The Role of Mediation in Peacemaking and Peacekeeping Negotiations*

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Abstract

We develop a model of bargaining that provides a rationality for the difference in the method of negotiation, depending on the nature of the conflict. We distinguish negotiations previous to a potential conflict, and negotiations during a conflict. In these contexts, we study the role of a mediator that tries to achieve a certain balance between the efficiency of the agreement and the equality of the sharing. We show that the credibility of the mediator comes from his willingness to impose delays in the negotiation, even if that implies costs. We also find how the "weak" player in the conflict can strategically profit from the mediator's quest for equity. Finally, we show how the capacity of the mediator to induce a higher equality in the sharing is always higher in a peacemaking situation than in a peacekeeping one.

Keywords: bargaining, mediation, Rubinstein.

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1 Introduction

Negotiations among potentially conflicting parties (for instance, international agreements) are a clear example of the application of Game Theory and, in particular, of Bargaining Theory. This Theory has been, throughout the years, a very successful area of research. The theoretical advances, as well as the applications, have shed a lot of light on the way agents or institutions negotiate, in order to achieve a mutually beneficial agreement.

There is a crucial feature in many real-life economic negotiations, however, that is still poorly understood. This is the role played by arbitrators and mediators in the negotiation processes. While arbitrators impose an agreement, mediators facilitate the reaching of an agreement by the players (Muthoo, 1999). The importance of these professional negotiators is clear. In most international negotiations, the United Nations (UN) sends a group of diplomats (supervised by a main negotiator) whose aim is to help the parties involved in the conflict to achieve a successful agreement. In other domains, like for instance domestic conflicts that are caused by a severe and damaging strike, the legislation usually allows governments to impose an arbitrator to the parties.[1]

In spite of this importance, the literature on Bargaining has seldom approached the role of these negotiators, and has essentially done it by treating them as passive agents with an exogenously predetermined role. There is, on the one hand, the approach followed by Compte and Jehiel (1995) and Manzini and Mariotti (2001), who analyze the role of arbitrators as pre-fixed outside options to the bargaining process. On the other hand, Jarque, Ponsatí and Sákovics (2003) and Copic and Ponsatí (2003) study mediators as information filters in a context of two-sided asymmetric information, where the role of the mediators is to make the agreement public when the parties have made mutually acceptable offers.

An exception to these approaches are the works by Ponsatí (2001) and Manzini and Ponsatí (2002), in which third-parties (other than the agents directly involved in the conflict) take strategic decisions that may affect the negotiation processes. In their models, these parties are stake-holders (agents indirectly affected by the outcome of the bargaining process) and their intervention is made through the promise of monetary transfers to the agents in order to ease the termination of the conflict.

In this work, we are interested in analyzing the role of mediators who have the capacity to strategically intervene in a conflict, but from a very different perspective to the analyses of previous works. In our framework, the mediator does not benefit from transfers of the parties involved in the conflict, and his intervention is driven by his interest in achieving a certain balance between the efficiency and the equality of the final agreement. Moreover,

[1]For an example of this arbitration, see the strike in the Spanish Flight Company IBERIA in July 2001, that was ended after a compulsory arbitration process imposed by the Spanish Government.

his mediation activity is not conducted through the promise of monetary transfers to the parties, but through his capacity to alter the way in which negotiations are conducted. In particular, the mediator can choose between allowing the agents involved to conduct direct (face-to-face) negotiations or forcing them to undertake indirect (mediated) negotiations in which the offers of the players go through him, who decides whether to convey them to the other party, or not.

Another important issue of this paper is that it allows us to analyze and compare the role of the mediators in conflicts of different nature, where the two parties negotiate to share a fixed surplus. Often, a distinction is made between *peacekeeping* negotiations (pre-conflict) and *peacemaking* negotiations (in-conflict). This distinction is common in war conflicts (*peacekeeping* versus *peacemaking*), but also in labor conflicts (*before strike* versus *in-strike*). The former, corresponds to situations in which the negotiation tries to achieve an agreement that avoids the declaration of a potential conflict. In the latter, the conflict has already started and the aim of the negotiations is to find a way to end it. In peacemaking, each player suffers a cost each period until they reach an agreement.[2]

The following cite, quoted from an interview with Françesc Vendrell (a UN negotiator with more than 30 years of experience in international conflicts) can serve to illustrate the important link between the way in which negotiations are conducted, and the nature of the conflict and, therefore, to highlight the relevance of the approach proposed in this article:[3]

> "I would rather negotiate pendularly with each party, than with both sides face-to-face. (.....) I am talking about negotiation processes to conquer the peace, *peacemaking*, (.....) in which there is a primacy of rounds of contacts over multilateral meetings. This is different to what happens with *peacekeeping* negotiations."

The aim of the paper is to analyze a model of bargaining under complete information that provides a rationality for the difference in the method of negotiation, depending on the nature of the conflict, that is, *peacemaking* or *peacekeeping* negotiations. The most important point is the modelization of the mediator. In this sense, we parametrize the mediator's willingness to sacrifice joint surplus (efficiency) in order to achieve a greater equality.

We find important differences in the mediator's intervention depending on the environment of the negotiation. For a peacemaking scenario we show that, first, even if the

[2]For an analysis of a negotiation mechanism under these two alternative environments see Porteiro (2003).

[3]Inteview appeared in the magazine "El País Semanal" (number 1.318, 30^{th} December 2001).

mediator is willing to sacrifice completely the efficiency in order to achieve a higher equality, he is not able to induce a fully egalitarian sharing. The reason for this is that the mediator's capacity to influence the players is undermined by the existence of a conflict that generates a flow of damages for the players. Second, we also prove that in this case, more equality can be achieved by giving the right to start the negotiation process to the ex-ante strong player (in contrast with unmediated bargaining processes). If the mediator gives the initiative to the weak player, this agent will use strategically the higher costs that the continuation of the conflict will impose on him.

When moving to a peacekeeping scenario, the position of the mediator, rather than improving, worsens. Even if the conflict is only a potential outcome of the process, the agents will use strategically the threat of declaring it. We show that the players will, at equilibrium, link the non-acceptance of their offers to the start of the conflict and that this will severely undermine the mediator's capacity to increase the egalitarianism of the agreement. In particular, we find that in peacekeeping, it will never be possible to induce a more egalitarian sharing in indirect (mediated) negotiations, than in direct (face-to-face) ones. As a conclusion, from the analysis of the two different scenarios, we observe that the capacity of the mediator to induce a higher equality in the sharing is always higher in the peacemaking situation than in the peacekeeping one.

The paper is organized as follows: in Section 2 we introduce the economic environment and the model. Section 3 analyzes the *peacemaking* situation, that is, the mediation activity when the conflict has already started, and in Section 4 we study the *peacekeeping* scenario, which corresponds to a situation previous to a potential conflict. Section 5 concludes and comments on possible extensions. We provide an Appendix in Section 6.

2 The Economic Environment

We model negotiations between two parties in the presence of a potential conflict. The players bargain *à la Rubinstein,* under complete information, over the division of a fixed surplus with value $s \in \mathbb{R}_+$. The bargaining process is infinitely repeated and players have an homogeneous discount factor $\delta \in (0, 1)$. There are two types of negotiations that differ in the timing of the bargaining process with respect to the conflict: *peacemaking* and *peacekeeping* situations.

In peacemaking processes, the conflict has already started. The negotiation takes place inside the conflict and the players bargain to try to stop it. From a game-theoretical perspective, these situations could be named as *negotiations with no-cooperation as the status-quo.* In this case, the players suffer a cost of conflict each period. The cost suffered by player i each period of conflict is $(1-\delta)c_i$. This cost is normalized so that the discounted cost of being in continuous conflict is $c_i > 0$ $(i = 1, 2)$. To simplify, we suppose that $c_1 = c > c_2 = 0$. We call player 1 the "weak" player and player 2 the "strong" player.

We also assume that $s > c$. This means that the conflict does not destroy the whole surplus that can be divided among the two players.[4]

In peacekeeping negotiations, the objective of the bargaining process is to divide the surplus s between the parties and to avoid the declaration of the conflict. Since the conflict has not already started, neither player suffers a cost each period. From a game-theoretical perspective, these can be denoted as *negotiations with no-cooperation as an outside-option.* In this bargaining process, each player has the option of breaking up the negotiations and go into the conflict. In case of opting out, negotiations will continue in a peacemaking environment.

The objective is to analyze the role of mediation in these negotiations processes. We study mediators that have the capacity to strategically intervene in the conflict in the following sense: the mediator can choose between allowing the agents involved in the conflict to conduct *direct negotiations*, or forcing them to undertake *indirect negotiations.* In the direct negotiation case (face-to-face), the players bargain *à la Rubinstein* and the only role of the mediator is to choose who is the player that has the right to start the negotiation process. In indirect negotiations (mediated), the mediator takes an active role in the process, by deciding whether to submit an offer or not. If the mediator can credibly commit not to submit a proposal (that, if submitted, would be accepted), then he will be able to alter the outcome of the bargaining process. The credibility of the threats depends crucially on the preferences of the mediator. His main trade-off is equality versus efficiency. In terms of efficiency, the best the mediator can do is not to block any proposal that would be accepted if submitted. If the mediator wants to affect the sharing, that is, to increase equality by reducing the first mover advantage of the initial proposer, he has to credibly threaten with "blocking" proposals, even if this implies (out of equilibrium) a delay in the agreement.

We parametrize the mediator's willingness to sacrifice joint surplus in order to achieve a greater equality by $\alpha \in \mathbb{R}_+$. In order to do it, we define the preferences of the mediator as follows.

Definition 1 *Consider two vectors of payoffs for players i and j, (P_i, P_j) and (P'_i, P'_j). We say that a type-α mediator, $m(\alpha)$, (weakly) prefers the first vector of payoffs to the*

[4] Note that this assumption is stronger than the necessary condition to reach an agreement, that in this case, is $s > -c$.

second one, if and only if:

$$\left|P_i' - P_j'\right| - \left|P_i - P_j\right| \geq \frac{1}{\alpha}\left[(P_i' + P_j') - (P_i + P_j)\right].$$

The left-hand side of this expression denotes how much more egalitarian is (P_i, P_j) with respect to (P_i', P_j'), and the right-hand side expresses how much more efficient is (P_i', P_j') with respect to (P_i, P_j). This construction allows for two extreme cases:

- $\alpha \to +\infty$: Equality-seeking mediator. A sharing is preferred whenever is more egalitarian.

- $\alpha \to 0$: Efficiency-seeking mediator. He selects the most egalitarian sharing, but only among those that are equally efficient.

  Moreover, it allows for intermediate situations where the mediator is willing to sacrifice *some* efficiency, in order to achieve a higher equality.

To understand better the role of the mediator, take the following example from a domestic conflict: *Consider a wage negotiation between a firm and its workers. In case of conflict of these two parties, that is, workers go on strike, the firm will suffer much less from the conflict than the workers. Suppose that the negotiation takes place without the presence of a mediator: the "weaker" position of the workers makes that, at an eventual solution of the conflict, they will receive much less. But if the mediator intervenes in the negotiation process, and it is common knowledge that he will search for an equitable solution, the "weak" party can use its "weakness" as a strategic threat to the mediator. We will see this effect more clearly in the analysis of the model.*

In the following sections, we analyze independently the two different scenarios: *peacemaking* and *peacekeeping*. In each case, we study the decision of the mediator of conducting *direct* or *indirect* negotiations.

3 In-conflict Negotiations (*Peacemaking*)

The players are in a **peacemaking** setting, that is, the negotiation takes place inside the conflict. This implies that each player suffers a stream of costs while the negotiation takes place.

3.1 Direct (face-to-face) Negotiations

The two players, 1 and 2, bargain *à la Rubinstein* over the division of a fixed surplus, s. Time runs in discrete periods of equal length, numbered by the natural numbers. At a previous stage of the negotiation the mediator chooses which of the two players has the right to start the negotiation. Suppose that the player who starts is player i. In even (odd) periods player i (player j) makes an offer. The other party may accept, thus terminating the game with agreement at the proposed shares. If he rejects, bargaining goes on to the following round. Each period until an agreement is reached, player 1 suffers a cost $(1-\delta)c$ and player 2 a cost of 0.

Proposition 1 *For any $c \in \mathbb{R}_+$ and $\delta \in (0,1)$, there exists a unique subgame perfect equilibrium (SPE) of the game where player 1 starts the negotiation. The payoffs, P_1^* and P_2^*, for players 1 and 2 at equilibrium are:*

$$P_1^* = \frac{s-\delta c}{1+\delta},$$

$$P_2^* = \frac{\delta(s+c)}{1+\delta}.$$

For the game where player 2 starts the negotiation, the unique subgame perfect equilibrium payoffs, $P_1^{'}$ and $P_2^{'}$, are:

$$P_1^{'} = \frac{\delta s - c}{1+\delta},$$

$$P_2^{'} = \frac{s+c}{1+\delta}.$$

Proof. See Appendix. ■

Now we come to analyze the difference in payoffs between the two players and the decision of the mediator of choosing who starts the negotiation.

Corollary 1 *The SPE sharing has the following distributional properties:*

- *If the "strong" player (player 2) starts the negotiation, then he receives a bigger share than the "weak" player (player 1).*
- *If the "weak" player (player 1) starts the negotiation, then he receives a bigger share than the "strong" player (player 2) if and only if*

$$\frac{s}{c} > \frac{2\delta}{1-\delta}.$$

Proof. Direct from the payoffs in Proposition 1. ■

This Corollary highlights the trade-off faced by the "weak" player (player 1). The fact he has a higher conflict cost ($c > 0$) gives him a worse bargaining position, while the first-mover advantage is beneficial for him. The effect that dominates is determined by the value of the parameters of the model. If the overall size of the surplus to share is relatively high with respect to the conflict cost, then the positive effect associated with having the initiative in the negotiation dominates, giving a higher payoff to player 1.

Proposition 2 *In direct negotiations under peacemaking, the mediator will always choose the "weak" player (player* 1*) to start the negotiation. Formally,*

$$\left(P_1^{Dpm}, P_2^{Dpm}\right) = \left(P_1^*, P_2^*\right).$$

Proof. Direct from Proposition 1. ■

3.2 Indirect (mediated) Negotiations

The basic framework is that of indirect (mediated) bargaining with complete information. The two players bargain *à la Rubinstein.* The main difference with the direct (face-to-face) case is the role of the mediator. He meets independently with each party and decides whether or not to submit this player's proposal to the other party.

The process is as follows: at any stage t the mediator meets with the party that has the right to make a proposal (player i). This player makes a proposal. The mediator meets with player j and decides whether or not to submit to him the proposal of player i. If player j receives the proposal, he can either accept it or reject it. If he accepts, the game ends. If he rejects, or if he does not receive the proposal from i, he has the right to make a counter-proposal to the mediator at stage $t + 1$, and the process is repeated.

Characterization of the Equilibrium with Mediator We first solve the equilibrium for all possible types of mediators, that is, for all the possible values of α. Then, we comment on the two extreme cases and the evolution of the equilibrium payoffs depending on the preferences of the mediator.

We restrict the analysis to stationary strategies. The following stationary strategies for the two agents that are involved in the negotiation, are the only stationary strategies that can be optimal for these players: player 1 always offers $(x, s - x)$ and rejects anything less than w. Player 2 always offers $(y, s - y)$ and rejects anything less than z. The reason is that if any of the players decides to reject an offer, then he must reject any offer that is strictly worse for him. Moreover, at equilibrium, each player will offer the minimum amount that the other agent will accept and that the mediator still submits. This implies that $w = y$ and $z = s - x$.

The following conditions are necessary for the strategies above mentioned to be a stationary subgame perfect equilibrium (SSPE). The optimal strategy of the mediator is determined by conditions (M1) and (M2). The mediator, when a player proposes an offer, has to decide whether to submit it to the other party or not. He will submit the proposal whenever the trade-off between efficiency and equality of the final sharing is preferred to the sharing that induces the continuation of the game. Condition (M1) ensures that the offer made by player 1 will not be blocked by the mediator, that is, the mediator prefers $(P_i, P_j) = (x, s - x)$ to $(P'_i, P'_j) = (\delta y - (1 - \delta)c, \delta(s - y))$. Condition (M2) is the analogous condition for the offer of player 2. We do not need to specify the mediator's strategy because the equilibrium payoffs that arise from these conditions are unique.

(1) $y \geq \delta x - (1 - \delta)c$.

(2) $s - x \geq \delta(s - y)$.

(3) $x \geq \delta y - (1 - \delta)c$.

(4) $s - y \geq \delta(s - x)$.

(M1) $\frac{(1-\delta)}{\alpha}(s + c) \geq |2x - s| - |\delta(2y - s) - (1 - \delta)c|$.

(M2) $\frac{(1-\delta)}{\alpha}(s + c) \geq |2y - s| - |\delta(2x - s) - (1 - \delta)c|$.

The equilibrium of this game has the following properties: the SSPE payoffs can be cathegorized in three branches. First, if α (i.e., the capacity of the mediator to sacrifice efficiency) is sufficiently low the mediator is, at equilibrium, *completely inactive*. The negotiation is conducted as if there was no filter to the agents' proposals. The equilibrium sharing, therefore, coincides with the one under direct negotiations, since the binding conditions in this case are (1) and (2). On the other extreme, if α is sufficiently high and at the same time the damage caused by the conflict is relatively low (i.e. $\frac{s}{c}$ sufficiently large), the equilibrium is *fully mediated*. This means that the threat of blocking the proposal received is binding for both agents (conditions (M1) and (M2)). This way, neither of them can fully benefit from his position as a proposer. For intermediate cases, the equilibrium is *partially mediated*. In this range of parameter values, the mediator's position is not strong enough to drive completely the negotiation but, still, it has influence over the outcome. At equilibrium, the mediator, actively threats the strong player in order to reduce his advantage as a proposer, but allows the weak player to profit completely from his position when proposing (the binding conditions in this case are (2) and (M2)).

In the following Proposition we describe the equilibrium of this game. The different equilibrium payoffs for each of the branches are described in the proof of the Proposition that is provided in the Appendix. We use the following notation:

$$\alpha_1 = \frac{(1+\delta)^2(s+c)}{s(1-\delta^2)-c(1-\delta)^2},\ \alpha_2 = \frac{(1-\delta^2+2\delta)(s+c)}{s(1+\delta^2)-c(1-\delta^2+2\delta)},$$
$$\alpha_3 = \frac{\delta^2(s+c)}{\delta s - c},\ \alpha_4 = \frac{(1-\delta^2)(s+c)}{(1-\delta)^2 s+(3+\delta^2)c}.$$

Proposition 3 *In indirect negotiations with a type-α mediator, the role of the mediator and the main characteristics of the SSPE are as follows:*

- *If $\alpha \leq \min\{1, \alpha_3\}$, then the mediator is inactive.*
- *If $\alpha \geq \max\{\alpha_1, \alpha_2\}$ and $\frac{s}{c} > \frac{1+2\delta-\delta^2}{1+\delta^2}$, then the equilibrium is fully mediated.*
- *If $\alpha \geq 1$ and $\frac{s}{c} < \frac{1+2\delta-\delta^2}{1+\delta^2}$, or $\max\{1, \alpha_4\} \leq \alpha \leq \max\{\alpha_1, \alpha_2\}$ and $\frac{s}{c} > \frac{1+2\delta-\delta^2}{1+\delta^2}$, then the equilibrium is partially mediated.*
- *If $\alpha_3 \leq \alpha \leq \alpha_4$ and $\frac{s}{c} > \frac{1+2\delta-\delta^2}{1+\delta^2}$, then the mediator is inactive or partially mediated depending on the equilibrium chosen.*

Proof. See Appendix. ∎

Figure 1 presents an illustration of this equilibrium configuration.

[Insert Figure 1]

Some insights can be extracted from this general characterization. First, note that the mediator's willingness to sacrifice efficiency on the grounds of a higher equality (parametrized in the model by α) crucially determines the outcome of the negotiations. This commitment to achieve equality, even at the cost of destroying resources, if necessary, gives credibility to the mediator's threat to block proposals and, hence, alters the equilibrium sharing. Second, an important element that affects the mediator's capacity to influence the negotiation is how costly the conflict is. The relative damage of continuing the conflict with respect to the total amount of resources to share (i.e., $\frac{s}{c}$) measures how costly it is for the mediator to actually intervene in the negotiation.

In order to extract more insights on the implications of an active mediator, we consider now the prediction that Proposition 3 makes for the two extreme cases of the preferences of the mediator, that is, *efficiency-seeking mediator* ($\alpha \to 0$), and *equality-seeking mediator* ($\alpha \to +\infty$).

Proposition 4 *If $\alpha = 0$ (fully efficiency-seeking mediator), the two negotiation processes, that is, direct and indirect, are equivalent.*

Proof. Direct from Proposition 3. ■

This Proposition confirms our previous claim that only mediators that can credibly commit to delay the negotiations (and, hence, destroy resources), do have an impact on the final sharing. Otherwise, they become simple passive actors. This can be interpreted, in fact, as an illustration of the "outside option" principle: *only threats that are credible will have an effect on the outcome.*[5]

Let us move now to the opposite, and more interesting, case.

Proposition 5 *In indirect negotiations with an equality-seeking mediator* ($\alpha \longrightarrow +\infty$)*, there exists a unique SSPE in which the offers made by the players are the following:*

- *If* $\frac{s}{c} \geq \frac{1+2\delta-\delta^2}{1+\delta^2}$ *then,*

$$x = \frac{s}{2} + \frac{(1-\delta^2)}{2\left(1+\delta^2\right)}c,$$
$$y = \frac{s}{2} - \frac{(1-\delta)^2}{2\left(1+\delta^2\right)}c.$$

- *If* $\frac{s}{c} \leq \frac{1+2\delta-\delta^2}{1+\delta^2}$ *then,*

$$x = \frac{s}{2} + \frac{s-\delta c}{2(1+\delta)},$$
$$y = \frac{s}{2} - \frac{c-\delta s}{2(1+\delta)}.$$

Proof. Direct from the Proof of Proposition 3 in Appendix. ■

Several insights emerge from this Proposition. The first, and most obvious one is that, even if the mediator is willing to completely sacrifice efficiency in order to achieve a higher equality, he is not able to induce a fully egalitarian sharing. The reason is that the existence of conflict costs that affect more one player than the other, is a source of inequality that undermines the capacity of the mediator. The tool the mediator has to increase the egalitarianism of the sharing is to threaten the proposer with blocking his offer (implying a delay in the resolution of the conflict). The larger asymmetry of the conflict costs, the more inequality this blocking will generate and, therefore, the weaker the position of the mediator (the less credible his threat). This, together with the fact that some first-mover advantage still exists, implies that, even in this extreme case in which the mediator would be willing to postpone indefinitely the agreement if needed, he is not capable of forcing any of the players to make a fully egalitarian offer.

[5] See, for instance, Sutton (1986), for a clear and intuitive explanation of this principle.

Second, the structure of the equilibrium is different, depending on the values of the parameters. When $\frac{s}{c} \geq \frac{\left(1+2\delta-\delta^2\right)}{1+\delta^2}$, the equilibrium offers are completely determined by the mediator's threats to block their proposals (see in the proof that in this case the binding conditions that determine the equilibrium offers are (M1) and (M2)). In this case, neither of the players can, at equilibrium, fully benefit from his position when having the initiative in the negotiation. However, if s is relatively low with respect to the conflict cost (c) (that is, when $\frac{s}{c} < \frac{\left(1+2\delta-\delta^2\right)}{1+\delta^2}$), the equilibrium configuration is different. In this case, the position of player 1 is very weak with respect to player 2 and, hence, the mediator allows this player to fully use his advantage when he is the proposer (player 1's offer is determined by the acceptance-rejection decision of player 2). At the same time, the mediator restricts the first-mover-advantage of player 2 (the strongest) by using his power to block proposals.

Focusing on the first case, we can see the more striking and, at first sight, more counter-intuitive result of the intervention of the mediator. *When the weak player is the proposer, his share of the surplus is increasing in his own conflict costs.* In the direct negotiation case, the result is completely the opposite. The reasoning is the following: the larger the conflict cost of one player, the smaller his bargaining power and, hence, the smaller share of the surplus he gets. However, this reasoning fails here, because the player's equilibrium offers are not determined by the other player's reaction, but by the mediator's blocking threat.

In a sense, the "competitor" of player 1 is not directly player 2, but rather the mediator. The stronger the position of the mediator, the less capacity will have player 1 to exploit his first mover advantage as a proposer, and vice versa. The higher c, the less credible will be the mediator's threat to block proposals, as it would imply an important source of inequality in the final sharing. As a result of this, the larger c, the bigger the share that player 1 can ask for himself, without triggering a block from the mediator. This paradoxical result can also be restated in other terms: the quest for equality makes the mediator become an agent for the weak player.

The example of the wage negotiation that we referred to in Section 2 may help to clarify this point: as we already argued, in this setting, and in the absence of a mediator,the position of the workers is much weaker and, hence, in the eventual solution of the conflict it will receive much less. When a mediator steps in, if it is common knowledge that this mediator will search for an equitable solution, things change dramatically. Now, the weak party, the workers, use their high losses, if the conflict continues, as a strategic threat. The fact that they know that the mediator will not allow the conflict to continue, gives them a very strong position at the expense of the strongest party, the firm.

As a consequence of this effect, we have the following result.

Corollary 2 *When $\alpha \to \infty$, the mediator achieves a* ***higher equality*** *by giving the* ***first-mover advantage to the strongest player*** *in the conflict. Formally,*

$$\left(P_1^{Ipm}, P_2^{Ipm}\right) = (y, s-y).$$

Proof. See Appendix. ■

Once again we see how the presence of an active mediator has very important implications. The fact that the mediator's intervention greatly over-reinforces the negotiation position of the weak player makes that now, and contrary to the case with direct negotiations, more equality can be achieved by giving the initiative in the negotiation to the ex-ante strong player, as he has a weaker bargaining position with respect to the mediator.

Finally, one can see how the mediator's activity, instead of improving over the direct negotiations in terms of equality, can turn out to be detrimental. To show it, we compare the payoffs achieved with **direct** and **indirect** negotiations.

From the Proposition above, we know that the payoffs of the **indirect** (mediated) negotiation (with the strongest player (player 2) proposing first) are:

- If $\frac{s}{c} \geq \frac{1+2\delta-\delta^2}{1+\delta^2}$ then,

$$\begin{aligned} P_1^{Ipm} &= \frac{s}{2} - \frac{(1-\delta)^2}{2\left(1+\delta^2\right)}c, \\ P_2^{Ipm} &= \frac{s}{2} + \frac{(1-\delta)^2}{2\left(1+\delta^2\right)}c. \end{aligned}$$

- If $\frac{s}{c} \leq \frac{1+2\delta-\delta^2}{1+\delta^2}$ then,

$$\begin{aligned} P_1^{Ipm} &= \frac{s}{2} - \frac{c-\delta s}{2(1+\delta)}, \\ P_2^{Ipm} &= \frac{s}{2} + \frac{c-\delta s}{2(1+\delta)}. \end{aligned}$$

From the analysis already performed, we know that the payoffs of the **direct** (face-to-face) negotiations are:

$$P_1^{Dpm} = \frac{s-\delta c}{1+\delta} \text{ and } P_2^{Dpm} = \frac{\delta(s+c)}{1+\delta}.$$

In Corollary 3 below, we compare the degree of inequality in the presence and in the absence of the mediator:

Corollary 3 *There exist two thresholds in the discount factor,* (δ_1, δ_2), $0 < \delta_1 < \delta_2 < 1$ *such that:*

- *If $\delta \leq \delta_1$ the **equality** achieved is **higher** in the **presence** of an active mediator.*
- *If $\delta \in (\delta_1, \delta_2)$ the **equality** achieved is **lower** in the **presence** of an active mediator.*
- *If $\delta \geq \delta_2$ the **equality** achieved is **higher** in the **presence** of an active mediator.*

Proof. See Appendix. ■

[Insert Figure 2]

The result of this Corollary is, at first sight, quite surprising: the presence of an active mediator that threatens the parties with blocking "unequal" proposals, may result, ex-post, in a higher degree of inequality. The explanation for this has to do with the nature of the mediator's intervention. The presence of conflict costs reduces the effective capacity of the mediator to equalize payoffs, and this weakness will be strategically used by the players in their own benefit.

As a result, for extreme cases of δ, the intervention of the mediator always improves over the face-to-face situation. When the value of δ is very low, then the first-mover advantage of the initial proposer is very important and the "blocking" capacity of the mediator effectively reduces its impact on the final sharing. On the other extreme, when δ is sufficiently high, the advantage for the "weak" player of starting the negotiation is almost inexistent and his share is substantially reduced by the fact of having higher conflict costs. In this case, the mediator's intervention also recovers the equality lost. Moreover, the higher is the degree of patience of the players, the higher is the capacity of the mediator to induce an egalitarian sharing.

However, for intermediate values of δ, the inequality of the face-to-face negotiation is relatively low as the first-mover-advantage of the "weak" player allows him to compensate for his higher conflict costs in such a way that the final sharing is relatively balanced. The intervention of the mediator in this case is not able to improve the equality achieved in a direct negotiation, since in the indirect negotiations the players use strategically the "weakness" of the mediation activity, that comes from the presence of conflict costs, in their own benefit.

4 Preconflict Negotiations (*Peacekeeping*)

The players are in a **peacekeeping** setting, where negotiations take place previous to a potential conflict. These games correspond to situations with no-cooperation as an outside option. The players that bargain over the division of the surplus have the option of breaking out negotiations and go to the conflict. If this happens, they receive their outside option payoffs, that will be denoted by (P_i^*, P_j^*). These payoffs will be determined by the outcome of the peacemaking negotiations. We state the general results in the following Lemma and Proposition, for any outside payoffs that satisfy $s \geq P_i^* + P_j^*$. In a posterior analysis, we concentrate on a particular case of payoffs to be able to extract more interesting implications of the mediator's activity.

4.1 Direct (face-to-face) Negotiations

Player 1 and player 2 bargain over the division of s. Call player i the one that has the right to start the negotiation. Each player has the option of breaking up the negotiations and go into the conflict. In even (odds) periods player i (player j) makes an offer. The other party may accept and the game ends with agreement at the proposed shares. Alternatively, if he rejects, either of the two parties may decide to start the conflict, in which case both receive their outside payoffs, (P_i^*, P_j^*). If the offer is rejected but neither player opts out, then bargaining goes on to the following round.

The framework and solution is the one used in Ponsatí and Sákovics (1998), but we restrict attention to stationary strategies (independent of t) and we allow the outside payoffs of the players to be negative. We first prove the following Lemma.

Lemma 1 *In a direct negotiation under a peacekeeping scenario, for any outside payoffs* $(P_i^*, P_j^*) \in \mathbb{R}^2$, *such that* $s \geq P_i^* + P_j^*$, *immediate agreement at* $(s - P_j^*, P_j^*)$ *is an outcome that can be supported by a SSPE, when player* i *is the first to propose. Moreover, it is the unique SSPE when* $P_i^* > \delta^2 \left(s - \frac{P_j^*}{\delta}\right)$ *and* $P_j^* > \delta^2 \left(s - \frac{P_i^*}{\delta}\right)$. *Otherwise, the outcomes that can be supported by a SSPE are immediate efficient agreements that give player* i *a payoff in* $\left[s(1-\delta) + P_i^*, s - P_j^*\right]$.

Proof. Consider the following strategies: if player i is the proposer he always asks for $s - P_j^*$; the responder accepts any proposal that is not worse than the (candidate) equilibrium proposal; if the proposer asks for more, then the responder rejects and takes his outside option; if the responder does not accept a proposal, the proposer opts out. It is straightforward to verify that these strategies constitute a SSPE.

To prove the second part of the Lemma, note that for the lowest possible share for player i to get at equilibrium we need the following: player i should (weakly) prefer continuing to opting out when player j rejects his proposal. Otherwise, the only candidate equilibrium offer will be for player i to ask for $s - P_j^*$, because player $j's$ threat to reject would not be credible. From this, we know that the continuation value of player i from the next period must be at least $\frac{P_i^*}{\delta}$. Since player j should be indifferent between accepting and waiting for next period, his maximum possible share is $\delta\left(s - \frac{P_i^*}{\delta}\right)$. By symmetry, the maximum possible share for player i is $\delta\left(s-\frac{P_j^*}{\delta}\right)$, and therefore, $\frac{P_i^*}{\delta}\leq\delta\left(s-\frac{P_j^*}{\delta}\right)$ and $\frac{P_j^*}{\delta}\leq\delta\left(s-\frac{P_i^*}{\delta}\right)$. Given these conditions, both players (weakly) prefer this agreement to their option. It is straightforward to see that these conditions are also sufficient. ■

Note that a necessary condition for having multiplicity of equilibria is that $\delta s > P_i^* + P_j^*$. In the posterior analysis, we concentrate on a particular vector of outside payoffs from the peacemaking negotiation to extract more meaningful implications of the mediator's activity. Since these payoffs do not satisfy this condition, we study in what follows the case where the unique SSPE payoffs are $(s-P_j^*, P_j^*)$.

From the Lemma above, we observe that, in this case, the possibility of both players to opt out, implies that the threat of the proposer of going to the outside option if his offer is rejected is always credible. From here, we can conclude the following about the relationship between **peacemaking** and **peacekeeping**: *in a peacekeeping setting, where the agents face a direct negotiation with the outside option of going to a peacemaking situation, the outcome is completely determined by the outcome of the peacemaking process.*

This implies that the equilibrium sharing with peacekeeping is, in fact, the same as with peacemaking. This means that in the absence of an active mediator, even if the players are not actually in conflict, the sharing is as if they were in it.

4.2 Indirect (mediated) Negotiations

The game that we analyze is as follows: at any stage t, the mediator meets with the party that has the right to make a proposal (player i). This player makes an agreement proposal. The mediator meets with player j and decides whether or not to submit to him the proposal of player i. If player j receives the proposal, he can accept and the game ends. If j rejects (or does not receive the offer), either of the two parties may decide to start the conflict. If the offer is rejected (or not submitted) but neither player opts out, bargaining goes on to the following round (the mediator meets with player j and so on).

There are interesting effects that may arise when we allow the mediator to intervene in the peacekeeping scenario, that is, before any of the players opts out and moves to the peacemaking situation. First, the lack of conflict costs per-period of delay in the peacekeeping negotiations, makes the mediator less constrained about efficiency considerations, which may ease his intervention in terms of achieving a higher equality, for a given level of α. Second, this effect may be possibly outweighed, as a greater equality may increase the incentives of the player that loses by this increased equality to opt out and start the conflict, moving to a peacemaking situation. The formal statement of the results is the subject of the following Proposition.

Proposition 6 *For the mediated game in a peacekeeping situation with outside options,* *where the outside payoffs are such that* $s \geq P_i^* + P_j^*$, *the SSPE payoffs* $(x_i^*, s - x_i^*)$ *are:*

- *If* $P_i^* \leq P_j^* - \left|s - 2P_j^*\right|$, *and if not, whenever* $\alpha \leq \alpha_i^0 \equiv \frac{\left(s-(P_i^*+P_j^*)\right)}{\left|s-2P_j^*\right|-\left|P_i^*-P_j^*\right|}$, *then*

$$x_i^* = s - P_j^*.$$

- *Otherwise,*

$$x_i^* = \frac{s}{2} + \frac{1}{2}\left(\frac{1}{\alpha}\left(s - \left(P_i^* + P_j^*\right)\right) + \left|P_i^* - P_j^*\right|\right).$$

Proof. See Appendix. ■

From this general case, we can extract some preliminary insights on the role of a mediator in a peacekeeping negotiation. First, and fully consistent with our previous results, a necessary condition for a mediator to have an active role in the negotiation process is that he is actually willing to sacrifice efficiency in order to induce a higher equality, otherwise, he becomes a purely passive observer. However, and contrary to the peacemaking case, this necessary condition is not sufficient. In this setting it can be the case that even a fully equality-seeking mediator ($\alpha \to \infty$) is incapable of altering the equilibrium sharing. This occurs when the sharing that results from the outside options is very unbalanced against the proposer as, thus, the mediator loses its capacity to effectively threaten him.

In those cases in which the mediator actually alters the distribution, his intervention is of a different nature than in a peacemaking environment. In a peacekeeping negotiation, the threat of the mediator is not to reduce the proposer's first-mover advantage and pass it to the responder (as in the peacemaking setting), but rather to actually enforce the implicit threat made by the proposer himself (to move to the conflict). This is the best the mediator can aim at achieving since the outside option is a safe outcome the players can always ensure for themselves.

Let us now move to a more specific case in which we can extract more meaningful implications of the mediator's activity: the equality-seeking mediator ($\alpha \to \infty$). Moreover, when negotiations are broken, the outside option is, in fact, the start of the conflict. This means that the agents will suffer one round of damages (with costs $(1 - \delta)c$ for player 1 and 0 for player 2) and will continue negotiating under a peacemaking environment.[6]

[6] We assume that there is no delay in the start of the negotiations after the conflict breaks out. This allows us to eliminate a purely artificial source of equity given by discounting (that reduces the present value of the differences in payoffs across agents).

With this, the actual value of the outside option is:[7]

$$\begin{aligned} P_1^* &= P_1^{Ipm} - (1-\delta)\, c, \\ P_2^* &= P_2^{Ipm}. \end{aligned}$$

Note that this is the most natural case of study. In it, the parties try to negotiate in the absence of a conflict and know that, if eventually, one of the parties decides to start the conflict, this will mean a flow of damages (unevenly distributed among the players) and the continuation of the negotiations in a new scenario (peacemaking). Focusing on this case, we can find the following corollaries:

Corollary 4 *An equality-seeking mediator ($\alpha \to \infty$) will, at equilibrium:*

- *Always be able to affect the equality of the sharing when the "strong" player moves first.*
- *Never be able to affect the equality of the sharing when the "weak" player moves first.*

Proof. When $\alpha \to \infty$, the mediator can alter the equilibrium proposal of the player with the right to start the negotiation (player i) if and only if

$$P_i^* > P_j^* - \left| s - 2P_j^* \right|.$$

Consider first the case $i = 2$ (i.e., the strong player). Using the fact that $P_1^* < P_2^*$ it is straightforward to see that the above condition is always fulfilled for the equilibrium values.

Conversely, consider the case $i = 1$ (i.e., the weak player). Substituting the equilibrium values for P_1^* and P_2^*, we rewrite the above condition as

$$P_1^{Ipm} - (1-\delta)\, c > s - P_2^{Ipm}.$$

And this condition never holds, since $P_1^{Ipm} + P_2^{Ipm} = s$. ■

Again, and analogous to a peacemaking situation, we observe how giving the initiative to the weak player is a source of problems for the mediator. Recall that in a peacemaking negotiation, the reason was that the weak player was able to pass more demanding

[7] The posterior analysis and results correspond to the following cases: $\delta < \delta_1$, $\delta > \widetilde{\delta}$, and $\delta \in [\delta_1, \widetilde{\delta}]$ and $\frac{s}{c} < \frac{1-\delta^2+2\delta}{2(1-\delta)}$. For the case where $\delta \in [\delta_1, \widetilde{\delta}]$ and $\frac{1-\delta^2+2\delta}{(1-\delta)} > \frac{s}{c} > \frac{1-\delta^2+2\delta}{2(1-\delta)}$, the result obtained in Corollary 4 is the opposite, but the results of Corollary 5 and Proposition 7 remain the same. When $\delta \in [\delta_1, \widetilde{\delta}]$ and $\frac{s}{c} > \frac{1-\delta^2+2\delta}{(1-\delta)}$, the results obtained in Corollaries 4 and 5 and Proposition 7 are the opposite. We disregard these last cases since the range of parameters that satisfy them is little significant.

proposals through the mediator, exploiting the fact that, if they were blocked, this would imply high damages for this player. In a peacekeeping setting, the result, although related, is of a different nature. Here, the effect is not quantitative (passing more uneven proposals) but rather qualitative (when the mediator is really active). The quantitative aspect is lost, since the players always link their proposals to fixed quantities (the outside option). What the mediator loses by giving the first-mover advantage to the weak player is the capacity to effectively threaten the proposer with executing the outside option, as this sharing, is a very unbalanced one.

Corollary 5 *In* ***peacekeeping,*** *when the mediator's intervention alters the behavior of the players, the equality achieved is always smaller than in* ***peacemaking.***

Proof. First, in peacekeeping, if the mediator is active, the payoff of the proposer is

$$x_i^* = \frac{s}{2} + \frac{1}{2}\left(\frac{1}{\alpha}\left(s - \left(P_i^* + P_j^*\right)\right) + \left|P_i^* - P_j^*\right|\right).$$

It is straightforward to see that the equality induced is higher, the higher is α. For $\alpha \to \infty$ we have (substituting the equilibrium values for the outside option payoffs):

$$x_i^* = \frac{s}{2} + \frac{1}{2}\left|P_1^{Ipm} - (1-\delta)\,c - P_2^{Ipm}\right|.$$

Since $x_i^* > \frac{s}{2}$, then the inequality of the sharing is given by:

$$2x_i^* - s = \left|P_1^{Ipm} - (1-\delta)\,c - P_2^{Ipm}\right|.$$

As in peacemaking we have shown that $P_1^{Ipm} \leq P_2^{Ipm}$, then:

$$2x_i^* - s = P_2^{Ipm} - P_1^{Ipm} + (1-\delta)\,c > P_2^{Ipm} - P_1^{Ipm}.$$

This completes the proof. ■

Even if under peacemaking we saw that the fact of being inside a conflict was a source of weakness for the mediator, we see how the situation in pre-conflict environments, instead of improving, turns out to be even worse. Even if, a priori, in peacekeeping when the mediator delays the solution of the conflict this does not imply imposing conflict costs for the parties, it is constrained by the fact that the agents link the rejection of their proposals, to the start of the conflict. This severely undermines his capacity to induce an egalitarian solution to the conflict.

It is direct to see that, if the mediator can actually affect the proposer's strategy and, hence, the sharing resulting from the negotiation, the distribution is completely determined by the outside option of the players. This implies, therefore, that the equilibrium level of inequality is the same independently of which player starts the negotiation.[8] This is in contrast with the result in peacemaking where the mediator strictly preferred to give the first-mover advantage to the ex-ante "strong" player.

However, so far, we have not seen when the presence of an active mediator is really beneficial in terms of equality. That is, we have not compared the degree of inequality obtained with indirect negotiations with that resulting from a direct (face-to-face) negotiation in which the mediator does not have the capacity to block the proposals made by the players. By doing so, we find:

Proposition 7 *In terms of the equality of the final sharing and in a peacekeeping environment, the presence of an active mediator never improves over a direct negotiation in which the weak player is the first proposer.*

Proof. Follows directly from comparing the equilibrium sharing in Lemma 1 and Proposition 6. ■

This result shows the extent to which the position of the mediator is weakened in a peacekeeping scenario. His intervention threatening the parties with blocking their proposals is completely unable to induce a more egalitarian sharing than simply letting the parties negotiate directly (giving the weak player the first-mover advantage). Moreover, it can be checked that, whenever the mediator is really active at equilibrium, i.e., actually alters the proposals of the players, then his presence is detrimental for the equality.

This extreme negative result is driven by the fact that in a peacekeeping situation, the parties use the threat to start the conflict as a strategic device (even the weak player uses this threat). This puts the mediator in a very weak position as any intervention aimed at reducing the advantage of the proposer will generate a break out of the conflict.

The intuition for this result is the following: in the direct negotiation case (Ponsatí and Sákovics, 1998), the proposer can credibly commit to opting out and this prevents the responder from using all his bargaining power. When the mediator intervenes (indirect negotiation case), the proposer has to credibly threat, not only the responder, but also the mediator. Since the mediator has preferences over equality and efficiency, the proposers "lose" bargaining power with respect to the responders when α increases, given that a sufficiently high α implies that the mediator prefers not to submit a given offer and go to the outside option.

[8] Note that this does not mean that the equilibrium payoffs are the same, the initial proposer always gains more.

5 Conclusion

We have proposed a model of bargaining that allows us to analyze and compare the role of mediation in conflicts of different nature, where two parties negotiate to share a fixed surplus.

We have distinguished between *peacemaking* (in-conflict) negotiations and *peacekeeping* (pre-conflict) negotiations. The main difference is that in a peacemaking scenario, since the conflict has already started, the players bear costs per-period of conflict. Since these costs are different, the players are asymmetric.

We deeply analyze the negotiations in a peacemaking scenario in order to study the role of a mediator that strategically intervenes in the process of bargaining. We observe two important implications of this analysis: first, even if the mediator is willing to sacrifice completely efficiency in order to achieve a higher equality, he is not able to induce a fully egalitarian sharing. Second, we also prove that in this case, more equality can be achieved by giving the right to start the negotiation process to the ex-ante strong player (which is contrary to the unmediated bargaining processes). This can be explained by the fact that the "competitor" of the weakest player is not directly the other player, but rather the mediator.

In a peacekeeping scenario, the conflict is not active yet and, therefore, is only a potential outcome of the process. We find that, even if in this setting the absence of conflict costs would, in principle, help the mediator in achieving a higher equality, the result is completely the opposite. In a peacekeeping negotiation, the agents will always use the threat of breaking out the conflict (and moving to a peacemaking environment) in their negotiation strategy. As a result, the position of the mediator, and its capacity to induce an egalitarian sharing, is always weaker than in peacemaking.

At this point we can bring back the interview with a UN international negotiator, quoted in the Introduction. In it, Françesc Vendrell supported the option of using indirect (mediated) meetings to conduct peacemaking negotiations and direct (face-to-face) meetings for peacekeeping negotiations. Our results are consistent with Vendrell's choice. First, we have seen how, in many instances, a mediated negotiation can achieve more equality than a direct one in a peacemaking environment. In this situation, the capacity of the mediator to "filter" the communications among the parties can be a useful distributive tool. Secondly, we have shown that in a peacekeeping negotiation, this is not the case. Using the mediator as a filter between the two parties can never increase, and in some cases decreases, the resulting egalitarianism of the sharing and, hence, never improves over a direct negotiation.

Finally, it is worth mentioning that this model has to be seen as a first step in a new line of research. We believe that studying the impact that a mediator (understood as a strategic player) can have over an ongoing negotiation process, is a very interesting and potentially fruitful research. These issues, however, have received little attention so far in the literature. In this model we have dealt with the simplest interesting case: mediation in complete information bargaining. The next natural step is to study the same issues in a richer framework in which parties negotiate with two-sided asymmetric information. This would allow to analyze how a mediator can have effects not only in terms of the equality of the sharing, but also in the efficiency achieved.

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6 Appendix

Proof. (Proposition 1)

(following Fudenberg and Tirole (1996))

Consider the following (stationary) strategies: player 1 always offers $(x, s-x)$ and rejects anything less than y. Player 2 always offers $(y, s-y)$ and rejects anything less than $s-x$. The following conditions are necessary for the strategies above mentioned to be a subgame perfect equilibrium:

(1) $y \geq \delta x - (1-\delta)c$.

(2) $s - x \geq \delta(s-y)$.

(3) $x \geq \delta y - (1-\delta)c$.

(4) $s - y \geq \delta(s-x)$.

Note that conditions (1) and (2) have to be satisfied with equality since each player will offer the lowest share that the other will accept. Taking (1) and (2) together with equality, we get the following:

$$x = \frac{s - \delta c}{1+\delta};\ s - x = \frac{\delta(s+c)}{1+\delta};$$

$$y = \frac{\delta s - c}{1+\delta};\ s - y = \frac{s+c}{1+\delta};$$

With these offers, also conditions (3) and (4) are satisfied, since $s > -c$.

To prove uniqueness of equilibria, call $\overline{s}_1$ the supremum of the payments of player 1 at any subgame perfect equilibrium and $\underline{s}_1$, the infimum of these payoffs. Similarly, call $\overline{s}_2$ and $\underline{s}_2$, for player 2.

Player 1 offers $(x, s-x)$. Player 2 accepts for sure if

$$s - x \geq \delta \overline{s}_2 \Leftrightarrow x \leq s - \delta \overline{s}_2.$$

But any x lower than this value cannot be part of a subgame perfect equilibrium, since player 1 would be better off offering exactly that value. Therefore,

$$\underline{s}_1 \geq s - \delta \overline{s}_2. \tag{1}$$

We use the same argument for player j, and obtain

$$\underline{s}_2 \geq s - \delta \overline{s}_1 + c(1-\delta). \tag{2}$$

Since player 2 can obtain at least $\delta \underline{s}_2$, he would not accept anything less, so the most that player 1 can guarantee himself when making an offer is $s - \delta \underline{s}_2$. Therefore,

$$\overline{s}_1 \leq s - \delta \underline{s}_2, \tag{3}$$

and, similarly,

$$\overline{s}_2 \leq s - \delta \underline{s}_1 + c(1-\delta). \tag{4}$$

Together (1) and (4) bring us to

$$\underline{s}_1 \geq \frac{s - \delta c}{1+\delta}, \tag{5}$$

and, (2) and (3) to

$$\overline{s}_1 \leq \frac{s - \delta c}{1+\delta}. \tag{6}$$

We can conclude with (5) and (6) that

$$\underline{s}_1 = \overline{s}_1 = \frac{s - \delta c}{1+\delta},$$

and the same for player 2

$$\underline{s}_2 = \overline{s}_2 = \frac{\delta(s + c)}{1+\delta}.$$

And since this is the unique subgame perfect equilibrium, each player should accept when his shares are the ones at equilibrium. ■

Proof. (Proposition 3)

We first prove the following intermediate result that will help us to characterize the equilibrium outcomes:

- If at equilibrium (1) is not binding, then $y \leq \frac{s}{2}$.
- If at equilibrium (2) is not binding, then $x \geq \frac{s}{2}$.

Consider, first, the case in which (1) is not binding and assume that the equilibrium is such that $y > \frac{s}{2}$. Consider the following deviation for player 2 when he is the proposer: $y' = y - \varepsilon$, with $\varepsilon > 0$ and sufficiently small. Since (1) was not binding, there exist values of $\varepsilon > 0$ for which this condition still holds and, therefore, player 1 still finds the offer acceptable. Moreover, since $y > \frac{s}{2}$, the condition M2 is still fulfilled for y' and the new offer of player 2 is not blocked by the mediator. Therefore, player 2 has a profitable deviation since the new offer gives him a larger share of the surplus. An analogous argument allows to show that $x < \frac{s}{2}$ cannot be sustained as an equilibrium when (2) is not binding.

Now we prove that the equilibrium outcome cannot be such that conditions (1) and (M1) are binding. In this case, the only candidate equilibrium payoffs are the following (restricting to $y \leq \frac{s}{2}$ and $\delta(2x-s) \leq (1-\delta)c$, since the rest of the cases are eliminated because either condition (2) or (M2) are not satisfied):

$$\begin{aligned} x &= \frac{s(1-\delta+\alpha+\alpha\delta)}{2\alpha(1+\delta^2)} + \frac{c(1-\delta)(1+\alpha+2\alpha\delta)}{2\alpha(1+\delta^2)} \geq \frac{s}{2}, \\ y &= \delta x - (1-\delta)c \leq \frac{s}{2}. \end{aligned}$$

But in this case we can prove that these payoffs are not compatible with $\delta(2x-s) \leq (1-\delta)c$.

We are now in the position to fully characterize the outcome of the indirect (mediated) negotiations, in the presence of a type-α mediator. We use the following notation:

$$\begin{aligned} \alpha_1 &= \frac{(1+\delta)^2(s+c)}{s(1-\delta^2)-c(1-\delta)^2};\ \alpha_2 = \frac{(1-\delta^2+2\delta)(s+c)}{s(1+\delta^2)-c(1-\delta^2+2\delta)} \\ \alpha_3 &= \frac{\delta^2(s+c)}{\delta s - c};\ \alpha_4 = \frac{(1-\delta^2)(s+c)}{(1-\delta)^2 s + (3+\delta^2)c};\ \alpha_5 = \frac{\delta(1+\delta)(s+c)}{c(1-\delta)}; \end{aligned}$$

The offers made by the players in the SSPE are the following:

- For $\alpha \leq \min\{1, \alpha_3\}$,

$$\begin{aligned} x &= \frac{s-\delta c}{1+\delta}, \\ y &= \frac{\delta s - c}{1+\delta}. \end{aligned} \tag{IM}$$

- For $\alpha_1 \leq \alpha \leq \alpha_5$,

$$\begin{aligned} x &= \frac{s}{2} + \frac{s+c(1+\alpha)}{2\alpha} - \frac{\delta c}{1+\delta}, \\ y &= \frac{s}{2} - \frac{s+c(1+\alpha)}{2\alpha} + \frac{c}{1+\delta}. \end{aligned} \tag{FM$_1$}$$

- For $\alpha_2 \leq \alpha \leq \alpha_1$,

$$\begin{aligned} x &= \frac{s(2\alpha-\delta-\alpha\delta+\alpha\delta^2+\delta^2)-\delta c(1-\alpha)(1-\delta)}{2\alpha(1+\delta^2)}, \\ y &= \frac{s(\alpha+\delta-\alpha\delta+2\alpha\delta^2-1)-c(1-\alpha)(1-\delta)}{2\alpha(1+\delta^2)}. \end{aligned} \tag{PM$_1$}$$

- If $\frac{s}{c} \geq \frac{1+2\delta-\delta^2}{1+\delta^2}$ and $\alpha \geq \max\{\alpha_2, \alpha_5\}$,

$$\begin{aligned} x &= \frac{s}{2} + \frac{(1-\delta^2)(s+c(1+\alpha))}{2\alpha(1+\delta^2)}, \\ y &= \frac{s}{2} - \frac{(1-\delta)^2(s+c(1+\alpha))}{2\alpha(1+\delta^2)}. \end{aligned} \tag{FM$_2$}$$

- If $\frac{s}{c} \geq \frac{1+2\delta-\delta^2}{1+\delta^2}$ and $1 \leq \alpha \leq \min\{\alpha_2, \alpha_3\}$, or if $\frac{s}{c} \leq \frac{1+2\delta-\delta^2}{\delta(1+\delta)}$ and $\alpha \geq 1$,

$$\begin{aligned} x &= \frac{s(2\alpha + \alpha\delta - \delta) - \delta c(1+\alpha)}{2\alpha(1+\delta)}, \\ y &= \frac{s(\alpha + 2\alpha\delta - 1) - c(1+\alpha)}{2\alpha(1+\delta)}. \end{aligned} \tag{PM$_2$}$$

- If $\alpha_3 \leq \alpha \leq \alpha_2$, there is multiplicity of equilibria:
  - If $\alpha \geq \alpha_4$, the equilibria are (PM$_1$) and (PM$_2$).
  - If $1 \leq \alpha \leq \alpha_4$, the equilibria are (PM$_1$) and (PM$_2$) and (IM).
  - If $\alpha \leq 1$, the equilibria are (IM) and (PM$_1$).

Case 1 (Inactive mediator-IM)

Conditions (1) and (2) hold with equality. If this happens we obtain $(x, s-x)$ and $(y, s-y)$ from the direct negotiation case, that is,

$$\begin{aligned} x &= \frac{s - \delta c}{1+\delta}, \\ y &= \frac{\delta s - c}{1+\delta}. \end{aligned}$$

We have to check if there exists some value of α that makes that these payoffs satisfy conditions (M1) and (M2):

(M1) $\frac{(1-\delta)}{\alpha}(s+c) \geq |2x - s| - |\delta(2y - s) - (1-\delta)c| \iff$

$\frac{(1-\delta)}{\alpha}(s+c) \geq \frac{1}{1+\delta}|s(1-\delta) - 2\delta c| - (1-\delta)c + \delta s - 2\delta\left(\frac{\delta s - c}{1+\delta}\right).$

(M2) $\frac{(1-\delta)}{\alpha}(s+c) \geq |2y - s| - |\delta(2x - s) - (1-\delta)c| \iff$

$\frac{(1-\delta)}{\alpha}(s+c) \geq \frac{2c}{1+\delta} + \frac{(1-\delta)s}{1+\delta} - \frac{1}{1+\delta}\left|\delta s(1-\delta) - c(1+\delta^2)\right|.$

Case 1.1

$x \leq \frac{s}{2}$, that is, $|2x - s| = s - 2x = \frac{1}{1+\delta}(2\delta c - s(1-\delta))$. Then, we also have:

$$|\delta(2x - s) - (1-\delta)c| = (1-\delta)c + \delta(s - 2x) = \frac{1}{1+\delta}\left(c(1+\delta^2) - \delta s(1-\delta)\right).$$

In this case, we can prove that (M1) is always satisfied because

$$\frac{1}{1+\delta}(2\delta c - s(1-\delta)) - \frac{1}{1+\delta}\left(c(1+\delta^2) - \delta s(1-\delta)\right) < 0.$$

If we check for (M2), we get:

$$\begin{aligned}\frac{(1-\delta)}{\alpha}(s+c) &\geq |2y-s|-|\delta(2x-s)-(1-\delta)c| \Leftrightarrow \\ \frac{(1-\delta)}{\alpha}(s+c) &\geq \frac{2c}{1+\delta}+\frac{(1-\delta)s}{1+\delta}-\frac{1}{1+\delta}\left(c(1+\delta^2)-\delta s(1-\delta)\right)=(1-\delta)(s+c).\end{aligned}$$

And this is satisfied iff $\alpha \leq 1$.

Case 1.2

$x \geq \frac{s}{2}$, that is, $|2x-s| = 2x-s = \frac{1}{1+\delta}\left(s(1-\delta)-2\delta c\right)$.

Case (a):

$|\delta(2x-s)-(1-\delta)c| = \delta(2x-s)-(1-\delta)c = \frac{1}{1+\delta}\left(\delta s(1-\delta)-c(1+\delta^2)\right)$.

If we rewrite (M1):

$$\frac{(1-\delta)}{\alpha}(s+c) \geq s\left(\frac{1-2\delta+\delta^2}{1+\delta}\right)-c\left(\frac{1+4\delta-\delta^2}{1+\delta}\right)=z_1.$$

If we rewrite (M2):

$$\frac{(1-\delta)}{\alpha}(s+c) \geq s\left(\frac{1-2\delta+\delta^2}{1+\delta}\right)+c\left(\frac{3+\delta^2}{1+\delta}\right)=z_2>z_1.$$

So the necessary condition is (M2), and therefore we need:

$$\alpha \leq \frac{(1-\delta^2)(s+c)}{(1-\delta)^2 s+(3+\delta^2)c},$$

(note that this value is higher than 1 iff $\frac{s}{c} > \frac{1+\delta}{\delta(1-\delta)}$).

Case (b):

$|\delta(2x-s)-(1-\delta)c| = (1-\delta)c-\delta(2x-s) = \frac{1}{1+\delta}\left(c(1+\delta^2)-\delta s(1-\delta)\right)$.

Again, rewriting (M1) and (M2) we obtain,

$$\begin{aligned}\frac{(1-\delta)}{\alpha}(s+c) &\geq z_1, &\text{(M1)}\\ \frac{(1-\delta)}{\alpha}(s+c) &\geq (1-\delta)(s+c), &\text{(M2)}\end{aligned}$$

and since $(1-\delta)(s+c) > z_1$, the necessary condition is again (M2), which, to be satisfied, implies $\alpha \leq 1$.

We can conclude that there exists a minimum value of α, $\alpha_{\min} = \max\left\{1, \frac{(1-\delta^2)(s+c)}{(1-\delta)^2 s+(3+\delta^2)c}\right\}$, under which the candidate equilibrium payoffs are the ones obtained from the direct (face-to-face) negotiation.

Case 2 (Fully mediated-FM)

Conditions (M1) and (M2) hold with equality, that is,

$$\frac{(1-\delta)}{\alpha}(s+c) = |2x-s| - |\delta(2y-s)-(1-\delta)c|, \tag{M1}$$
$$\frac{(1-\delta)}{\alpha}(s+c) = |2y-s| - |\delta(2x-s)-(1-\delta)c|. \tag{M2}$$

Note first that, in this case, we must have $x \geq \frac{s}{2}$ and $y \leq \frac{s}{2}$. Otherwise, players 1 and 2, respectively, would have a beneficial deviation. Therefore, we can rewrite the conditions in the following way:

$$\frac{(1-\delta)}{\alpha}(s+c) = 2x-s-(1-\delta)c-\delta(s-2y), \tag{M1}$$
$$\frac{(1-\delta)}{\alpha}(s+c) = s-2y-|\delta(2x-s)-(1-\delta)c|. \tag{M2}$$

Case 2.1

$|\delta(2x-s)-(1-\delta)c| = (1-\delta)c-\delta(2x-s)$.

In this case, we obtain,

$$x = \frac{s}{2}+\frac{(1-\delta^2)(s+c(1+\alpha))}{2\alpha(1+\delta^2)},$$
$$y = \frac{s}{2}-\frac{(1-\delta)^2(s+c(1+\alpha))}{2\alpha(1+\delta^2)}.$$

We have to check if conditions (1) and (2) of equilibrium are satisfied.

We get the following conditions:

Necessary condition for (1) to be satisfied: $\alpha \geq 1$.

Necessary condition for (2) to be satisfied:

$\alpha\left(s(1+\delta^2)-c(1+2\delta-\delta^2)\right) \geq (s+c)(1+2\delta-\delta^2)$.

-If $\frac{s}{c} < \frac{1+2\delta-\delta^2}{1+\delta^2}$, this condition is never satisfied.

-If $\frac{s}{c} \geq \frac{1+2\delta-\delta^2}{1+\delta^2}$, we need the following condition for existence of equilibrium:

$$\alpha \geq \frac{(s+c)(1+2\delta-\delta^2)}{s(1+\delta^2)-c(1+2\delta-\delta^2)}. \tag{ii}$$

We can easily prove that condition (ii) implies (i).

Since we are in the case where $(1-\delta)c \geq \delta(2x-s)$, we need the following condition for this to hold:

$$\alpha \geq \frac{\delta(1+\delta)(s+c)}{c(1-\delta)}. \tag{iii}$$

Therefore, if $\alpha > \max\left\{\frac{\delta(1+\delta)(s+c)}{c(1-\delta)}, \frac{(s+c)(1+2\delta-\delta^2)}{s(1+\delta^2)-c(1+2\delta-\delta^2)}\right\}$ and $\frac{s}{c} \geq \frac{1+2\delta-\delta^2}{1+\delta^2}$, the candidate equilibrium offers are the previous ones.

Case 2.2

$|\delta(2x-s)-(1-\delta)c| = \delta(2x-s)-(1-\delta)c.$

In this case, we obtain,

$$\begin{aligned} x &= \frac{s}{2} + \frac{s+c(1+\alpha)}{2\alpha} - \frac{\delta c}{1+\delta}, \\ y &= \frac{s}{2} - \frac{s+c(1+\alpha)}{2\alpha} + \frac{c}{1+\delta}. \end{aligned}$$

Note that we do not have always $y \leq \frac{s}{2}$ and $x \geq \frac{s}{2}$. If we had $y \leq \frac{s}{2}$, then we would also have $x \geq \frac{s}{2}$. The necessary condition for $y \leq \frac{s}{2}$ is:

$$\alpha \leq \frac{(1+\delta)(s+c)}{c(1-\delta)}. \tag{iv}$$

A sufficient condition for this to be satisfied is $\alpha < \frac{1+\delta}{1-\delta}$.

The necessary condition to be in the case $(1-\delta)c \leq \delta(2x-s)$ is

$$\alpha \leq \frac{\delta(1+\delta)(s+c)}{c(1-\delta)}. \tag{v}$$

We check the conditions for (1) and (2) to be satisfied.

Necessary condition for (1) to be satisfied is

$$\alpha \geq \frac{(s+c)(1+\delta)^2}{s(1-\delta^2)+c(3-2\delta-\delta^2)}. \tag{vi}$$

The necessary condition for (2) to be satisfied is

$$\alpha \geq \frac{(1+\delta)^2(s+c)}{s(1-\delta^2)-c(1-\delta)^2}.$$

Then, if $\frac{(1+\delta)^2(s+c)}{s(1-\delta^2)-c(1-\delta)^2} \leq \alpha \leq \frac{\delta(1+\delta)(s+c)}{c(1-\delta)}$, and $\frac{s}{c} \geq \frac{1+2\delta-\delta^2}{\delta(1+\delta)}$, the mentioned offers are a potential equilibrium.

Case 3 (Partially mediated-PM)

We study the case when conditions (2) and (M2) are binding, which again implies that, at equilibrium, $y \leq \frac{s}{2}$:

$$\begin{aligned} s-x &= \delta(s-y), && (2) \\ \frac{(1-\delta)}{\alpha}(s+c) &= s-2y-|\delta(2x-s)-(1-\delta)c|. && (\text{M2}) \end{aligned}$$

Case 3.1

We analyze first the case where $|\delta(2x-s)-(1-\delta)c| = (1-\delta)c-\delta(2x-s)$. In this case the candidate equilibrium offers are the following:

$$\begin{aligned} y &= \frac{s(\alpha+2\alpha\delta-1)-c(1+\alpha)}{2\alpha(1+\delta)}, \\ x &= s(1-\delta)+\delta y. \end{aligned}$$

For the case $2x \leq s$, we can prove that condition (M1) always holds. The necessary condition for (1) to hold is $y \geq \frac{\delta s - c}{1+\delta}$, and we also need $x \leq \frac{s}{2}$ (which implies $y \leq \frac{s}{2}$). This implies that $1 \leq \alpha \leq \frac{\delta(s+c)}{s-\delta c}$.

For the case $2x \geq s$, we have to add a necessary condition for (M1) to hold, which is $y \leq \frac{s(1-\delta-\alpha+3\alpha\delta)}{4\alpha\delta}$, and impose $x \geq \frac{s}{2}$ and $(1-\delta)c \geq \delta(2x-s)$. If we put all these necessary conditions together, we obtain the following:

If $\frac{s}{c} \leq \frac{1-\delta^2+2\delta}{1+\delta^2}$, we need $\alpha \geq 1$.

If $\frac{s}{c} \geq \frac{1-\delta^2+2\delta}{1+\delta^2}$, we need $1 \leq \alpha \leq \frac{(1-\delta^2+2\delta)(s+c)}{s(1+\delta^2)-c(1-\delta^2+2\delta)}$.

Case 3.2.

We analyze now the case where $|\delta(2x-s)-(1-\delta)c| = \delta(2x-s)-(1-\delta)c$. The candidate equilibrium offers are:

$$\begin{aligned} y &= \frac{s(\alpha+\delta-\alpha\delta+2\alpha\delta^2-1)-c(1-\alpha)(1-\delta)}{2\alpha(1+\delta^2)}, \\ x &= s(1-\delta)+\delta y. \end{aligned}$$

Checking for all the conditions required we get the following necessary conditions for the above offers to be an equilibrium:

$\frac{s}{c} > \frac{1}{\delta}$ and $\frac{\delta^2(s+c)}{\delta s-c} \leq \alpha \leq \frac{(1+\delta)^2}{s(1-\delta^2)-c(1-\delta)^2}$.

Given all these candidate equilibria, we can check that, for the cases 1 and 3, there exists an overlapping of equilibrium offers for some regions.

This completes the proof. ∎

Proof. (Corollary 2)

Given the payoffs obtained for the case when $\alpha \to +\infty$, we can easily prove the following:

- For the case $\frac{s}{c} \geq \frac{(1-2\delta-\delta^2)}{1+\delta^2}$, the difference in payoffs if the weakest player (player 1) starts the negotiation is $\frac{(1-\delta^2)c}{(1+\delta^2)}$, while the difference if it is the strongest player the one who starts is $\frac{(1-\delta)^2c}{(1+\delta^2)} < \frac{(1-\delta^2)c}{(1+\delta^2)}$.
- For the case $\frac{s}{c} \leq \frac{(1-2\delta-\delta^2)}{1+\delta^2}$, the difference in payoffs if the weakest player (player 1) starts the negotiation is $\frac{s-\delta c}{1+\delta}$, while the difference if it is the strongest player the one who starts is $\frac{c-\delta s}{1+\delta} < \frac{s-\delta c}{1+\delta}$.

This completes the proof. ∎

Proof. (Corollary 3)

First we can compute ΔP^{Dpm} as follows:

$$\Delta P^{Dpm} \equiv \left|P_1^{Dpm} - P_2^{Dpm}\right| = \left|\frac{s(1-\delta)-2\delta c}{1+\delta}\right| = \begin{cases} \frac{s(1-\delta)-2\delta c}{1+\delta}, \text{ if } \delta \leq \widetilde{\delta} \\ \frac{2\delta c - s(1-\delta)}{1+\delta}, \text{ if } \delta \geq \widetilde{\delta} \end{cases}, \text{ with } \widetilde{\delta} = \frac{s}{s+2c}.$$

Take the case $\frac{s}{c} \geq \frac{1+2\delta-\delta^2}{1+\delta^2}$. In this case,

$$\Delta P^{Ipm} \equiv \left| P_1^{Ipm} - P_2^{Ipm} \right| = \frac{(1-\delta)^2}{(1+\delta^2)} c.$$

Take the case $\frac{s}{c} \leq \frac{1+2\delta-\delta^2}{1+\delta^2}$. In this case,

$$\Delta P^{Ipm} = \frac{c - \delta s}{1+\delta}.$$

Note that $\Delta P^{Dpm} = 0$ for $\delta = \widetilde{\delta}$, and it is decreasing in δ for $\delta \leq \widetilde{\delta}$ and increasing in δ for $\delta \geq \widetilde{\delta}$. In the same way, note that ΔP^{Ipm} decreases with δ. ■

Proof. (Proposition 6)

When the outside payoffs are such that $s = P_i^* + P_j^*$, note that, since the outside payoffs sum up to s, and the players have the possibility of opting out at any time of the game, the only equilibrium agreement is exactly their outside payoffs, because neither player will accept anything less.

In the case where $s > P_i^* + P_j^*$, since we concentrate in the case where the threat of breaking the negotiations and moving to the outside option is always credible for the proposer, the optimal strategy of the proposer will always be as follows: *propose* $(x_i, s - x_i)$ *and break the negotiations and move to the outside-option in case this offer is not accepted (or not submitted by the mediator).* By the same reasoning, the responder will accept this offer whenever submitted.

The proposal has to be such that the following restrictions are fulfilled:

1. $|2x_i - s| \leq \frac{1}{\alpha}\left(s - (P_i^* + P_j^*)\right) + \left|P_i^* - P_j^*\right|$. Otherwise, the mediator prefers to block the proposal and induce the players to move to the outside option.

2. $s - x_i \geq P_j^*$. Otherwise, the responder, even if the mediator submits the proposal, will prefer to reject it and ensure the outside option payoffs.

3. $x_i \geq P_i^*$. The proposer has to gain, at least, the same payoffs as in the outside option.

Taking this into account, let us see when $x_i^* = s - P_j^*$ constitutes an equilibrium offer. Note that this corresponds to the Ponsatí-Sákovics offer and (by 2.) is the maximum the proposer can aim at achieving.

First, it is straightforward, since $s > P_1^* + P_2^*$, that this offer fulfills 3.

It fulfills 1. if and only if

$$\left|s - 2P_j^*\right| \leq \frac{1}{\alpha}\left(s - (P_i^* + P_j^*)\right) + \left|P_i^* - P_j^*\right|. \qquad (M_P)$$

First, if $\left|s - 2P_j^*\right| \leq \left|P_i^* - P_j^*\right|$ this condition always holds. It can be checked that this condition is equivalent to

$$P_i^* \leq P_j^* - \left|s - 2P_j^*\right|. \qquad (C_P)$$

It can be checked, moreover, that a sufficient condition for (C_P) to hold is that $P_j^* \geq \frac{s}{2}$.

When (C_P) does not hold, (M_P) is fulfilled, provided $\alpha \leq \alpha_i^0$, with

$$\alpha_i^0 \equiv \frac{\left(s - (P_i^* + P_j^*)\right)}{\left|s - 2P_j^*\right| - \left|P_i^* - P_j^*\right|}.$$

Summarizing, $x_i^* = s - P_j^*$ constitutes an equilibrium offer if and only if: (C_P) holds or, in case it does not hold, if $\alpha \leq \alpha_i^0$.

Otherwise, this proposal will not be submitted by the mediator. In this case, the proposer will ask for himself, the maximum share that is compatible with the block-pass decision of the mediator. Therefore, x_i^* will be such that:

$$x_i^* = \frac{s}{2} + \frac{1}{2}\left(\frac{1}{\alpha}\left(s - \left(P_i^* + P_j^*\right)\right) + \left|P_i^* - P_j^*\right|\right).$$

It can be easily checked that, first $x_i^* \geq P_i^*$ and, hence 3. is fulfilled and that 2. is fulfilled as well since $x_i^* \leq s - P_j^*$ (given $P_j^* < \frac{s}{2}$, in this case).

This constitutes, therefore, an equilibrium offer. ■

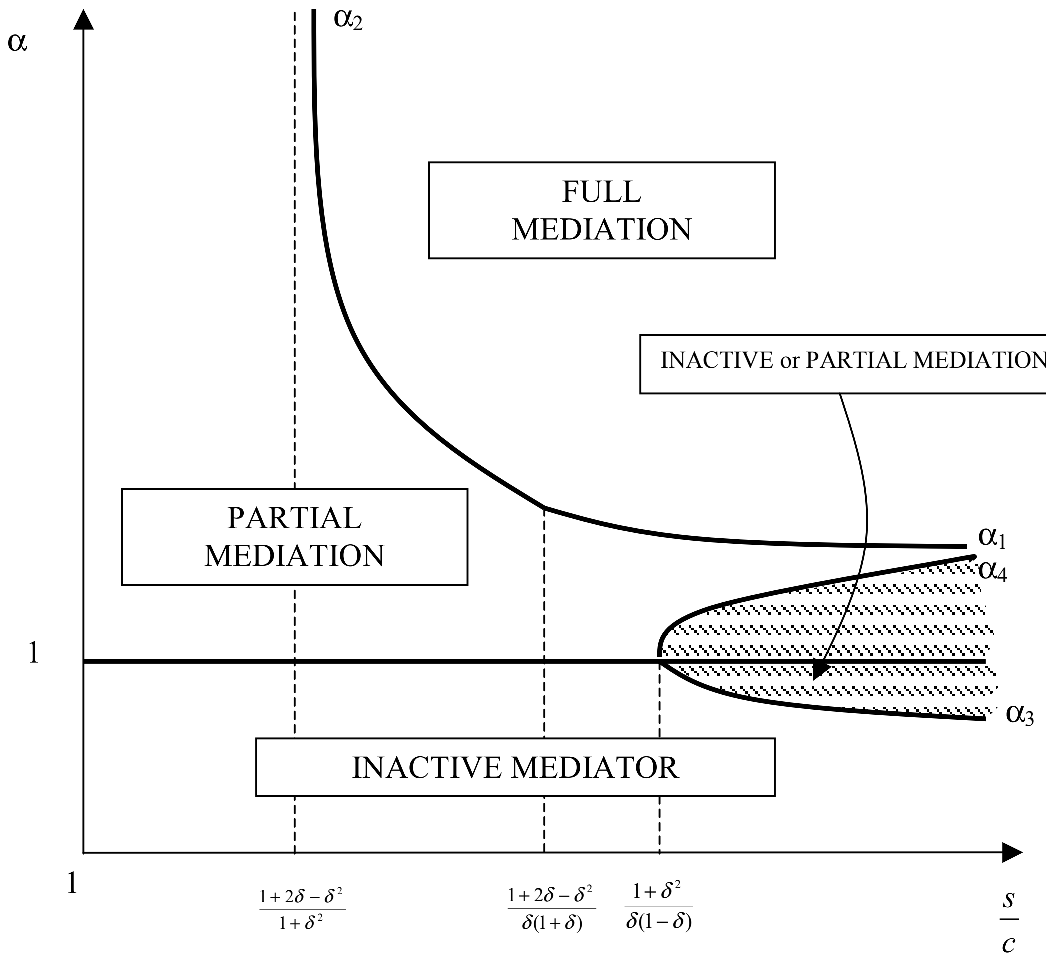

Figure 1: Structure of the equilibria in indirect negotiations under peacemaking.

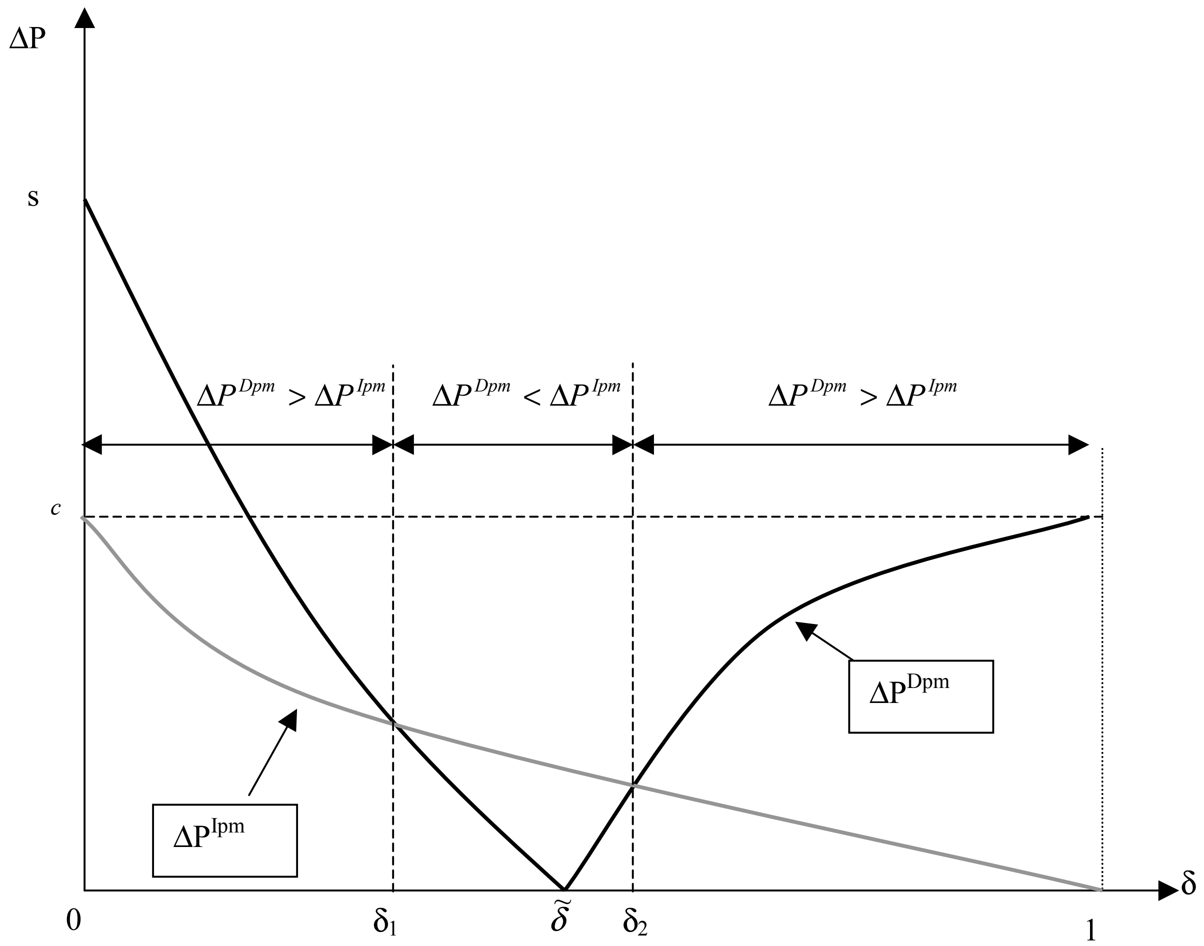

Figure 2: Comparison of the equality achieved in a direct and in an indirect negotiation by an equality-seeking ($\alpha \to \infty$) mediator.