Efficiency of Competitive Equilibria with Hidden Action: the Role of Separable Preferences*

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Abstract

In the present paper we study the efficiency properties of competitive equilibria in economies with hidden action and multiple goods. We borrow the description of the economy from Lisboa [3] and we apply a method of proof close in spirit to the one used in the literature on incomplete financial markets economies. We are then able to show that Lisboa's original result of constrained efficiency rests crucially on the assumption of separable preferences and on the structure of uncertainty.

Keywords: hidden action, separable preferences, constrained efficiency.

JEL Classification: D52, D61, D82.

1 Introduction

In the present paper we study the efficiency properties of competitive equilibria in a pure exchange economy with idiosyncratic uncertainty, multiple goods and hidden action. The economy last two periods, today and tomorrow. Tomorrow, each individual can be in one out of two idiosyncratic states and we assume that the uncertainty affects the endowments only. The probability of being in one state is affected by an hidden action chosen by each consumer today.

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Consumers are risk averse. Today they are offered insurance contracts by a couple of perfectly competitive intermediaries. The action chosen by consumer is affected by the level of utility they can achieve tomorrow, which in turn depends on the insurance contracts offered *and* the prices arising in the spot markets for goods.

Intermediaries know the consumers' preferences and their ex ante distribution of endowments. Yet, they take prices as given. They can anticipate the choice of action made by consumers, but they can only take into account its dependence on offered contracts.

Because of the hidden action, the economy is characterized by asymmetric information. The functioning of financial markets is then affected in a way which may conflict with efficiency. Indeed, from the literature on incomplete financial markets, we know that a planner's intervention can have important welfare effects, even when it has to take into account imperfections in the existing markets. This result hinges crucially on the possibility for the planner to take into account those price effects that are ignored by markets participants. The presence of multiple consumption goods is then a necessary condition for the result.

In the economy discussed here markets are indeed incomplete because of the hidden action and there are multiple consumption goods. In spite of this, competitive equilibria are constrained efficient, as shown in the original work of Lisboa [3], where it is suggested that this result follows from the assumption of endogenous asset structure, as opposed to the exogenous asset structure commonly assumed in models with incomplete financial markets.

In the present paper we propose a different interpretation of the original result of Lisboa [3]. We use a proof close in spirit to the one developed in the literature on incomplete financial markets and we find indeed that no planner's intervention can improve upon a competitive equilibrium. We make clear that this result follows because price effects induced by the planner's intervention are neutralized. The wealth effects, as it is well known, are then insufficient to induce a welfare improvement. What is more important, the proof we propose highlights the crucial role of the assumption on consumers' preferences in obtaining the result, as opposed to the role of the endogenous market structure.

The paper is organized as follows. In section **2** we introduce the main characteristics of the economy. In section **3** we describe the insurance market. In section **4** we study the consumer's choice problem. In section **5** we define the equilibrium concept. All the content of these sections is borrowed from Lisboa [3]. Finally in section **6** we propose our original proof of the constrained efficiency of competitive equilibria and we comment on the new insights resulting from the analysis.

2 The economy

Let us assume a pure exchange economy with multiple goods and idiosyncratic uncertainty populated with a large number N of consumers all ex ante equal.[1] The economy last two periods, $t = 0, 1$. There is no consumption nor endowments in the first period. In the second period uncertainty is resolved and each consumer will then be in a state $s \in \mathcal{S} = \{1, 2\}$. Uncertainty affects only the individual endowments e_s and not the preferences. In particular, we assume that state-contingent endowments are $e_s \in \mathbb{R}^{\mathrm{L}}_{++}$, with $e_1 >> e_2$. State $s = 1$ hence qualifies as the good state.

Let π_s be the probability of being in state s at $t = 1$. As we assume N to be large, we make appeal to the law of large numbers to conclude that at $t = 1$ a share π_s of consumers will be in state s. It is then possible to make use of expected feasibility constraints: for an allocation $x = (x_1, x_2)$ to be feasible it is required that

$$\pi_1(x_1 - e_1) + \pi_2(x_2 - e_2) = 0\,,$$

where $x_s \in \mathbb{R}^{\mathrm{L}}_{++}$. Notice that the above condition implies that there is no aggregate uncertainty.

In order to capture the effect of the hidden action on the contractual relationship between the insurance companies and the consumers, we assume that probabilities of idiosyncratic states are not fixed, but can be altered by an action chosen by consumers in a given set $\mathcal{A}$. The action is not observable by any agent but the one who chooses it. Information is therefore asymmetric.[2] The action set is $\mathcal{A} = [0, 1]$ and the probability of state s is given by $\pi_s(a) : \mathcal{A} \to (0, 1)$, a function satisfying the following[3]

Assumption 1 : $\partial_a \pi > 0 > \partial_a^2 \pi$.

Notice that $\mathcal{S} = \{1, 2\}$ implies $\partial_a \pi_1 = -\partial_a \pi_2$. Therefore in the rest of the paper we use the notation $\partial_a \pi$, where it is understood that $\partial_a \pi := \partial_a \pi_1$. Since $\partial_a \pi > 0$, choosing an higher level of action therefore increases the probability of state 1, but this is costly: the utility function of a consumer who chooses a bundle $x = (x_1, x_2)$ and an action a is

$$v(x, a) := \pi_1(a)\, u(x_1) + \pi_2(a)\, u(x_2) - c(a)\,,$$

[1] Thanks to this assumption, all consumers will behave in the same way. Therefore we omit any index on the consumers' choice variables.

[2] This is a standard way to model the analysis of hidden action, following the tradition of insurance economics. It is not the only possible: one could assume that probabilities are fixed and outcomes depend on the action chosen. See e.g. Marshall [6] for an analysis of the different formalizations.

[3] We let $\partial_n g(n, m)$ and $\partial_n^2 g(n, m)$ be, respectively, the first and the second derivative of g with respect to n.

where the function $c(a) : \mathcal{A} \to \mathbb{R}_+$ measures the cost of choosing a given action. It is assumed to satisfy the following

Assumption 2 : $\partial_a c(0) = 0\,,\ \partial_a c\,, \partial_a^2 c > 0\,.$

Notice that the utility function v is assumed to be separable in the action, in the sense that, for a given bundle x, the expected benefit and the cost of a are additively separable. This is a strong hypothesis, which drastically simplifies the analysis of the consumer's problem, for it implies that the choice of consumption bundles and action are not interrelated. Finally, we assume that the function $u : \mathbb{R}^{\mathrm{L}}_{++} \to \mathbb{R}$ satisfies the following

Assumption 3 : *u is continuous on $\mathbb{R}^L_+$ and $\mathcal{C}^\infty$ on $\mathbb{R}^{\mathrm{L}}_{++}$ and it satisfies*

- $\{x'_s \in \mathbb{R}^{\mathrm{L}}_+ \mid u(x'_s) \geq u(x_s)\} \subset \mathbb{R}^{\mathrm{L}}_{++}\,,$
- $\nabla u(x_s) \in \mathbb{R}^{\mathrm{L}}_{++}\,,$
- $h' D^2 u(x_s) h < 0$ *for every* $h \neq 0$ *such that* $\nabla u(x_s) h = 0\,,$

for each $x_s \in \mathbb{R}^{\mathrm{L}}_{++}$.

3 Insurance Markets

At date $t = 0$, before idiosyncratic uncertainty resolves, consumers can sign a contract with an insurance company, fully committing to pay an amount $\tau_s < 0$ (respectively, to receive an amount $\tau_s > 0$) if and only if state s obtains tomorrow. Contracts allow to transfer purchasing power from one idiosyncratic state to another. The role of a contract is indeed analogous to that of contingent commodities or securities with payoffs conditional on the idiosyncratic state, but there are conceptual differences.

Given a single contract $\tau = (\tau_1, \tau_2)$, terms of exchange of income are defined implicity: τ_2/τ_1 is the relative price of income for that single contract. More generally, a contract can be of the form $\tau = (\tau_1(\tau_2), \tau_2)$, where the function $\tau_1(\tau_2)$ specifies the units of income that consumers have to give up in state 1 to get τ_2 units of income in state 2. This interpretation implies that τ_2 is a positive number and $\tau_1(\tau_2)$ a negative number. If $\tau_1(\tau_2)$ is linear, e.g. $\tau_1(\tau_2) = -\beta\tau_2$, with $\beta > 0$, then the relative price of income is constant for any level of τ_2. If the consumer is just given the price $\tau_1(\tau_2)$ from an insurance company, then the market structure is analogous to one with contingent commodities or securities, with possibly non-linear prices. If the consumer can only choose a contract offered by the insurance company, as we assume, then the specification of an admissible set of contracts does suffice.

In the rest of the paper, we will maintain the following assumptions about the characteristics of the insurance market:

Assumption 4 (Exclusivity) : *each consumer can accept only one contract.*

Assumption 5 (Full commitment) : *contracts are honored by consumers and insurance companies.*

Assumption 6 (Common knowledge) : *at $t = 0$, preferences and endowments of consumers are known by all market participants; in particular they are known by the insurance companies.*

Suppose two intermediaries, α and β, compete in offering insurance contracts to consumers before the resolution of uncertainty. As each insurance company can always offer no contract and make zero profits, it will never offer a contract that does not at least break even. Since assumptions **4**, **5** and **6** imply that the intermediaries can anticipate the choices made by consumers, and that they can calculate the function $a(\tau)$, it is reasonable to assume that they will use this information to identify the set of contracts guaranteeing non-negative profits. It follows that the set of admissible contracts is

$$\mathcal{T} = \{(\tau_1, \tau_2) \in \mathbb{R}^2 \,|\, \pi_1(a)\,\tau_1 + \pi_2(a)\,\tau_2 \leq 0\}\,.^4$$

4 Consumers' problem

At $t = 0$, consumers choose a contract τ among those offered by the insurance companies. After signing the contract, they choose the action a. At $t = 1$, they choose a bundle of goods, given prices $p \in \mathbb{R}^{\mathrm{L}}_{++}$ and income $p \cdot e_s + \tau_s$. Since preferences are separable across idiosyncratic states and in the cost of action, it is possible to solve the consumers' problem stepping backwards from $t = 1$.

In the first step, taking prices p and a contract τ as given, consumers choose a state-contingent consumption vector $x_s \in \mathbb{R}^{\mathrm{L}}_{++}$ such that

$$x_s \in \arg\max\{u(x_s)\,|\,p \cdot x_s = p \cdot e_s + \tau_s\,, \quad s = 1, 2\}\,. \tag{1}$$

Let $x_s(\tau_s) := x_s(p, p \cdot e_s + \tau_s)$ be the consumer's demand in state s if he accepts a contract τ. The maximum utility he can get in state s is therefore $\tilde{u}(\tau_s) = u(x_s(\tau_s))$. Notice that the proposed notation hides on purpose the dependence of indirect utility on prices so as to emphasize the fact that the reasoning is made for a fixed price vector.

In the second step, given $\tilde{u}(\tau_s)$, consumers choose an action such that

$$a \in \arg\max\{\pi_1(a)\,\tilde{u}(\tau_1) + \pi_2(a)\,\tilde{u}(\tau_2) - c(a)\,|\,a \in \mathcal{A}\}. \tag{2}$$

[4] With some abuse of notation we let $\pi_s(\tau) = \pi_s(a(\tau))$.

Given a price vector p and a contract τ, the function to be maximized is strictly concave in a. The optimal level of action is then completely characterized by the following first order condition

$$\partial_a \pi [\, \tilde{u}(\tau_1) - \tilde{u}(\tau_2)\,] - \partial_a c \leq 0\,, \qquad a \geq 0\,. \tag{3}$$

When (3) holds with equality, it defines an implicit function $a(\tau)$, which gives the incentive compatible level of action for any contract τ.

In the third step, given $\tilde{u}(\tau_s)$ and $a(\tau)$, consumers choose a contract

$$\tau \in \arg\max \{\pi_1(\tau)\,\tilde{u}(\tau_1) + \pi_2(\tau)\,\tilde{u}(\tau_2) - c(\tau) \,|\, \tau \in \mathcal{T}'\}\,, \tag{4}$$

where $\mathcal{T}' \subseteq \mathcal{T}$ is the set of contracts offered by the insurance companies.[5]

Remark. In the following discussion, we assume that the function $v(\tau) = \sum_s \pi_s(\tau)\,\tilde{u}(\tau_s) - c(\tau)$ is concave. We do not give the detailed conditions which are sufficient for this assumption to be satisfied. Yet, notice that it is easy to prove that, when prices are kept fixed, the concavity of u implies the concavity of $\tilde{u}$.[6] As for the concavity of $\pi_s(\tau)\,\tilde{u}(\tau_s)$, what is crucial is that the curvature of the indifference curve be higher that the elasticity of the probability with respect to the optimal level of action.[7]

5 Equilibrium

In order to define the competitive equilibrium, first of all we need the contracts offered by each insurance company to be a best response to the contracts offered by the opponent. Let $P(\tau^\alpha, \bar{\tau}^\beta)$ be the profit of insurer α when he offers a contract τ^α and his opponent offers a contract $\bar{\tau}^\beta$. The equilibrium concept for the competition among insurance companies is then described by the following

Definition 5.1 *A (Nash) equilibrium in the supply of insurance with hidden action is a pair of contracts* $(\bar{\tau}^\alpha, \bar{\tau}^\beta) \in \mathcal{T}'$ *such that* $P(\bar{\tau}^\alpha, \bar{\tau}^\beta) \geq P(\tau^\alpha, \bar{\tau}^\beta)$ *and* $P(\bar{\tau}^\alpha, \bar{\tau}^\beta) \geq P(\bar{\tau}^\alpha, \tau^\beta)$*, for all* $\tau^\alpha, \tau^\beta \in \mathcal{T}'$*.*

Notice that from assumption **4** it follows that

$$P(\tau^\alpha, \bar{\tau}^\beta) = \begin{cases} -(\pi_1(\tau^\alpha)\,\tau_1^\alpha + \pi_2(\tau^\alpha)\,\tau_2^\alpha) & \text{if} \quad v(\tau^\alpha) > v(\bar{\tau}^\beta) \\ -\frac{1}{2}\,(\pi_1(\tau^\alpha)\,\tau_1^\alpha + \pi_2(\tau^\alpha)\,\tau_2^\alpha) & \text{if} \quad v(\tau^\alpha) = v(\bar{\tau}^\beta) \\ 0 & \text{if} \quad v(\tau^\alpha) < v(\bar{\tau}^\beta)\,. \end{cases}$$

Equilibrium contracts are characterized by the following

[5] For consistency with the previous notation, we we let $c(\tau) = c(a(\tau))$.

[6] See Mas-Colell *et alii* [7], exercise 6.C.5.

[7] For a discussion of this property, see Arnott [1].

Proposition 1 *A pair of contracts* $\bar{\tau}^{\alpha} = \bar{\tau}^{\beta} = \bar{\tau}$ *is an equilibrium in the supply of insurance with hidden action if and only if it satisfies*

$$\bar{\tau} \in \arg\max \{\pi_1(\tau)\, \tilde{u}(\tau_1) + \pi_2(\tau)\, \tilde{u}(\tau_2) - c(\tau) \,|\, \tau \in \mathcal{T}'\}.$$

Proof. See the Appendix.[8] ■

Notice that at an equilibrium in the supply of insurance insurance companies make zero profits, i.e. $\pi_1(\bar{\tau})\, \bar{\tau}_1 + \pi_2(\bar{\tau})\, \bar{\tau}_2 = 0$.

In a competitive equilibrium with hidden action, consumers maximize their expected utility and insurance companies maximize their profits, given the incentive compatible level of action chosen by consumers. Prices then ensure that markets for goods clear at date 1. These properties are summarized by the following

Definition 5.2 *A competitive equilibrium with strategic insurance intermediaries is a 4-tuple* (x, a, p, τ) *such that*

- $x_s \in \arg\max \{u(x_s) \,|\, p \cdot (x_s - e_s) = \tau_s\}$ *for* $s = 1, 2$,
- $a \in \arg\max \{\pi_1(a)\, \tilde{u}(\tau_1) + \pi_2(a)\, \tilde{u}(\tau_2) - c(a) \,|\, a \in \mathcal{A}\}$,
- $\tau \in \arg\max \{\pi_1(\tau)\, \tilde{u}(\tau_1) + \pi_2(\tau)\, \tilde{u}(\tau_2) - c(\tau) \,|\, \tau \in \mathcal{T}'\}$,
- $\pi_1(a)\,(x_1 - e_1) + \pi_2(a)\,(x_2 - e_2) = 0$.

Notice that we have made use of a market clearing condition in expected terms, following the reasoning proposed at the beginning of section **2**.

6 Constrained efficiency

We now propose a new proof of the constrained efficiency of a competitive equilibrium in the above economy with hidden action close in spirit to the one used in models with incomplete financial markets.[9]

Imagine that, starting from a competitive equilibrium, the planner redistributes income across consumers under an appropriate balanced budget constraint. Consumers then adjust their demands and spot markets clear. Following the literature on incomplete financial markets, we say that a competitive equilibrium is constrained efficient if this planner's intervention cannot increase the consumers' welfare.

[8] The proposition is borrowed from Lisboa [3], while the proof is original.

[9] This type of analysis dates back to Stiglitz [9]. For a survey of its applications to economies with incomplete financial markets, see Magill and Shafer [4].

Consider then a competitive equilibrium (x, a, p, τ) and let $(\mathrm{d}\tau_1, \mathrm{d}\tau_2)$ be change in income transfers, to which is associated a change $\mathrm{d}p$ in the equilibrium price vector.[10] The income transfers are budget balanced, or *feasible*, if

$$\pi_1 \mathrm{d}\tau_1 + \pi_2 \mathrm{d}\tau_2 + \psi \mathrm{D}a = 0\,, \tag{5}$$

where $\psi := \partial_a \pi\,(\tau_1 - \tau_2)$ is the change in expected transfer due to the change in probabilities and $\mathrm{D}a = \partial_{\tau_1} a\, \mathrm{d}\tau_1 + \partial_{\tau_2} a\, \mathrm{d}\tau_2 + \partial_p a\, \mathrm{d}p$ is the total change in the optimal action.[11]

To any $(\mathrm{d}\tau_1, \mathrm{d}\tau_2)$ we can associate the total change in expected utility $\pi_1 \mathrm{D}u_1 + \pi_2 \mathrm{D}u_2$, where $\mathrm{D}u_s$ is the change in utility in state s evaluated at the competitive equilibrium.[12] Feasible income transfers improve upon a competitive equilibrium if $\pi_1 \mathrm{D}u_1 + \pi_2 \mathrm{D}u_2 > 0$. We summarize these properties in the the following

Definition 6.1 *A competitive equilibrium (x, a, p, τ) is constrained efficient if there does not exist any feasible income transfer which improves upon it.*

The efficiency properties of competitive equilibria are then summarized in the following

Proposition 2 *Every competitive equilibrium is constrained efficient.*

Proof. We proceed in steps. The idea of the proof is to show that if $(\mathrm{d}\tau_1, \mathrm{d}\tau_2)$ satisfies (5), then it satisfies

$$\pi_1 \mathrm{D}u_1 + \pi_2 \mathrm{D}u_2 = 0\,. \tag{6}$$

• Let z_s be the excess demand in state s. By using Roy's identity, first of all we notice that at equilibrium the following equality holds true

$$\pi_1 \partial_p u_1 + \pi_2 \partial_p u_2 = -(\pi_1 \partial_{\tau_1} u_1 z_1 + \pi_2 \partial_{\tau_2} u_2 z_2)\,. \tag{7}$$

• We now show that, when preference are separable, the price effect and the income effect on the optimal choice of action are related via the following equality

$$\partial_p a = -(\partial_{\tau_1} a z_1 + \partial_{\tau_2} a z_2)\,. \tag{8}$$

[10] Notice that $\mathrm{d}p = \sum_s \partial_{\tau_s} p\, \mathrm{d}\tau_s$, where in this section $\partial_n g(n, m)$ is always evaluated at equilibrium. For $\mathrm{d}p$ to be well defined, we assume that the economy is regular at the equilibrium under consideration. See Siconolfi and Villanacci [8] for an analysis of regular economies with idiosyncratic uncertainty and symmetric information.

[11] Recall that $\partial_a \pi := \partial_a \pi_1 = -\partial_a \pi_2$. Therefore $\psi = \frac{\partial \pi_1}{\partial a} \tau_1 + \frac{\partial \pi_2}{\partial a} \tau_2$.

[12] Notice that the first order effect of $\mathrm{D}a$ on expected utility is zero at a competitive equilibrium and therefore it does not appear in the above expression.

Recall that the optimal action is the implicit solution of the single equation (3), from which it follows that

$$\partial_p a = -\iota(\partial_p u_1 - \partial_p u_2)\,,$$

where $\iota := \frac{\partial_a \pi}{\partial_a^2 \pi\,(u_1-u_2) - \partial_a^2 c}$. Using again Roy's identity, we find that

$$-\iota(\partial_p u_1 - \partial_p u_2) = (\iota \partial_{\tau_1} u_1) z_1 - (\iota \partial_{\tau_2} u_2) z_2\,.$$

Since (3) also implies that $\partial_{\tau_1} a = -\iota \partial_{\tau_1} u_1$ and $\partial_{\tau_2} a = \iota \partial_{\tau_2} u_2$, we can conclude that (8) is indeed true when preference are separable.[13]

• Let $\bar{\partial}_p u := \pi_1 \partial_p u_1 + \pi_2 \partial_p u_2$. We now use (7) to rewrite the total change in utility as follows

$$\begin{aligned}
\pi_1 \mathrm{D} u_1 + \pi_2 \mathrm{D} u_2 &= (\pi_1 \partial_{\tau_1} u_1 + \bar{\partial}_p u\, \partial_{\tau_1} p)\mathrm{d}\tau_1 + (\pi_2 \partial_{\tau_2} u_2 + \bar{\partial}_p u\, \partial_{\tau_2} p)\mathrm{d}\tau_2 \\
&= \pi_1 \partial_{\tau_1} u_1 \mathrm{d}\tau_1 + \pi_2 \partial_{\tau_2} u_2 \mathrm{d}\tau_2 + \bar{\partial}_p u \mathrm{d}p \\
&= \pi_1 \lambda_1 \mathrm{d}\tau_1 + \pi_2 \lambda_2 \mathrm{d}\tau_2 - (\pi_1 \lambda_1 z_1 + \pi_2 \lambda_1 z_2)\mathrm{d}p\,,
\end{aligned}$$

where λ_s is the multiplier associated to the constraint in (1). Let λ_0 be the multiplier associated to the constraint in (4). Then $\hat{\lambda}_s := \frac{\lambda_s}{\lambda_0}$ is the value of one unit of income in state s measured in units of income at $t = 0$. In the Appendix we show that at a competitive equilibrium $\pi_s \hat{\lambda}_s = \pi_s + \psi \partial_{\tau_s} a$. It follows that

$$\begin{aligned}
\pi_1 \hat{\lambda}_1 \mathrm{d}\tau_1 + \pi_2 \hat{\lambda}_2 \mathrm{d}\tau_2 &= \pi_1 \mathrm{d}\tau_1 + \pi_2 \mathrm{d}\tau_2 + \psi(\partial_{\tau_1} a\, \mathrm{d}\tau_1 + \partial_{\tau_2} a\, \mathrm{d}\tau_2)\,, \\
(\pi_1 \hat{\lambda}_1 z_1 + \pi_2 \hat{\lambda}_2 z_2)\mathrm{d}p &= (\pi_1 z_1 + \pi_2 z_2)\mathrm{d}p + \psi(\partial_{\tau_1} a\, z_1 + \partial_{\tau_2} a\, z_2)\mathrm{d}p\,.
\end{aligned}$$

• Let $\mathrm{D}\hat{u} := \frac{1}{\lambda_0}(\pi_1 \mathrm{D} u_1 + \pi_2 \mathrm{D} u_2)$. We now use (8) and the fact that at a competitive equilibrium $\pi_1 z_1 + \pi_2 z_2 = 0$ to write the total change in utility as follows

$$\mathrm{D}\hat{u} = \pi_1 \mathrm{d}\tau_1 + \pi_2 \mathrm{d}\tau_2 + \psi \mathrm{D} a\,.$$

The above expression implies that when $(\mathrm{d}\tau_1, \mathrm{d}\tau_2)$ satisfies (5), it also satisfies (6), so that no welfare improvement is possible. ∎

[13] To our knowledge, (8) has never been used in the way proposed here. It has been discussed by Arnott and Stiglitz [2] in their analysis of optimal taxation with moral hazard.

7 Interpretation

Following the planner's intervention, there are two effects on the utility, a direct one, due to the change in the state-contingent transfers $\mathrm{d}\tau_s$, and an indirect one, due to the change in the price level $\mathrm{d}p$. After recognizing that the substitution effect has no impact on utility, we find that the indirect effect depends only on the two state-contingent income variations $z_s\mathrm{d}p$, see (7). Following this intuition, we can rewrite the total change in utility as

$$\begin{aligned}\pi_1\mathrm{D}u_1 + \pi_2\mathrm{D}u_2 &= \pi_1\partial_{\tau_1}u_1\mathrm{d}\tau_1 + \pi_2\partial_{\tau_2}u_2\mathrm{d}\tau_2 + \bar{\partial}_p u\mathrm{d}p \\ &= \pi_1\partial_{\tau_1}u_1(\mathrm{d}\tau_1 - z_1\mathrm{d}p) + \pi_2\partial_{\tau_2}u_2(\mathrm{d}\tau_2 - z_2\mathrm{d}p) \\ &= \pi_1\partial_{\tau_1}u_1\mathrm{d}\tilde{\tau}_1 + \pi_2\partial_{\tau_2}u_2\mathrm{d}\tilde{\tau}_2\,,\end{aligned}$$

where $\mathrm{d}\tilde{\tau}_s := \mathrm{d}\tau_s - z_s\mathrm{d}p$ is the net wealth change in state s.

Since preferences are separable, the action depends on the level of utility in both states, see (3). By applying the reasoning of the previous paragraph, we find that the planner's intervention has two effects on the choice of action and that the indirect one depends again only on the two state-contingent income variations, see (8). It follows that $\mathrm{D}a$ can be rewritten as

$$\begin{aligned}\partial_{\tau_1}a\,\mathrm{d}\tau_1 + \partial_{\tau_2}a\,\mathrm{d}\tau_2 + \partial_p a\,\mathrm{d}p &= \partial_{\tau_1}a\,(\mathrm{d}\tau_1 - z_1\mathrm{d}p) + \partial_{\tau_2}a\,(\mathrm{d}\tau_2 - z_2\mathrm{d}p) \\ &= \partial_{\tau_1}a\,\mathrm{d}\tilde{\tau}_1 + \partial_{\tau_2}a\,\mathrm{d}\tilde{\tau}_2\,.\end{aligned}$$

Let $\mathrm{D}\tilde{a} := \partial_{\tau_1}a\,\mathrm{d}\tilde{\tau}_1 + \partial_{\tau_2}a\,\mathrm{d}\tilde{\tau}_2$ and rewrite the planner's budget constraint (5) as follows

$$\begin{aligned}0 &= \pi_1\mathrm{d}\tau_1 + \pi_2\mathrm{d}\tau_2 + \psi\mathrm{D}a \\ &= \pi_1\mathrm{d}\tilde{\tau}_1 + \pi_2\mathrm{d}\tilde{\tau}_2 + \psi\mathrm{D}\tilde{a} + (\pi_1 z_1 + \pi_2 z_2)\mathrm{d}p \\ &= \pi_1\mathrm{d}\tilde{\tau}_1 + \pi_2\mathrm{d}\tilde{\tau}_2 + \psi\mathrm{D}\tilde{a}\,,\end{aligned}$$

where we used the fact that at equilibrium $\pi_1 z_1 + \pi_2 z_2 = 0$ because of market clearing. Substituting once again for $\pi_s\lambda_s = \lambda_0(\pi_s + \psi\partial_{\tau_s}a)$, we get that

$$\begin{aligned}\pi_1\mathrm{D}u_1 + \pi_2\mathrm{D}u_2 &= \pi_1\partial_{\tau_1}u_1\mathrm{d}\tilde{\tau}_1 + \pi_2\partial_{\tau_2}u_2\mathrm{d}\tilde{\tau}_2 \\ &= \lambda_0(\pi_1\mathrm{d}\tilde{\tau}_1 + \pi_2\mathrm{d}\tilde{\tau}_2 + \psi\mathrm{D}\tilde{a})\,,\end{aligned}$$

which again implies that a feasible planner's intervention cannot improve upon a competitive equilibrium.

Remark. If there is only one consumption good, then $\mathrm{d}p \equiv 0$ and therefore $\mathrm{d}\tilde{\tau}_s \equiv \mathrm{d}\tau_s$. As it is still true that at a competitive equilibrium $\pi_s \hat{\lambda}_s = \pi_s + \psi \partial_{\tau_s} a$, the above proof is still valid and we conclude that the competitive equilibrium is constrained efficient.

Remark. If there is no hidden action, then $\pi_s(a) \equiv \pi_s$, an exogenously given probability. Information is then symmetric and the model corresponds to a simplified version of Malinvaud's [5].[14] By repeating the above proof with $\psi \equiv 0$, it is possible to conclude that competitive equilibria are *unconstrained* efficient. Notice that in this case we get $\hat{\lambda}_1 = \hat{\lambda}_2$, i.e. full insurance.

8 Appendix

(**Proposition 1**) Let $\psi := \partial_a \pi\,(\tau_1 - \tau_2)$, $\theta_s := \pi_s \lambda_s$ and $\zeta_s := \lambda_0(\pi_s + \psi \partial_{\tau_s} a)$, where λ_s is the multiplier associated to the constraint in (1) and λ_0 is the multiplier associated to the constraint in (4). As both the objective and the constraint functions are continuous, it is easy to prove that the constraint of the maximization problem must hold with equality, i.e. $\pi_1(\bar{\tau})\,\bar{\tau}_1 + \pi_2(\bar{\tau})\,\bar{\tau}_2 = 0$ for any

$$\bar{\tau} \in \arg\max \{\pi_1(\tau)\,\tilde{u}(\tau_1) + \pi_2(\tau)\,\tilde{u}(\tau_2) - c(\tau) \,|\, \tau \in \mathcal{T}'\} \tag{9}$$

As for the admissible deviations with respect to a given contract, we consider only local deviations, in the sense that, given $\bar{\tau}$, a deviating contract will be of the type $\tau = \bar{\tau} + \mathrm{d}\tau$, where $\mathrm{d}\tau = (\mathrm{d}\tau_1, \mathrm{d}\tau_2)$.

Notice that the first order necessary and sufficient conditions for an interior maximum of the above problem are

$$\pi_s \lambda_s = \lambda_0(\pi_s + \psi \partial_{\tau_s} a)\,. \tag{10}$$

• We first prove that if $\bar{\tau}$ is an equilibrium in the supply of insurance, then it satisfies (9). If $\bar{\tau}$ does not satisfy (9), then there exist a deviation $\mathrm{d}\tau$ such that

$$\sum_s (\pi_s + \psi \partial_{\tau_s} a)\,\mathrm{d}\tau_s < 0\,, \tag{11}$$

and

$$\sum_s \pi_s \partial_{\tau_s} \tilde{u}\,\mathrm{d}\tau_s > 0\,, \tag{12}$$

[14] Notice that Malinvaud introduced insurance intermediaries in his model, but he assumed they were bound to an institutional zero-profits constraints, whereas in the present model they make zero profit at equilibrium as a consequence of competition.

where all the functions are evaluated at $\bar{\tau}$. Notice that, thanks to the envelope theorem, the first order effect on utility of a change in the optimal action is zero and therefore does not appear in the above expression. Fix a $\gamma < 0$ and consider a $\mathrm{d}\tau$ such that $\zeta_1 \mathrm{d}\tau_1 + \zeta_2 \mathrm{d}\tau_2 = \gamma$. Rewrite (12) as

$$\theta_1 \mathrm{d}\tau_1 - \theta_2 \frac{\zeta_1}{\zeta_2} \mathrm{d}\tau_1 + \gamma \frac{\theta_2}{\zeta_2} . \tag{13}$$

If $\frac{\theta_1}{\theta_2} > \frac{\zeta_1}{\zeta_2}$, then (13) is positive for $\mathrm{d}\tau_1 > 0$ and γ sufficiently small, while if $\frac{\theta_1}{\theta_2} < \frac{\zeta_1}{\zeta_2}$, then (13) is positive for $\mathrm{d}\tau_1 < 0$ and γ sufficiently small. Therefore if $\bar{\tau}$ does not satisfy (9), there exists a feasible deviation, which is a contradiction with $\bar{\tau}$ being an equilibrium in the supply of insurance.

• We now prove that if $\bar{\tau}$ satisfies (9), then it is an equilibrium in the supply of insurance. If $\bar{\tau}^\alpha = \bar{\tau}^\beta = \bar{\tau}$ is not an equilibrium in the supply of insurance, then there is a feasible deviation τ such that $P(\tau, \bar{\tau}^\beta) > P(\bar{\tau}^\alpha, \bar{\tau}^\beta)$. Moreover, since a deviating contract is accepted only if it gives a higher utility, τ must also satisfy (12). It then follows that $\bar{\tau}$ does not solve (10) and hence it does not satisfy (9).

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