_E}\right)^{\frac{1}{1-\alpha_K}} \tag{9}$$

The following definition follows:

Definition 1 *An economy is said to be resource-conservative (resp. resource consuming) if its steady resource stock is larger (resp. smaller) than the stock z^* which maximizes consumption per capita.*

A resource-conservative economy could increase the consumption of all generations, including the present one, by just raising resource extraction. The unextracted resource closely parallels the unconsummed numeraire: it is invested to restore the next period stock. For this reason, we could label this case "over-accumulation" of resource. The inverse holds for a resource-consuming economy. Resource exploitation should be temporarily reduced to let the resource reach the higher z^* level. At that level, exploitation is eventually run at a higher level than initially. This case could be labelled "under-accumulation" of resource. In this case, the economy could reach a higher a level of consumption per head, but at the expense of the current generation.

We have defined a sustainable future in the case of a stationary economy. Another question follows immediately: how can that sustainable future be reached? To put it in another way: do there exist forces driving the economy on a transition towards that sustainable future? A long run state of the economy with a preserved level of consumption says nothing about the transition path of consumption. The transition could be one in which generations do not enjoy a fair level of consumption. This is typically the issue of sustainable *development*: in the words of the Brundtland report (WECD, 1987), sustainable development is a development that "meets the need of the present without compromising the ability of future generations to meet their own needs". In a sense, what is required is to reconcile the future generations' interests with the present generations' interests.

It is probably fake to say that all individuals are purely selfish. On the other hand, it may be equally unrealistic to assume that they have a perfectly universal concern for the entire posterity. It seems however reasonable to assume that they enjoy the idea that they accomplish their duty regarding future generations because they experience a "warm glow" or a "joy-of-giving" from fulfilling their duty. Andreoni (1989) used this idea to model the so-called joy-of-giving bequest motive and applied it to charities giving and transfers inside the family. In the next section we investigate the micro-foundations of a growth model and build a framework in which individuals have some dose of interest for future generations. We use the joy-of-giving bequest motive to model the bequest of resource from one generation to the other. With this model we shed light on three puzzles questioning the very concept of sustainability. These are to be presented in section 4.

3 A decentralized economy with a bequest motive

The economy is of Diamond's (1965) type with a constant population, but with the two extensions of an extracted resource and a joy-of-giving bequest motive. The young households at time $t = 0$ hold the global stock of resource Z_{-1}. This stock is shared equally between the N first young: $z_{-1} = Z_{-1}/N$. This section presents the agents' and the firms' behavior and characterize the equilibrium.

3.1 Households' behavior

Each individual lives for two periods: youth and old age. He is endowed with one unit of labor which he supplies inelastically during his first period of life for a real wage w_t. He is also endowed with the total available individual resource stock $H(z_{t-1}) z_{t-1}$, composed of his parents' bequest z_{t-1} augmented by it natural return $Nh(z_{t-1}) z_{t-1}$. He decides how much to extract of this inherited stock. Extraction is costless. He provides the amount extracted e_t to the production process for a real price q_t. There are two possible uses for his first-period total income, $w_t + q_t e_t$: consumption c_t and savings s_t. When old, the individual bequeathes the unextracted resource stock z_t to his heir, invests his savings in productive capital and receives capital income $R_{t+1} s_t$, where $R_{t+1} = 1 + r_{t+1}$ is the interest factor. He consumes all his second-period income and then dies. This is summarized by the youth and old-age budget constraints

$$w_t + q_t e_t = c_t + s_t \tag{10}$$

$$R_{t+1} s_t = d_{t+1} \tag{11}$$

and by the equation of motion of the individual resource stock with extraction

$$H(z_{t-1}) z_{t-1} = e_t + z_t \tag{12}$$

The individual's preferences are defined on youth and old-age consumption, c_t and d_{t+1}, and on the level of the unextracted resource stock bequeathed to his heir, z_t. They are represented by the following additively separable utility function

$$U_t = (1-\beta)\log c_t + \beta \log d_{t+1} + \gamma \log z_t \tag{13}$$

The parameter $\beta \in (0,1)$ reflects the weight attached to consuming when old while $\gamma > 0$ is the degree of the joy-of-giving bequest motive.

As we hinted at in the previous section, the individual has the feeling of doing his duty by abstaining from consuming the whole family good. He feels he has to preserve the resource for the sake of his heir. By doing so he makes sure that he does not threaten the opportunities of his descendant. It should be emphasized that the bequest motive we assume here is substantially different from a concern for the resource or the environment as a whole. Indeed, not only the individual does not care about the other individuals' resource stocks, but also he gets utility only from his own bequest.

Two decisions characterize the individual's problem: the saving decision and the extraction decision. Considering prices as given, the individual chooses s_t and e_t in order to maximize his utility. By substituting c_t, d_{t+1} and z_t by their respective expressions, we get the following maximization problem:

$$\max_{\{s_t, e_t\}} (1-\beta)\log(w_t + q_t e_t - s_t) + \beta \log(R_{t+1} s_t) + \gamma \log(H(z_{t-1}) z_{t-1} - e_t) \tag{14}$$

and the first-order conditions write

$$\frac{1-\beta}{w_t + q_t e_t - s_t} = \frac{\beta}{s_t} \tag{15}$$

$$\frac{(1-\beta) q_t}{w_t + q_t e_t - s_t} \leq \frac{\gamma}{H(z_{t-1}) z_{t-1} - e_t} \tag{16}$$

with equality if $e_t \geq 0$. Solving the first equation for s_t as a function of e_t yields: $s_t = \beta(w_t + q_t e_t)$. If extraction is unconstrained, the solution to the maximization problem is given by

$$e_t = \frac{H(z_{t-1}) z_{t-1}}{1+\gamma} - \frac{\gamma}{1+\gamma}\frac{w_t}{q_t} \tag{17}$$

$$s_t = \frac{\beta}{1+\gamma}[w_t + q_t H(z_{t-1}) z_{t-1}] \tag{18}$$

If optimal extraction is constrained, the saving decision simply writes $s_t = \beta w_t$. It is never optimal for the individual to extract all the resource since we would then get an infinitely low utility.

The extraction decision depends on two elements. First, it is increasing in the stock, including the natural return. Second it is decreasing in the relative price of labor with respect to the price of the resource, w_t/q_t. The condition of non-negativity of e_t is given by

$$\gamma \leq \frac{q_t H(z_{t-1}) z_{t-1}}{w_t} \tag{19}$$

The right-hand side of the non-negativity constraint on e_t is the ratio of the inherited resource stock valued at price q_t on the wage income. This ratio reflects the relative importance of the two individual's sources of income when young. It increases as the individual's dependence on the bequeathed resource increases.

3.2 Firms' behavior

There is a representative firm which produces the consumption/investment good. The technology of production displays constant returns to scale of the three production factors: capital K, labor L and extracted resource E. It is represented by a linearly homogeneous production function: $F(K_t, L_t, E_t)$. The profit of the representative firm is $\pi_t = F(K_t, L_t, E_t) - R_t K_t - w_t L_t - q_t E_t$. The firm maximizes its profit with respect to K_t, L_t and E_t considering prices as given. The first-order conditions are given by: $F'_K(K_t, L_t, E_t) = R_t, F'_L(K_t, L_t, E_t) = w_t$ and $F'_E(K_t, L_t, E_t) = q_t$. We shall assume a CES specification for the production function,

$$F(K_t, L_t, E_t) = A\left(\alpha_K K_t^{-\rho} + \alpha_L L_t^{-\rho} + \alpha_E E_t^{-\rho}\right)^{-1/\rho} \quad (20)$$

In intensive terms, the FOC's read as follows

$$\frac{\alpha_K}{A^\rho}\left[\frac{f(k_t, e_t)}{k_t}\right]^{1+\rho} = R_t \quad (21)$$

$$\frac{\alpha_E}{A^\rho}\left[\frac{f(k_t, e_t)}{e_t}\right]^{1+\rho} = q_t \quad (22)$$

$$\frac{\alpha_L}{A^\rho} f(k_t, e_t)^{1+\rho} = w_t \quad (23)$$

where $f(k_t, e_t) = A\left(\alpha_K k_t^{-\rho} + \alpha_L + \alpha_E e_t^{-\rho}\right)^{-1/\rho}$.

3.3 Equilibrium

We now study the equilibrium of period t. What is given in period t is the inherited resource stock z_{t-1} and the productive capital k_t. We want to determine the following time t variables: the prices w_t, R_t and q_t, the individuals' resource supply, the bequeathed stock and consumptions: e_t, z_t, c_t and d_t and the representative firm's factor demands and output supply K_t, L_t, E_t and Y_t. The labor market equilibrium implies $L_t = N$. Hence, $k_t = K_t/N$ and $e_t = E_t/N$ in equilibrium and the equilibrium expressions of factor prices are given by the marginal productivities valued at these k_t and e_t.

Proposition 1 *(i) In equilibrium, the individual's optimal extraction is unconstrained.*
(ii) Individual's optimal extraction does not depend on capital:

$$e_t = e\left(\underset{+/-}{z_{t-1}}, \underset{-}{\gamma}, \underset{-}{\rho}\right) \tag{24}$$

(iii) An increase in the inherited resource stock z_{t-1} increases the extraction if the inherited stock is low enough ($z_{t-1} < \bar{z}$). Beyond the threshold value $\bar{z}$, an increase in the inherited resource stock decreases extraction.
(iv) An increase in the degree of bequest motive γ decreases extraction.
(v) As substitutability between factors decreases (increasing ρ), extraction decreases.

Proof. See appendix "Extraction in equilibrium". ■

The fact that the equilibrium extraction does not depend on capital is due, at first, to the fact that the relative price w_t/q_t is independent of k_t in equilibrium. Indeed the ratio of the marginal productivities of labor and resource only depends on e_t. Second, the additive separability of the log-linear utility function is also responsible for this feature.

The main implication of this proposition is the following. As long as the inherited individual resource stock is positive there will always be extraction and the resource will always be sold by individuals to the firm for use in production.

We now turn to the analysis of the steady state and the dynamics of this economy. At a steady state equilibrium, the economy reproduces itself each period. Extraction is simply equal to the natural return which is added each period to steady stock, *i.e.* per capita:

$$e(z, \gamma, \rho) = Nh(z)\, z \tag{25}$$

The dynamics of the economy is as follows. At each period, we solve for e_t as a function of z_{t-1} and we determine z_t and k_{t+1}. The dynamics of z_t and k_{t+1} are given by

$$z_t = H(z_{t-1})\, z_{t-1} - e(z_{t-1}, \gamma, \rho) \tag{26}$$

$$k_{t+1} = \frac{\beta}{(1+\gamma) A^{\rho}} \Big[\alpha_L f \left[k_t, e\left(z_{t-1}, \gamma, \rho\right)\right]^{1+\rho}$$

$$+ \alpha_E f \left[k_t, e\left(z_{t-1}, \gamma, \rho\right)\right]^{1+\rho} \frac{H\left(z_{t-1}\right) z_{t-1}}{e\left(z_{t-1}, \gamma, \rho\right)^{1+\rho}} \Big] \tag{27}$$

The dynamics of z_t are independent of capital. Given the initial conditions, *i.e.* given z_{-1} and k_0, we determine the intertemporal equilibrium. We can illustrate the equilibrium by using a Cobb-Douglas production function.

Example 3 ***The Cobb-Douglas-quadratic example***
Assume the Cobb-Douglas special case of the CES production function : $f(k_t, e_t) = Ak_t^{\alpha_K} e_t^{\alpha_E}$*. Then equilibrium prices read* $R_t = \alpha_K A k_t^{\alpha_K - 1} e_t^{\alpha_E}$*,* $w_t = \alpha_L A k_t^{\alpha_K} e_t^{\alpha_E}$ *and* $q_t = \alpha_E A k_t^{\alpha_K} e_t^{\alpha_E - 1}$*. Together with a quadratic resource dynamics* $e_t + z_t = [1 + N(\mu - \nu z_{t-1})] z_{t-1}$*,* e_t*,* z_t *and* k_{t+1} *write as follows*

$$e_t = \varepsilon(\gamma) [1 + N(\mu - \nu z_{t-1})] z_{t-1} \tag{28}$$

$$z_t = [1 - \varepsilon(\gamma)] [1 + N(\mu - \nu z_{t-1})] z_{t-1}$$

$$k_{t+1} = \frac{\beta}{1-\beta} \varepsilon(\gamma)^{\alpha_E} \left(\alpha_L + \frac{\alpha_E}{\varepsilon(\gamma)}\right) A k_t^{\alpha_K} \left[\left(1 + N(\mu - \nu z_{t-1})\right) z_{t-1}\right]^{\alpha_E}$$

where $\varepsilon(\gamma) = \alpha_E (\alpha_E + \alpha_E \gamma + \alpha_L \gamma)^{-1} \in (0,1)$*.*

At a steady state equilibrium we have $N(\mu - \nu z) z = \varepsilon(\gamma) [1 + N(\mu - \nu z)] z$*. We can solve for* z *and deduce* e

$$z = \frac{\mu}{\nu} - \frac{1}{N\nu} \frac{\alpha_E}{\gamma(1 - \alpha_k)} \tag{29}$$

$$e = \frac{\alpha_E}{(1 - \alpha_K)} z \tag{30}$$

The steady state equilibrium value of k *is the solution of* $s = k$ *where* $s = \beta(1 - \alpha_k) A k^{\alpha_K} e^{\alpha_E}$*:*

$$k = \left(\beta(1 - \alpha_K) A e^{\alpha_E}\right)^{\frac{1}{1-\alpha_K}} \tag{31}$$

4 Three puzzles about sustainability

The motivation of this paper is that sustainable development, as questioned by the Brundtland definition, might be even more puzzling than expected. Within the model depicted above three issues can be stressed out. First, it may happen that, despite the bequest motive towards the natural resource, this resource collapses, compromising the ability of forthcoming generations to fulfill their own needs. Second, the possibility for reaching the maximum steady state consumption level through decentralized preferences is not guaranteed. Third, the economy may experience oscillatory evolutions, some generations facing less favorable opportunities than the previous and the next ones.

4.1 Resource extinction despite altruism

The existence of a positive steady state resource stock at equilibrium may be regarded as a natural candidate for sustainability since each individual in each generation faces the same opportunities. Present behavior does not threaten the well-being of future generations. It may also be interpreted as an intergenerationally equitable outcome since each generation is treated identically.

However the possibility to reach a trivial equilibrium where the resource stock is equal to zero cannot be ruled out.

Proposition 2 *Let the resource own dynamics be quadratic* $z_t = z_{t-1} + N(\mu - \nu z_{t-1}) z_{t-1}$.
(i) in the case where factors are strong substitutes ($\rho \in (-1, 0)$)*, the resource extinction never occurs, whatever* $\gamma > 0$*;*
(ii) in the case where factors are poor substitutes ($\rho > 0$)*, the resource extinction never occurs only if the concern for the bequeathed resource is higher than the following threshold*

$$\underline{\gamma} = \frac{1}{N\mu} \tag{32}$$

Proof. See appendix "Dynamics of z_t". ■

Unexpectedly, the mere existence of a taste for bequeathing the resource is not always sufficient to avoid the extinction of the resource. When factors are poor substitutes the taste for bequest must not only be positive, but larger than the minimum threshold $\underline{\gamma}$. This minimum value depends on the technology of production (ρ), the resource's productivity (μ) and the population level (N).

Whenever the resource stock declines in the positively sloped section of the dynamics $z_t = H(z_{t-1}) z_{t-1} - e(z_{t-1}, \gamma, \rho)$, that is to say for low values of z, both natural return and extraction decrease. Our point is that the speed at which extraction decreases is slower when factors are poor substitutes than when they are high substitutes. In the latter case, any positive value of γ leads to resource preservation. In the former case, only a sufficiently strong bequest motive guarantees that the resource is preserved.

This proposition sheds light on the interplay between sustainability concepts and factors substitutability in an equilibrium analysis. When factors are substitutes within the production process, the resource is not essential to production. It may then be completely exploited without compromising consumption opportunities of future generations. This then satisfies a *weak sustainability* criterion. Our result shows that, in any case, a positive resource stock will be maintained in the long run, even though it is not essential to production. This holds true for any level of the bequest motive. Thus, *strong sustainability* will also be satisfied. When factors are poor substitutes, sustainability may be endangered because the resource is essential to production. In this case, *weak sustainability* implies *strong sustainability.* The resource stock must be preserved for sustainability to be reached. We derived a condition on preferences for this to hold in equilibrium.

< Figure 2. Phase diagram with extraction >

4.2 Preserved, but misused resource

We now study the case where there exists a single long run positive equilibrium level of the resource stock. Yet, what ensures that this preserved resource stock maximizes consumption level in the long run? This is our second puzzle: the following proposition establishes conditions on the preferences for this to hold with a Cobb-Douglas production function.

Proposition 3 *With a Cobb-Douglas production function and a quadratic resource dynamics, there exists a system of individuals' preferences* (β^*, γ^*) *which leads to the maximum steady consumption path. This system of preferences is such that*

$$\gamma = \gamma^* \equiv \frac{2\alpha_E}{N\mu\,(1-\alpha_K)} \tag{33}$$

$$\beta = \beta^* \equiv \frac{\alpha_K}{1-\alpha_K} \in (0,1) \tag{34}$$

This implies that $\alpha_K \in (0, \frac{1}{2})$

Proof. *See appendix "Conditions on preferences".* ■

We thus have established the existence of a system of preferences which decentralizes the unique long run consumption-maximizing path. In other words, the capital and the resource stocks are respectively equal to k^* and z^* as defined in section 2.

Most of the time, however, the preferences will drive the economy to another long run equilibrium. Assume a second-best economy with a tendency to under-accumulate the resource ($z < z^*$). This may come from a taste for bequeathing the resource lower than γ^*. Whatever the level of capital per head, consumption is not maximized. What is then the capital stock k which maximizes the consumption per head? According to the Hartwick rule, one should expect k to be larger than k^*.

Proposition 4 *Whenever an economy under-accumulate or over-accumulate its natural resource, the level of capital which maximizes consumption is always lower than* k^**.*

Proof. See appendix "Conservationists *vs* 'exploitationists". ■

What explains this result is that, as long as the resource stock is not equal to z^*, extracted resource is not maximized. Indeed, only z^* leads to the *maximum sustainable yield.* Since e is lower than e^*, the marginal productivity of the capital stock k^* is lower than the marginal cost of reproducing k^* each period. As a consequence, only a lower capital stock can make it.

The Hartwick rule addresses the issue of the role of substitutability between natural and man-made capital stocks (see *e.g.* Asheim *et al.* (2003)). This question, in fact, should be considered from two points of view: from a technological perspective, as done above, but also from the point of view of the contribution of each stock to sustainability, *i.e.* from their capacity to raise revenues.

Let consider a world where people are eager to fulfill their duty towards future generations by preserving a high level of the natural resource. Call this a *conservative* economy (*i.e.* conservative with respect to the natural resource). Formally, this economy is such $\gamma > \gamma^*$. The previous proposition shows that, when people are too conservative ($\gamma > \gamma^*$ and $z > z^*$), it follows that $e < e^*$ and that it is not necessary to maintain a capital stock as high as k^*: a smaller capital stock would maximise the consumption level ($\beta < \beta^*$). In other words, in a *conservative* world, it is as if the two capital stocks were substitutes with respect to their contribution to sustainability.

Let us now consider a world where people are relatively selfish or short-sighted in the sense that they neglect their duty towards the future generations. Call this an *exploitationist* economy (agents over-exploit the natural resource for their own welfare). Formally we have $\gamma < \gamma^*$. In this case, one should expect that it might be helpful to maintain a capital stock higher than k^* as a compensation. Actually, our proposition reveals that an excessive selfishness ($\gamma < \gamma^*$ and $z < z^*$), and thus a low resource stock, would recommend also a lower capital stock (through $\beta < \beta^*$). From the point of view of their contributions to income and consumption, there seems to be like a complementarity relation between the two stocks, unlike in the conservative world.

4.3 Preserved though unequal opportunities

We have shown that the steady state z may *a priori* be either on the increasing side of the dynamics of z_t or on its decreasing side. This may alter the dynamics of capital k. First, a limit cycle may appear in the long run and, second, the transition path may display fluctuations on the way to a steady state. A complete analytical characterization of all possible trajectories and steady states as a function of the different parameters and initial conditions would be tricky to develop with our model. This difficulty flows from the presence of two capital stocks, from the quadratic specification of the resource's dynamics and from the CES production function. So, in this section we shall investigate these puzzling transitions with the help of a computational version of our model.

4.3.1 Parameters and calibration

The model encompasses the dynamic equations at equilibrium for z_t (equation (26)), k_{t+1} (equation (27)) and e_t (equation (24)), its explicit form being given in appendix under equation (38)). The share parameters of the CES production function are $\alpha_L = 0.7$, $\alpha_K = 0.2$, $\alpha_E = 0.1$ and the elasticity of substitution is $\rho = 0.8$. The size of the population is $N = 1$. The law of motion for the renewable resource has a quadratic form with $\mu = 2.1$ and $\nu = 0.01$. Individuals preferences are characterized by $\beta = 0.25$ (weight attached to consumption when old) and $\gamma = 1$ (weight attached to the endowing of resource to their heirs). The scale parameter of the production function is the only parameter we have to calibrate: this is simply done by inverting equation (27) with parameters values and initial conditions as given. Let $t = 1$ be the first period of time.

The model is computed as follows. The dynamics of z_t and e_t are recursive and independent from the dynamics of k_t. Considering an initial value z_0, one is able to compute the whole time path for z_t and e_t. Then, k_t can be computed with k_{t-1}, z_{t-2} and e_{t-1} as given. Equation (27) is solved using the Newton-Raphson algorithm. The time path of the economy, and eventually the steady state itself, depend both on the values of the set of parameters and on the initial values z_0, and k_1.

4.3.2 Long run cyclical equilibrium

We shall now consider the case where the resource's own dynamics is such that deterministic cycles may occur. We know that these dynamics are strongly influenced by the value of the parameter μ and that this behavior, in relation with extraction e_t, may have severe impacts on the dynamics of the capital stock k_t. Simulations illustrate that feature. Initial values are $z_0 = k_1 = 100$. While keeping all the parameters value untouched, we test the implications of different values of μ on the dynamics. For $\mu \lesssim 2.2$, convergence is monotonic towards the steady state, as illustrated is Figure 3. For $2.2 \lesssim \mu \lesssim 3.2$, convergence is non-monotonic but the steady state is preserved; it may be noticed that the magnitude of the cycles is increasing with respect to the value of μ. Whenever $\mu \simeq 3.2$, the slop of the dynamics of the resource at the steady state equals -1 and the steady state becomes oscillatory: a deterministic limit-cycle is reached for z_t. It oscillates between 174.8 and 245.2 with a constant central value of 210. Furthermore, we observe that k_t reaches a steady state (at a value of 112.4) after a non-monotonic convergence. In other words, at the steady state, a unique level of capital stock is compatible with two levels of resource stock. Figure 3 displays the time paths for these two variables over 100 periods.

< Figure 3: Convergence and limit-cycle with μ=3.2 (k_1=z_0=100) >

4.3.3 Monotonic *vs* oscillatory transition

The set of parameters given above has been chosen so as to verify the theoretical conditions for an interior solution and the existence of a non-trivial steady state, namely $\gamma = 1 > 1/N\mu$. Let us first consider the following initial values $z_0 = k_1 = 90$ so that we have $z_0 < \bar{z} = (2N\nu)^{-1}(1 + N\mu) = 155$. Figure 4 displays the convergence of the two stocks starting from their initial values to the steady state. This one is such that $z = 104.8$ and $k = 111.0$. Convergence is monotonic and reached after 4 periods of time. Let us then consider 180 as an alternative initial value for z_0 : this is larger than $\bar{z}$. Naturally, the steady state is not altered. Figure 5 displays this new time path. We can see that, considering the high value of initial stock for the resource, an inversed over-shooting occurs during the transition path. Convergence is not monotonic and takes 5 periods to reach the steady state. Interestingly, the capital stock convergence is not monotonic either.

< figure 4: Monotonic convergence with $k_1 = z_0 = 90$ (μ=2.1) >

< figure 5: Non-monotonic convergence with k_1=180 and z_0=90 (μ=2.1) >

Whether on the transition or at a steady state equilibrium, fluctuations are puzzling because they seem to violate the requirement that each generation should be given the right to choose their own path. No generation would *a priori* choose to be in the position of those generations at the low steady state. Thus this is clearly a case where intertemporal equilibrium even with a bequest motive fails to guarantee equal opportunities to any generation, be it on the transition or in the long run. Such paths may only be judged with a *maximin* criterion[2].

5 Conclusion

In this paper we develop a framework for analyzing sustainable development issues in an overlapping generations economy. Our model encompasses the very concept of opportunity among generations through the interactions between two kinds of capital stock, the natural one (a renewable resource) and the man-made one, both being considered as a source of revenues in the economy and having their own low of motion. We model an overlapping generations economy in which individuals are privately-endowed with a renewable resource. This resource can be extracted at no cost by the young households and provided to production as a source of revenue. However, a *joy-of-giving* bequest motive motivates the transfer of the unexploited resource to the heirs so as to let them the opportunity to raise their own revenues from the resource. The purpose was to analyze whether a decentralized decision-making process with environmental constraint may fulfill the necessary condition for sustainability.

The main findings are the following. It is always optimal for individuals to extract the resource in equilibrium as long as the bequeathed resource is positive; extraction in equilibrium does not depend on physical capital.

In the long run, the bequest motive does not systematically guarantee sustainability. When production factors are high substitutes and thus when

[2]The authors are grateful to Robert Cairns for suggesting them the relevance of the maximin criterion in this case.

extracted resource is inessential to production, any degree of the bequest motive is compatible with a preserved resource. So, both weak (consumption preservation) and strong sustainability (resource stock preservation) are satisfied. On the contrary, when factors are poor substitutes, *i.e.* when the resource is essential to production, strong sustainability (resource preservation) is required in order to have weak sustainability. We derive a condition on the degree of the bequest motive for strong sustainability to hold.

There exists a system of preferences which decentralizes the target of the consumption-maximizing path in the long run. But in most cases preferences will differ from this and the economy will converge to a sub-optimal long run equilibrium. As we showed, resource-conservative economies, which run a high steady resource stock, should compensate with a lower capital stock to maximize the second-best consumption level (substitutability result). On the contrary, resource-consuming economies, which run a low level of steady resource stock, should also keep a lower capital stock to maximize second-best consumption per head (complementarity result).

Finally, as we demonstrated (calibration exercises), there exist fluctuating transitions and also a limit-cycle. These outcomes question the issue of sustainability in the sense that either we converge to a sustainable future but at the price of unequal opportunities at some points of time, *i.e.* for some sacrificed generations (non-monotonic transitions), either we do not even have a long run with equal opportunities for each generation (limit-cycle). From the point of view of sustainability, these equilibria cannot be judged with a criterion of non-declining welfare but rather with a *maximin* criterion.

All these results show that, first, the tool of the intertemporal equilibrium analysis, which is the core of OLG models, particularly fits this issue of sustainability and, second, going deep inside this issue with the help of OLG tools reveals that deeper theoretical insights are needed to bring a better understanding of sustainable development. Importantly, the design of policies should also benefit from the awareness of the puzzles analyzed in this paper.

6 Appendices

6.1 Extraction in equilibrium

Proof of point (i) - At an unconstrained-extraction time t equilibrium, there is a unique finite positive quantity e_t which equalizes the prices from the inverted resource supply and demand functions on the factor market and which is inferior to $H(z_{t-1}) z_{t-1}$. From the expression of aggregate resource supply $Ne_t = N(1+\gamma)^{-1} H(z_{t-1}) z_{t-1} - N(1+\gamma)^{-1} \gamma q_t^{-1} w_t$ and from the equilibrium value of the real wage rate $w_t = (\alpha_L / A^\rho) f(k_t, e_t)^{1+\rho}$, we derive the inverted resource supply

$$q_t = \frac{\gamma (\alpha_L / A^\rho) f(k_t, e_t)^{1+\rho}}{H(z_{t-1}) z_{t-1} - (1+\gamma) e_t} \tag{35}$$

and the inverted resource demand verifies

$$q_t = \frac{\alpha_E}{A^\rho} \frac{f(k_t, e_t)^{1+\rho}}{e_t^{1+\rho}} \tag{36}$$

Equating the above two expressions of the price q_t yields:

$$\frac{\gamma \alpha_L}{H(z_{t-1}) z_{t-1} - (1+\gamma) e_t} = \frac{\alpha_E}{e_t^{1+\rho}} \tag{37}$$

The LHS tends to $\alpha_L \gamma / H(z_{t-1}) z_{t-1}$ as e_t tends to 0, while the RHS tends to $+\infty$ as e_t tends to 0. The LHS is increasing in e_t until the value

$$(1+\gamma)^{-1} H(z_{t-1}) z_{t-1} \, (< H(z_{t-1}) z_{t-1})$$

at the limit of which it tends to $+\infty$; from the other side, as e_t tends to $(1+\gamma)^{-1} H(z_{t-1}) z_{t-1}$, the LHS tends to $-\infty$. Beyond $(1+\gamma)^{-1} H(z_{t-1}) z_{t-1}$, as e_t increases the LHS increases until 0 at the limit; but this is economically meaningless, since extraction cannot be larger than the stock. The RHS decreases as e_t increases and tends to 0 as e_t tends to $+\infty$. As a result, there always exists a finite positive $e_t \leq H(z_{t-1}) z_{t-1}$, such that the two curves cross.

Proof of point (ii) - Extraction, i.e. $e_t = (1+\gamma)^{-1} H(z_{t-1}) z_{t-1} - \gamma (1+\gamma)^{-1} w_t q_t^{-1}$, in equilibrium, is given by

$$e_t - \frac{H(z_{t-1}) z_{t-1}}{1+\gamma} + \frac{\gamma}{1+\gamma} \frac{\alpha_L}{\alpha_E} e_t^{1+\rho} = 0 \tag{38}$$

which is obtained by substituting $w_t q_t^{-1}$ with its equilibrium value, i.e.

$$\frac{\alpha_L A^{-\rho} f\left(k_t, e_t\right)^{1+\rho}}{\alpha_E A^{-\rho} f\left(k_t, e_t\right)^{1+\rho} e_t^{-(1+\rho)}} = \frac{\alpha_L}{\alpha_E} e_t^{1+\rho} \tag{39}$$

This equation in e_t is independent of capital. Its solution is a function $e\left(z_{t-1}, \gamma, \rho\right)$.

Proof of point (iii) - The solution of this equation is a function of z_{t-1}, γ and ρ: $e_t = e\left(z_{t-1}, \gamma, \rho\right)$. Let us study the derivative of this function w.r.t. z_{t-1}:

$$\frac{de_t}{dz_{t-1}} = \frac{(1+\gamma)^{-1}\left[H'\left(z_{t-1}\right) z_{t-1} + H\left(z_{t-1}\right)\right]}{1 + \gamma\left(1+\gamma\right)^{-1} \alpha_L \alpha_E^{-1} \left(1+\rho\right) e_t^{\rho}} \tag{40}$$

or

$$\frac{de_t}{dz_{t-1}} = \varepsilon\left(z_{t-1}, \gamma, \rho\right)\left[H'\left(z_{t-1}\right) z_{t-1} + H\left(z_{t-1}\right)\right] \tag{41}$$

where

$$\varepsilon\left(z_{t-1}, \gamma, \rho\right) = \frac{\alpha_E}{\alpha_E + \alpha_E \gamma + \alpha_L \gamma \left(1+\rho\right) e\left(z_{t-1}, \gamma, \rho\right)^{\rho}} \tag{42}$$

belongs to the interval $(0,1)$. Thus the derivative de_t/dz_{t-1} has the same sign as the derivative of the dynamics without extraction $z_t = \phi\left(z_{t-1},\right)$, i.e. $\phi'\left(z_{t-1}\right) = H'\left(z_{t-1}\right) z_{t-1} + H\left(z_{t-1}\right)$, i.e. first increasing for for values $z_{t-1} \in (0, \overline{z}]$ and then decreasing for $z_{t-1} \in \left(\overline{z}, H\left(z_{t-1}\right) z_{t-1}\right)$.

Proof of point (iv) - The derivative of $e\left(z_{t-1}, \gamma, \rho\right)$ w.r.t. γ is given by:

$$\frac{de_t}{d\gamma} = -\frac{H\left(z_{t-1}\right) z_{t-1} \left(1+\gamma\right)^{-2} + \alpha_L \alpha_E^{-1} e_t^{1+\rho} \left(1+\gamma\right)^{-2}}{1 + \gamma\left(1+\gamma\right)^{-1} \alpha_L \alpha_E^{-1} \left(1+\rho\right) e_t^{\rho}} < 0 \tag{43}$$

Proof of point (v) - The derivative of $e\left(z_{t-1}, \gamma, \rho\right)$ w.r.t. ρ is given by:

$$\frac{de_t}{d\rho} = -\frac{\gamma\left(1+\gamma\right)^{-1} \alpha_L \alpha_E^{-1} \left(1+\rho\right) e_t^{\rho}}{1 + \gamma\left(1+\gamma\right)^{-1} \alpha_L \alpha_E^{-1} \left(1+\rho\right) e_t^{\rho}} < 0 \tag{44}$$

6.2 Dynamics of z_t

The dynamics of the individual resource stock with extraction in equilibrium is $z_t - H\left(z_{t-1}\right) z_{t-1} + e\left(z_{t-1}, \gamma, \rho\right) = 0$. They have a bell shape, increasing on $(0,\overline{z})$ and decreasing on $(\overline{z},z_{\max})$. The slope of these dynamics are given by:

$$\frac{dz_t}{dz_{t-1}}=[1-\varepsilon(z_{t-1},\gamma,\rho)]\left[H'(z_{t-1})z_{t-1}+H(z_{t-1})\right] \tag{45}$$

It is therefore a fraction of $H'(z_{t-1})z_{t-1}+H(z_{t-1})$. This last expression is the derivative of the function $\phi(z_{t-1})$ which is the dynamics of the resource without extraction. It is positive for $z_{t-1}\in(0,\overline{z})$ and negative for $z_{t-1}\in(\overline{z},z_{\max})$. The limits are:

$$\lim_{z_{t-1}\to 0} z_t=0 \tag{46}$$

$$\lim_{z_{t-1}\to z_{\max}} z_t=0 \tag{47}$$

We consider the slope of the dynamics as z_{t-1} tends to 0 in the case of quadratic resource own dynamics given by $z_t=z_{t-1}+N(\mu-\nu z_{t-1})z_{t-1}$. Since

$$\varepsilon(z_{t-1},\gamma,\rho)=\frac{\alpha_E}{\alpha_E+\alpha_E\gamma+\alpha_L\gamma(1+\rho)e(z_{t-1},\gamma,\rho)^{\rho}} \tag{48}$$

and

$$H'(z_{t-1})z_{t-1}+H(z_{t-1})=1+N(\mu-2\nu z_{t-1}) \tag{49}$$

we have

$$\lim_{z_{t-1}\to 0}\left[H'(z_{t-1})z_{t-1}+H(z_{t-1})\right]=1+N\mu \tag{50}$$

$$\lim_{z_{t-1}\to 0}\varepsilon(z_{t-1},\gamma,\rho)=\begin{cases}0 & \text{if } \rho\in(-1,0)\\ \frac{1}{1+\gamma} & \text{if } \rho>0\end{cases} \tag{51}$$

Hence, in the quadratic case, the slope of the dynamics as $z_{t-1}\to 0$ is given by

$$\lim_{z_{t-1}\to 0}[1-\varepsilon(z_{t-1},\gamma,\rho)]\left[H'(z_{t-1})z_{t-1}+H(z_{t-1})\right]=\begin{cases}1+N\mu & \text{if } \rho\in(-1,0)\\ \frac{\gamma(1+N\mu)}{1+\gamma} & \text{if } \rho>0\end{cases} \tag{52}$$

If $\rho \in (-1, 0)$ this slope $(1 + N\mu)$ is greater than 1 independently of γ. If $\rho > 0$ this slope is greater than 1 iff

$$\gamma > \frac{1}{N\mu} \tag{53}$$

Since the dynamics are continuous and concave and end up with negative slope, starting with positive slope larger than 1, there exists a non-trivial steady state z.

6.3 Conditions on preferences

We have $z = z^*$ if and only if

$$z = \frac{\mu}{\nu} - \frac{1}{N\nu} \frac{\alpha_E}{\gamma^* (1 - \alpha_K)} = \frac{\mu}{2\nu} = z^* \tag{54}$$

which leads to the following condition

$$\gamma^* = \frac{2\alpha_E}{N\mu (1 - \alpha_K)} \tag{55}$$

and then, taking $e = e^* = \alpha_E (1 - \alpha_K)^{-1} z^*$ we have $k = k^*$if and only if

$$k = (\beta^* (1 - \alpha_K) A (e^*)^{\alpha_E})^{\frac{1}{1-\alpha_K}} = (\alpha_K A (e^*)^{\alpha_E})^{\frac{1}{1-\alpha_K}} = k^* \tag{56}$$

which leads to the following condition

$$\beta^* = \frac{\alpha_K}{1 - \alpha_K} \tag{57}$$

The condition for a positive stationary natural stock z is given by

$$\gamma^* > \underline{\gamma} \Leftrightarrow \alpha_E > \frac{1 - \alpha_K}{2} \tag{58}$$

and having $\beta \in (0, 1)$ requires

$$0 < \frac{\alpha_K}{1 - \alpha_K} < 1 \Leftrightarrow 0 < \alpha_K < \frac{1}{2} \tag{59}$$

6.4 Conservationists vs ‘exploitationists’

Let $\tilde{z} \neq z^*$, then by definition $\widetilde{e} < e^*$. With a CES production function we have $f''_{ke} > 0$ and so $f'_k (k^*, \widetilde{e}) < 1$. As a result, $\tilde{k}$ solution of $f'_k \left(\widetilde{k}, \widetilde{e}\right) = 1$ is such that $\tilde{k} < k^*$.

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Figure 1. Phase diagram of the natural resource

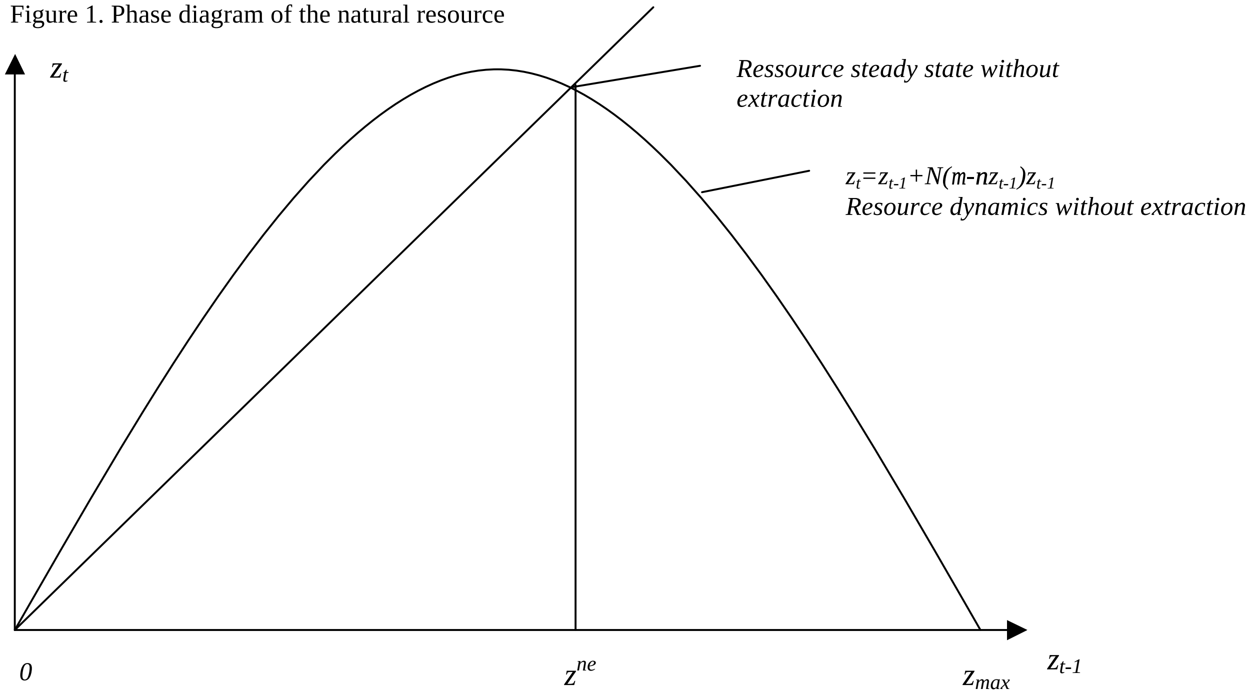
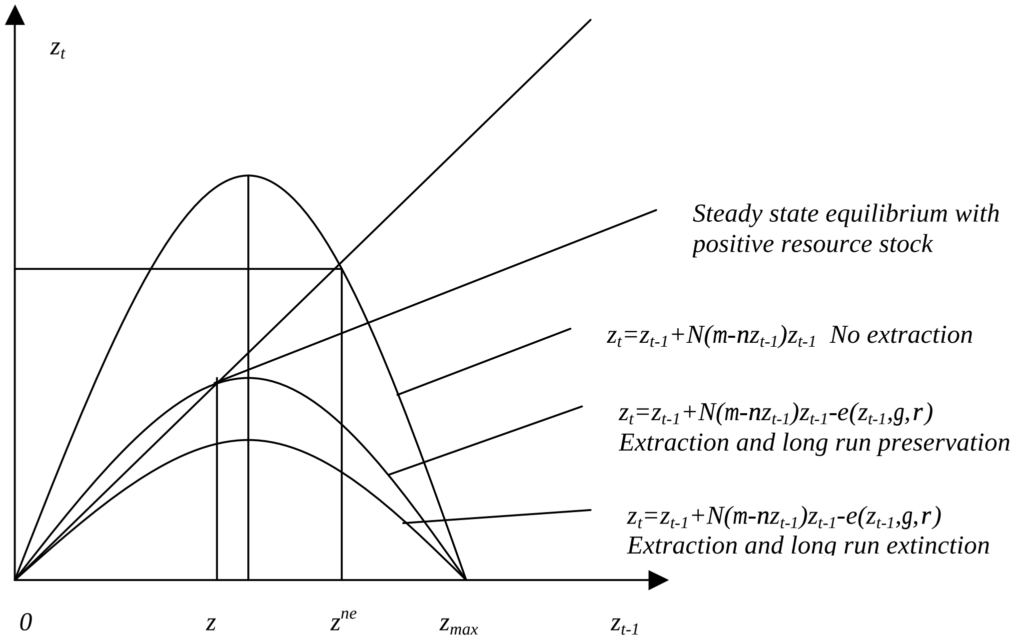


Figure 2. Phase diagram with resource extraction



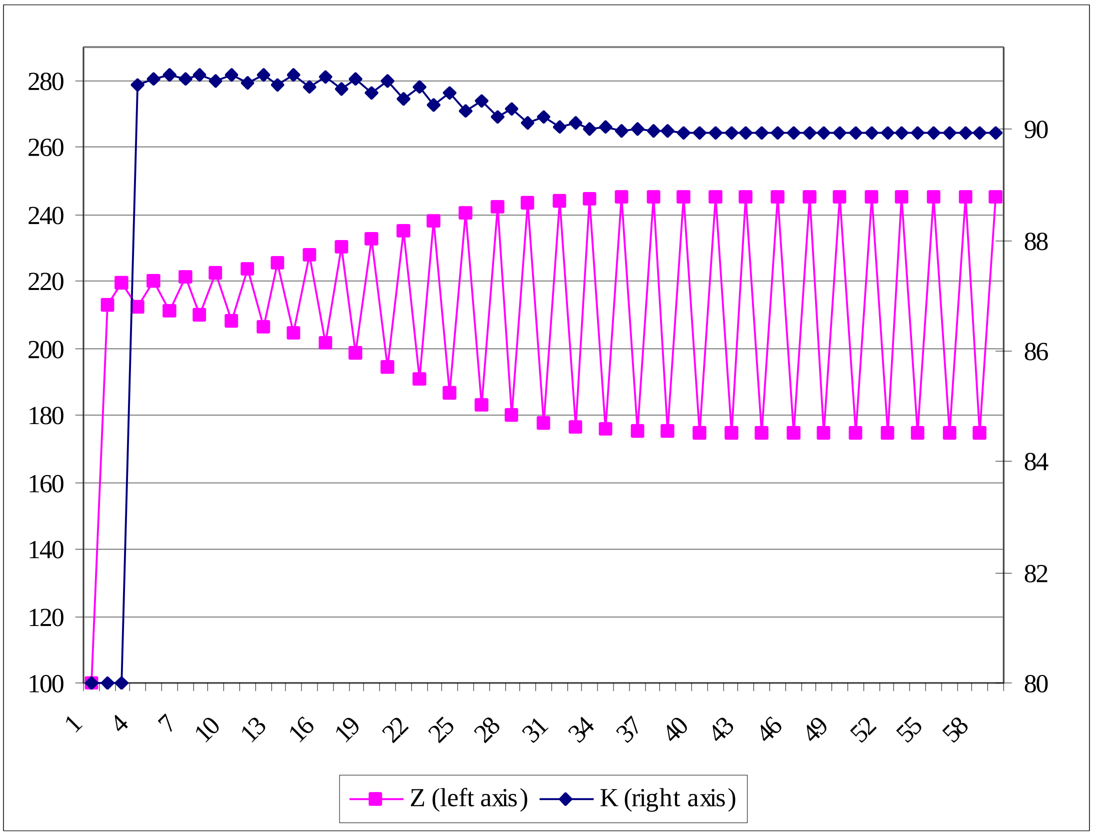


Fig 3. Convergence and limit cycle with m **= *3.2 (***k_0 **=** z_{-1} **= *100)***

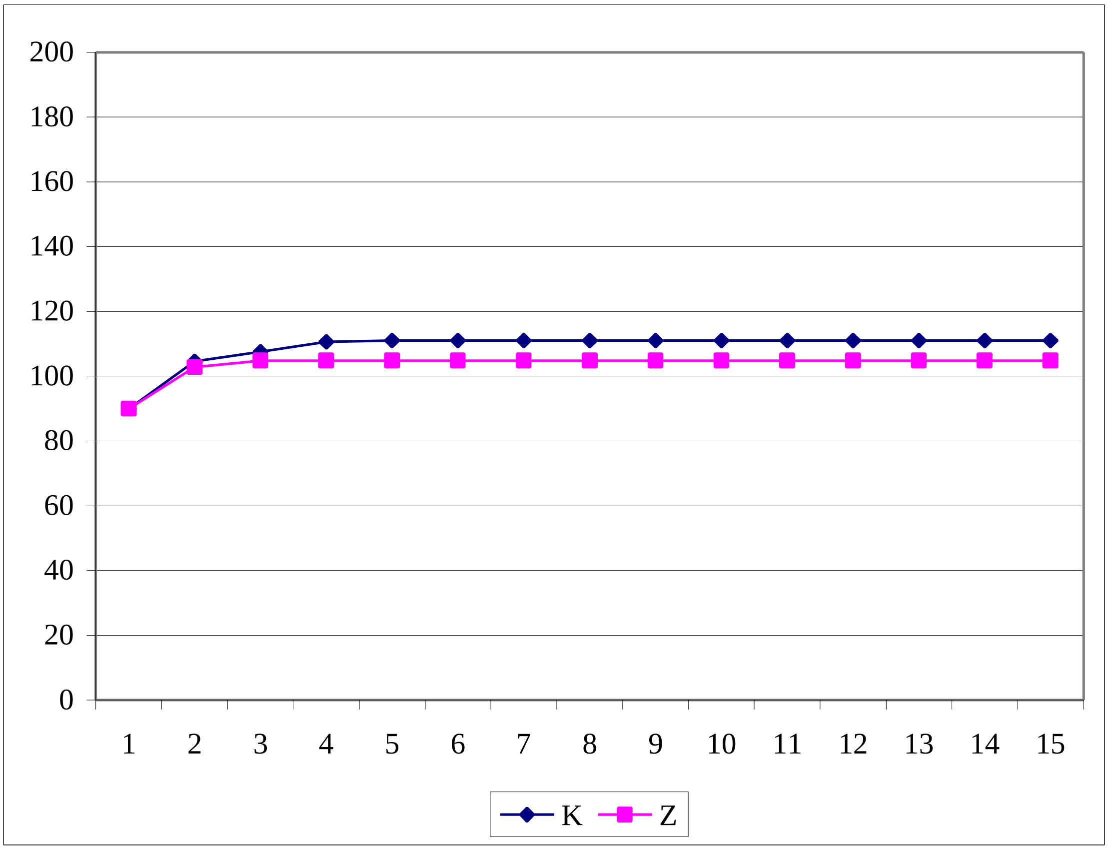


Fig 4 Monotonic convergence with k_0 **=** z_{-1} **=** ***90*** **(***m* **= *2.1*)**

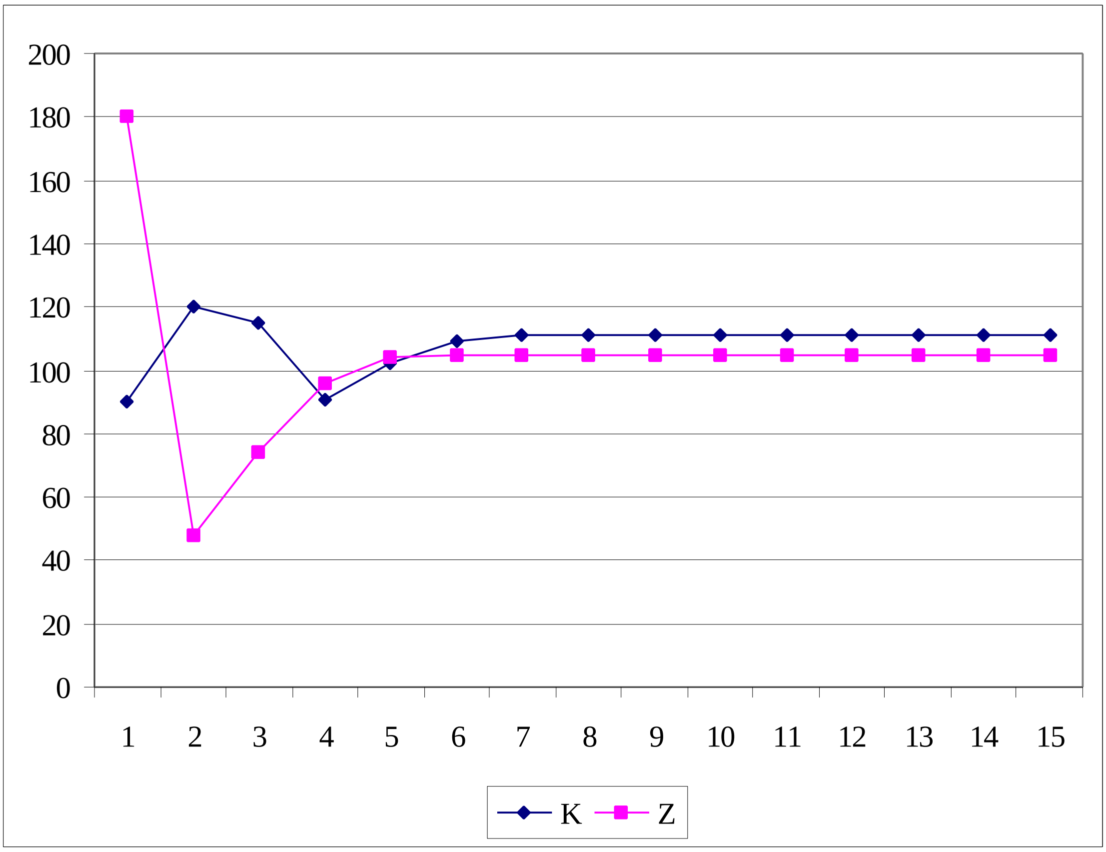


Fig 5. Non-monotonic convergence with $z_{-1} = 180$ and $k_0 = 90$ ($m = 2.1$)