On Conservative Stable Standard of Behaviour in Situations with Perfect Foresight[1]

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Abstract

In this note we show that the solution notion called *conservative stable standard of behaviour* (CSSB), introduced by Greenberg (1990) has very little predictive power in environments with farsighted players although intuitively it is quite nice. First we show that CSSB can make no prediction at all in a large class of environments that are commonly encountered (like normal form games, social networks etc.), i.e., the entire set of social states is stable with respect to this notion. Next we find that even with some feasibility restrictions on the paths, the set of outcomes stable with respect to CSSB is a superset (some times a strict superset) of the largest consistent set (LCS) in a class of environments that includes voting games with a finite number of outcomes, even though for such environments the LCS itself may contain many intuitively unreasonable outcomes.

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1 Introduction

The solution notion called *conservative stable standard of behaviour* (CSSB), introduced by Greenberg (1990), has been used by Xue (1998) to predict stable social states in situations where players are farsighted. The motivaton of Xue was to get rid of certain conceptual shortcomings prevalent in the literature concerning coalitional stability with farsighted players. Most notably, he attempted to improve upon the concept of the largest consistent set (LCS) introduced by Chwe (1994). He correctly recognized that

> ...the inclusiveness of the LCS...stem[s] from the fact that indirect dominance defined on Z [the set of outcomes] fails to capture perfect foresight since it ignores the possible deviation along the way from one alternative (e.g., a) to another (e.g., d).

He tried to incorporate a "credibility" restriction on a coalition's deviations from an outcome. Since the paths along which coalitions can move can be of arbitrary length, the framework of *social situations* developed by Greenberg (1990) was quite useful for his analyses. He obtained his notion of coalitional stability through the concept of a stable standard of behaviour. Toward the end of his paper he claims that

> In his concluding remarks Chwe recognizes several issues that the notion of LCS fails to address, yet no constructive solution was offered. The notion suggested in this paper resolves most if not all of these issues.

Below we agree that the CSSB applied to the situation with perfect foresight is an intuitively appealing stability notion.

However, we find that unfortunately, in a large subclass of social environments representable by games in effectivity function form, CSSB has *no predictive power at all* if every path is feasible. This is a serious drawback of the solution concept because this subclass contains environments like normal form games, voting games, games in characteristic function form, social networks (as in the framework of Jackson and Wolinsky (1996)) etc.. Furthermore, in Theorem 4.5 of his paper Xue compares CSSB with the LCS and shows that under some assumptions, his stability notion refines the LCS. However, we find that with a somewhat similar restriction of feasibility on paths, the reverse is true for a class of social environments which includes the class of voting games with a finite number of outcomes.

2 Preliminary Definitions and Remarks

We would follow almost all of Xue's definitions and notation.

A *social environment* is represented by $\mathcal{G} = (N,\ Z,\ \{\preceq_i\}_{i\in N},\ \{\underset{S}{\rightarrow}\}_{S\subseteq N})$. Here N is the finite set of players, Z is the set of social states or outcomes, $\preceq_i$ is the preference relation for $i \in N$ on Z and $\underset{S}{\rightarrow}$ is the effectivity relation for $S \subseteq N$. For each $i \in N$, $a \preceq_i b$ means that player i weakly prefers outcome b to outcome a. The strict part of $\preceq_i$ is denoted by $\prec_i$.[2] Thus, for every $i \in N$, $\prec_i$ is irreflexive on Z. For any $a,\ b \in Z$, $a \underset{S}{\rightarrow} b$ implies that the coalition S can enforce outcome b from outcome a. A number of examples of the games that can be written in this form is provided by Chwe (1994) and Xue (1998).

Definition 1 *Given a social environment $\mathcal{G}$, a path*[3] *is a singleton sequence of the form $\{a_1\}$ or an ordered sequence of the form $\{a_1,\ S_1,\ a_2,\ S_2, \ldots,\ S_{k-1},\ a_k\}$ where for each i, $a_i \in Z$, $S_i \subseteq N$ and for every $j = 1, \ldots, k-1$, $a_j \underset{S_j}{\rightarrow} a_{j+1}$.*

If an outcome $a \in Z$ lies on a path α then that is denoted as $a \in \alpha$. The set Π denotes the set of all *possible* paths. For $a \in Z$, Π_a denotes the set of all possible paths that originate from a, i.e., the possible paths which have a as the first element. Below, some time we shall impose a feasibility restriction and consider the set of *feasible* paths only instead of considering all possible paths. The set of feasible paths is generically denoted by Π^f. For $a \in Z$, Π^f_a denotes the set of all feasible paths that originate from a including $\{a\}$ itself. For a path α, $t(\alpha)$ denotes its terminal outcome. Individual preferences are extended on Π as follows. For $i \in N$ and for any two paths α and β, $\alpha \preceq_i \beta$ if and only if $t(\alpha) \preceq_i t(\beta)$ (and similarly for $\prec_i$). This implies that for a sequence of coalitional moves described by a path, the players receive the pay-offs corresponding to the terminal element of the path. For some coalition S and $a,\ b \in Z$, if $a \prec_i b$ for all $i \in S$ then that is written as $a \prec_S b$. Similarly, for paths α and β, if $t(\alpha) \prec_S t(\beta)$ then it is also

[2]In Xue's specification, the individual preferences are assumed to be strict but the results following do not change with this added restriction.

[3]Here we have slightly modified Xue's definition of a path. In our definition of a path, we also specify the coalitions which enforce one outcome from another along the path. This is similar to the definition of history given in Herings *et al.* (2000).

written as $\alpha \prec_S \beta$.

Definition 2 *Suppose* Π^f *is given as the set of feasible paths for environment* $\mathcal{G}$. *A standard of behaviour*[4] *(SB)* σ *is a map,* $\sigma : Z \mapsto \Pi^f$ *such that for every* $a \in Z$, $\sigma(a) \subseteq \Pi^f_a$.

Definition 3 *Suppose* Π^f *is given as the set of feasible paths for environment* $\mathcal{G}$. *An SB* σ *is:*

(i) A conservative internally stable standard of behaviour (CISSB) for $\mathcal{G}$ *if for all* $a \in Z$, $\alpha \in \sigma(a) \implies$ *there do not exist* $S \subseteq N$, $b \in \alpha$ *and* $c \in Z$ *such that* $\{b,\ S,\ c\} \in \Pi^f_b$, $\sigma(c) \neq \emptyset$ *and* $\alpha \prec_S \beta$ *for all* $\beta \in \sigma(c)$;

(ii) A conservative externally stable standard of behaviour (CESSB) for $\mathcal{G}$ *if for all* $a \in Z$, $\alpha \in \Pi^f_a \setminus \sigma(a) \implies$ *there exist* $S \subseteq N$, $b \in \alpha$ *and* $c \in Z$ *such that* $\{b,\ S,\ c\} \in \Pi^f_b$, $\sigma(c) \neq \emptyset$ *and* $\alpha \prec_S \beta$ *for all* $\beta \in \sigma(c)$.

An SB σ *is a conservative stable standard of behaviour (CSSB) for* $\mathcal{G}$ *if it is both a CISSB and a CESSB.*

Therefore, for an outcome $x \in Z$, a CSSB specifies the set of "credible" paths from x. The underlying idea is that if a coalition makes a feasible deviation from an outcome x to an outcome y then, being farsighted, the players in the coalition examine all the "credible" paths that originate from y. A feasible path is not "credible" if some coalition can feasibly deviate from it to another outcome and if its members are strictly better-off at every credible path originating from that outcome. Thus, the intuition behind CSSB is quite appealing.

The corresponding set of stable outcomes is defined as follows.

[4]The *general* framework of Greenberg (1990) uses concepts like positions, situations, inducement correspondences etc.. A rigorous recasting of the present set-up into Greenberg's framework can be made following Mariotti and Xue (no date).

Definition 4 (Xue (1998)) *Given an environment $\mathcal{G}$, a set $X \subseteq Z$ is said to be a set of stable outcomes under conservatism or Xue-stable if there exists a CSSB σ for $\mathcal{G}$ such that $X = \{a \in Z | \ \{a\} \in \sigma(a)\}$.*

3 The Results

In this section, first we show that if every path is feasible, then CSSB has no predictive power at all for a large class of environments.

Let us specify the following condition.

Condition C: An environment $\mathcal{G}$ satisfies Condition C if for every pair $(a, b) \in Z \times Z$, there exists a path $\alpha \in \Pi_a$ such that $t(\alpha) = b$.

That is, an environment $\mathcal{G}$ satisfies Condition C if for any two outcomes in Z, there exists a path between them.

A large class of environments satisfies this condition; e.g., games in normal form, voting games, games in characteristic function form, social networks[5] (as in the framework of Jackson and Wolinsky (1996)) etc..

Theorem 1 *Suppose an environment $\mathcal{G}$ satisfies Condition C and let Π^f be Π (i.e., every path is feasible). Then, the following SB σ is a CSSB:*

$$\text{for every } a \in Z, \ \sigma(a) = \Pi_a.$$

Therefore, in this case, the entire Z is a Xue-stable set.

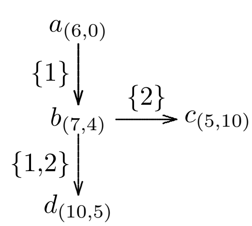
[5] In the appendix we have described how the environment of social networks falls within the present formalism

Proof: It suffices to show that σ as specified in the theorem is conservative internally stable. Suppose otherwise. Then, there exist $a \in Z$ and $\alpha \in \Pi_a$ for which the following is true:

(†) for some $b \in \alpha$, there exist $S \subseteq N$, and $c \in Z$ such that $\{b,\ S,\ c\} \in \Pi_b$, $\Pi(c) \neq \emptyset$ and $\alpha \prec_S \beta$ for every $\beta \in \Pi(c)$.

Therefore, by Condition C, for some $\beta \in \sigma(c)$, $t(\beta) = t(\alpha)$ and by irreflexivity of $\prec_i$, (†) can never hold. ∎

Remark 1 Xue has given some examples of environments where CSSB makes some non-trivial predictions. For example, see the game in Figure 2 of Xue (1998) given below. The unique CSSB for this environment is given by $\sigma(b) = \{b, \{2\}, c\}$ and $\sigma(a) = \{a\}, \sigma(c) = \{c\}$ and $\sigma(d) = \{d\}$. Therefore, for this environment the unique Xue-stable set is: $\{a, c, d\}$.



Note, however, that such environments do not satisfy Condition C. For example, in this game no outcome can be enforced from outcome c and outcome d.

Next, we state another condition.

Condition C': An environment $\mathcal{G}$ satisfies Condition C' if the following holds. For $S \subseteq N$, if there exist $(a, b) \in Z \times Z$, such that $a \underset{S}{\rightarrow} b$, then for every $(c, d) \in Z \times Z$, $c \underset{S}{\rightarrow} d$.

This condition is obeyed by simple games or voting games which we define below (see Peleg (1984)).

DEFINITION 5 *The social environment* $\mathcal{G}$ *is a simple game if there exists a non-empty set* $B \subset 2^N$ *called the set of winning coalitions such that*

(i) $T \in B,\ S \supseteq T \Longrightarrow S \in B$.

(ii) If $S \in B$ *then for any* $a, b \in Z,\ a \underset{S}{\rightarrow} b$ *and if* $S \notin B$ *then for no two* $a, b \in Z$ *is it the case that* $a \underset{S}{\rightarrow} b$.

Below we recapitulate a few concepts which will be useful.

DEFINITION 6 (Chwe (1994)) *For* $a,\ b \in Z$*,* b *indirectly dominates* a*, denoted as* $a \ll b$*, if there exist* $a_0,\ a_1, \ldots,\ a_m$ *in* Z *and coalitions* $S_0,\ S_1, \ldots,\ S_{m-1}$ *such that* $a_0 = a$ *and* $a_m = b$ *and for* $j = 0, \ldots,\ m-1$,

(i) $a_j \underset{S_j}{\rightarrow} a_{j+1}$,

(ii) $a_j \prec_{S_j} a_m$.

DEFINITION 7 (Chwe (1994)) *A set* $Y \subseteq Z$ *is said to be consistent if* $Y = \{a \in Z |\ \forall (S,\ d) \in (2^N \times Z)$ *for which* $a \underset{S}{\rightarrow} d$, $\exists\ e \in Y$ *such that* $[e = d$ *or* $d \ll e]$ *and* $a \not\prec_S e\}$. *The set* $L \subseteq Z$ *is said to be the largest consistent set (LCS) if it is consistent and it contains every consistent set.*[6]

For our subsequent discussion, we restrict the set of feasible paths as follows.

DEFINITION 8 *For* $a_1 \in Z$*, take a path* $\alpha \in \Pi_{a_1}$ *such that* $\alpha = \{a_1,\ S_1, \ldots,\ S_{k-1},\ a_k\}$. *The path* α *is feasible by domination only if for every* $j = 1, \ldots, k-1$, $a_j \prec_{S_j} a_k$. *Additionally, we assume that every singleton path is feasible by domination.*

Therefore, a non-singleton path from an outcome a is feasible only if the terminal element of this path indirectly dominates a along the path. Xue imposed a somewhat similar feasibility restriction in Theorem 4.5 of his paper for comparing CSSB with the LCS. He showed that under such a restriction, his stability notion

[6] Chwe (1994) showed that LCS exists for every environment.

refines the LCS. However, we show the following.

Theorem 2 *Take a social environment $\mathcal{G}$ for which $L \neq \emptyset$*[7] *and which satisfies Condition C'. Let Π^f be the set of paths feasible by domination. Then, $\mathcal{G}$ has some Xue-stable set X such that $L \subseteq X$. Moreover, there are some environments for which this inclusion is strict.*

To prove this theorem we first note the following lemmata.

Lemma 2.1 *Suppose for an environment $\mathcal{G}$, $L \neq \emptyset$ and Condition C' holds. Then, $a \in Z \setminus L$ implies that there exists $b \in L$ such that $a \ll b$.*

Proof: Suppose $L \neq \emptyset$ and $a \in Z \setminus L$. Then, by the definition of L, there exist $(S,\ d) \in (2^N \times Z)$ for which $a \underset{S}{\rightarrow} d$. Suppose that for no $b \in L$ is it true that $a \ll b$. Take any $b \in L$ and consider the pair (S, a). By C', $b \underset{S}{\rightarrow} a$. Since $a \notin L$ and $\{e \in L |\ a \ll e\} = \emptyset$, L cannot be consistent. But this is a contradiction. ■

Lemma 2.2 *Suppose there exists a CISSB σ for an environment $\mathcal{G}$ such that for every $a \in Z$, $\sigma(a) \neq \emptyset$. Then there exists a CSSB σ' for $\mathcal{G}$ such that for every $a \in Z$, $\sigma(a) \subseteq \sigma'(a)$.*

Proof: The proof is exactly similar to that of Theorem 3.4 in Greenberg *et al.* (1996). ■

Proof of Theorem 2: (i) Take an environment $\mathcal{G}$ for which $L \neq \emptyset$ and which satisfies Condition C'. Consider an SB σ as follows:

[7] This is ensured under quite weak conditions (see Chwe (1994), Xue (1997)). For example, every environment for which Z is finite admits a non-empty LCS.

$$\text{for every } a \in Z,\ \sigma(a) = \{\alpha \in \Pi_a^f |\ t(\alpha) \in L\}.$$

By Lemma 2.1, for every $a \in Z$, $\sigma(a) \neq \emptyset$.

Next we show that σ as defined above is conservative internally stable. Suppose otherwise. Then there exist $a \in Z$ and $\alpha \in \sigma(a)$ for which the following is true:

(‡) for some $b \in \alpha$, there exist $S \subseteq N$ and $c \in Z$ such that $\{b,\ S,\ c\} \in \Pi_b^f$, $\sigma(c) \neq \emptyset$ and $\alpha \prec_S \beta$ for all $\beta \in \sigma(c)$.

Now, consider the pair (S, c) with respect to $t(\alpha)$. By C', $t(\alpha) \underset{S}{\rightarrow} c$ and by (‡) for every $e \in L$, $e = c$ or $c \ll e$ implies that $t(\alpha) \prec_S e$. But then by the definition of LCS, $t(\alpha) \notin L$ which is a contradiction. Therefore, σ as defined above is conservative internally stable. Then, by Lemma 2.2, there exists a CSSB σ' for $\mathcal{G}$ such that for every $a \in Z$, $\sigma(a) \subseteq \sigma'(a)$. Define the Xue-stable set $X = \{a \in Z |\ \{a\} \in \sigma'(a)\}$. Then, $L \subseteq X$.

(ii) For proving the second part of the theorem, take the following proper [8] simple game, $\mathcal{G}$.

$N = \{1,\ \ldots,\ 7\}$, $Z = \{a, b, c, d, e, f\}$. Let the set of minimal winning coalitions be $W = \{S_1,\ \ldots,\ S_4\}$ where

$$S_1 = \{1, 2, 3\},\ S_2 = \{1, 4, 5\},\ S_3 = \{2, 4, 6\},\ S_4 = \{3, 5, 6, 7\}.$$

The players' preferences over Z are the following:

1	2	3	4	5	6	7
d	e	b	a	d	b	c
c	d	f	e	f	f	d
b	c	c	c	c	a	a
f	a	d	d	b	e	f
a	f	a	f	e	c	e
e	b	e	b	a	d	b

[8] A simple game (see Definition 5) is said to be proper if for every $S \in B$, $(N \setminus S) \notin B$.

This implies the following relations:

$$a \prec_{S_1} c,\ a \prec_{S_1} d,\ f \prec_{S_2} d,\ b \prec_{S_2} c,\ b \prec_{S_2} d,\ c \prec_{S_3} e,\ d \prec_{S_3} e,\ e \prec_{S_4} f.$$

It is easily checked that for no other $x, y \in Z$ and $S \in W$ is it true that $x \prec_S y$.

This environment satisfies the condition C'. Let Π^f be the set of paths feasible by domination.

In this framework the definition of a consistent set can be simplified as follows (Bhattacharya (2002)): a set $Y \subseteq Z$ is said to be consistent if $Y = \{a \in Z|\ \forall(S,\ d) \in (W \times Z),\ \exists\ e \in Y$ such that $[e = d$ or $d \prec e]$ and $a \not\prec_S e\}$. Then routine computation (see Chwe (1994)) yields that the LCS, L, for this game is $\{c, d, e, f\}$. However, Z is a Xue-stable set for this game. To see this, construct an SB σ such that for every $x \in Z$, $\sigma(x) = \{\alpha \in \Pi^f_x|\ t(\alpha) \in L\}$. By Lemma 2.2, there exists a CSSB σ' for $\mathcal{G}$ such that for every $a \in Z$, $\sigma(a) \subseteq \sigma'(a)$. We claim that $a \in \sigma'(a)$. Suppose otherwise. Then there exist $S \in W$ and $x \in Z$ such that $\{a,\ S,\ x\} \in \Pi^f_a$, $\sigma'(x) \neq \emptyset$ and $a \prec_S t(\beta)$ for all $\beta \in \sigma'(x)$. Check that $\{a,\ S,\ x\}$ is either $\{a,\ S_1,\ c\}$ or $\{a,\ S_1,\ d\}$. But note that $\{c,\ S_3,\ e\} \in \sigma'(c)$ and $\{d,\ S_3,\ e\} \in \sigma'(d)$. Since it is not the case that $a \prec_{S_1} e$, the claim is proved. Similarly it can be shown that $b \in \sigma'(b)$. ■

We obtain the following corollary from the proof of Theorem 2 which may be of independent interest.

Corollary 1 *Take a social environment $\mathcal{G}$ for which $L \neq \emptyset$ and which satisfies Condition C'. Let Π^f be the set of paths feasible by domination. Then, $\mathcal{G}$ has a non-empty valued largest CSSB.*

Remark 2 In this remark we point out a few conceptual drawbacks of the feasibility restriction of Xue which we wanted to remove in our feasibility restriction.

First, Xue merely requires that for a non-singleton path to be feasible, the terminal element should indirectly dominate the first element of the path but *not necessarily along the path*. Secondly, he does not impose this restriction on the entire set of paths but only on those in a non-empty valued CSSB. However, even with Xue's restriction it can be shown that for every environment we studied in Theorem 2, the LCS is contained in some Xue-stable set. However, the strict inclusion would not be true.

Remark 3 Theorem 2 shows that CSSB cannot refine the LCS in an important class of environments. However, in such environments the LCS itself suffers from a shortcoming. For the class of simple games with a finite number of outcomes, the LCS contains elements that are stable owing to incredible coalitional deviations (Bhattacharya (2002)).

Remark 4 An immediate question is that whether we can obtain a result like Corollary 1 if we replace the Condition C' by Condition C. The answer is negative. We give below an example of a social environment $\mathcal{G}$ for which $L \neq \emptyset$, which satisfies Condition C and for which we take Π^f to be the set of paths feasible by domination. However, the unique CSSB for $\mathcal{G}$ is not non-empty valued.

Example 1: (somewhat similar to Figure 4 in Xue (1998)) $N = \{1, 2\}$. $Z = \{a, b, c\}$. The players' preferences on Z are given as follows.

$$c \prec_1 a \prec_1 b \text{ and } b \prec_2 a \prec_2 c.$$

The effectivity relations are given as follows.

$a \underset{\{1\}}{\rightarrow} b,\ a \underset{\{1,2\}}{\rightarrow} b,\ a \underset{\{2\}}{\rightarrow} c,\ a \underset{\{1,2\}}{\rightarrow} c;$

$b \underset{\{1\}}{\rightarrow} a,\ b \underset{\{1,2\}}{\rightarrow} a,\ b \underset{\{1,2\}}{\rightarrow} c;$

$c \underset{\{2\}}{\rightarrow} a,\ c \underset{\{1,2\}}{\rightarrow} a,\ c \underset{\{1,2\}}{\rightarrow} b.$

This environment satisfies C.

Note that $x \prec_1 b$ for every $x \in Z \setminus \{b\}$. Since $1 \in S$ for every $(S, x) \in 2^N \times Z$ such that $b \underset{S}{\rightarrow} x$, the only feasible path from b is $\{b\}$ only. Similarly, note that $x \prec_2 c$ for every $x \in Z \setminus \{c\}$. Again, since $2 \in S$ for every $(S, x) \in 2^N \times Z$ such that $c \underset{S}{\rightarrow} x$, the only feasible path from c is $\{c\}$. Now, by using a similar reasoning as above, it can be checked that there are three feasible paths in Π_a^f, namely, $\{a\}$, $\{a, \{1\}, b\}$ and $\{a, \{2\}, c\}$. Let σ be any CSSB for this environment. By definition of a CSSB, $\sigma(b) = \{b\}$ and $\sigma(c) = \{c\}$. Now take, for example, the path $\{a, \{1\}, b\}$ from a. Consider the outcome a on this path. Then, $\{a,\ \{2\},\ c\} \in \Pi_a^f$, $\sigma(c) \neq \emptyset$ and $b \prec_2 c$. Therefore, the path $\{a, \{1\}, b\}$ cannot be in $\sigma(a)$. By using a similar reasoning for the other two paths, it can be shown that $\sigma(a) = \emptyset$.

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4 Appendix: Social Networks (Jackson and Wolinsky (1996)) in the Present Framework

Let N be the finite set of players and let g^N be the set of all doubleton subsets of N. A *bilateral network* g is a subset of g^N. Then, $Z = \{a| \ a \subseteq g^N\}$. Given a non-empty network $g \in Z$, an element $\{i,j\} \in g$ (where $i, j \in N$) is a link between players i and j in the network g. A value function $v : Z \mapsto \mathbf{R}$ assigns a real value to every network and the set of all value functions are denoted by V. Given a value function $v \in V$, an allocation rule $Y : Z \times V \mapsto \mathbf{R}^N$ allocates the value of a network to the players. Given a value function $v \in V$, an allocation rule $Y : Z \times V \mapsto \mathbf{R}^N$ induces a preference ordering $\preceq_i (v, Y)$ for each $i \in N$ on Z given as follows:

$$\text{for } a, b \in Z,\ a \preceq_i (v,Y)b \text{ if and only if } Y_i(a,v) \leq Y_i(b,v) \text{ and}$$

$$\text{for } a, b \in Z,\ a \prec_i (v,Y)b \text{ if and only if } Y_i(a,v) < Y_i(b,v).$$

Given a profile of players' preferences $\{\preceq_i\}_{i \in N}$, we assume that it has been induced by some underlying value function and allocation rule.

The coalitional effectivity relation is specified as follows.

Definition 9 (Jackson and van den Nouweland (2001)) *For $a, b \in Z$, and $S \subseteq N$, $a \underset{S}{\rightarrow} b$ if and only if*
(i) a link $\{i,j\} \in b \setminus a$ implies that $\{i,j\} \subseteq S$ and
(ii) a link $\{i,j\} \in a \setminus b$ implies that $\{i,j\} \cap S \neq \emptyset$.