Increasing Returns, Entrepreneurship and Imperfect Competition

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May 20, 2003

Abstract

We study a simple bilateral oligopoly model in which individual agents, who are initially endowed with capital, decide sequentially (i) whether they want to act as producers (entrepreneurs) or as capital lenders (rentiers) and, then (ii) which quantity of capital they would like to borrow or lend, though exchange of capital units against units of the produced good. Production takes place under increasing returns to scale. We show the existence of "natural equilibria", at which wealthier capital owners become entrepreneurs while the remaining ones decide to be rentiers. We also study the efficiency of equilibria which is shown to increase by replication of the economy, but sometimes to decrease as a consequence of wealth redistribution.

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1 Introduction

Schumpeter (1934, 1939) and Knight (1921) have both studied the concept of *entrepreneurship*. According to Schumpeter, the entrepreneurs are those who innovate by introducing new techniques of production and exchange, as well as new goods and services. Yet the reasons why some individuals choose to become entrepreneurs and some others rentiers or wage earners lies beyond the scope of his analysis. By contrast, Knight's approach tries precisely to penetrate the process through which the population of economic agents divides itself between these two activities. The explanation he provides is based upon an endogenous evaluation, by each economic agent, of the role which gives him the highest expected utility. According to Knight, three main reasons can explain why individuals distribute themselves between the two classes of entrepreneurs and non-entrepreneurs: *risk-aversion*, *ability* and *wealth*. The less risk-averse, the abler and the wealthier are the more likely to become entrepreneurs.

Several scholars have later developed models in order to formally establish the Knightian views. The first reason suggested by Knight has been developed by Kihlström and Laffont (1979), who have focused on the degree of risk-aversion as the key variable for explaining the equilibrium partition between entrepreneurs and non entrepreneurs. These authors also show that this partition generally fails to be Pareto-optimal and identify three reasons for this inefficiency. First, entrepreneurs who have different degrees of risk-aversion hire different numbers of workers whereas efficiency would require all firms to be of the same size. Second, the entrepreneurs bear all the risks whereas efficiency would require some risk-sharing among all the agents. Finally, there are too many firms at equilibrium. As for Laussel and Le Breton (1995)[1], they have used the second reason proposed by Knight for provid-

[1]See also Moresi (1998) for a slightly different model where only the individual's per-

ing an endogenous explanation of the partition between entrepreneurs and non-entrepreneurs, namely, different individual abilities. Assuming that individual abilities are unobservable, they are led to conclude that more efficient individuals become entrepreneurs and, as Kihlström and Laffont (1979) but for different reasons, they find that there are too many firms at equilibrium. The intuition underlying this result is that, as a worker, an individual obtains a wage which does not depend on his (her) ability, while, as an entrepreneur, his (her) income is an increasing function of his (her) ability. This creates, for individuals who would have been otherwise almost indifferent between the two activities, an incentive for becoming entrepreneurs rather than workers.

To the best of our knowledge, the third argument proposed by Knight, based on the old idea that those who are entrepreneurs are those who are the wealthier, has never been formally explored. The initial motivation for the present paper is precisely to study this conjecture in the simplest possible framework, but allowing for *increasing returns to scale* and *imperfect competition.* To this end, we consider a model with only two goods: a capital good (input) which serves for producing a single consumption good. Economic agents, while differing by their initial endowments of capital, have access to the same technology for producing the consumption good. There is a unique market in which these agents – who have beforehand decided whether to become entrepreneurs or rentiers – exchange units of capital against enforceable promises of delivering units of the consumption good, once manufactured. Assuming, in addition to the above, a constant returns to scale production function, would have made the model trivial and the solution undetermined.[2] In order to discard this indeterminacy, and allow

formance as an entrepreneur depends on his/her ability.

[2]The equilibrium price of capital, measured in units of the consumption good, would then be equal to its marginal/average product and the equilibrium number of firms, as well as the equilibrium partition of the set of agents between entrepreneurs and non-

thereby the different wealth levels of agents to play a role in explaining the partition between entrepreneurs and rentiers, we assume in our approach that production takes place under increasing returns to scale and that agents act strategically in an imperfectly competitive environment. We do this in the simplest possible way, supposing constant marginal productivity of capital and fixed costs on the production side, and non-cooperative behavior of the agents in the capital market. At a non-cooperative equilibrium, the price of capital becomes lower than its marginal product. Differences in wealth levels then entail larger differences in the payoffs of the entrepreneurs than in those of the rentiers: at equilibrium, it pays more to be better endowed with capital for an entrepreneur than for a rentier. This is the basic intuition that may support Knight's conjecture about the role of wealth differences in explaining firms' creation.

Our results confirm Knight's intuition. We show, indeed, that there always exists a "natural equilibrium" at which *every* economic agent who has chosen to be an entrepreneur is wealthier than *every* agent choosing to be a rentier. However there may exist simultaneously one or several "non natural" equilibria at which this property is not satisfied, with at least one rentier j wealthier than at least one entrepreneur s. The existence of such equilibria is made possible due to the upward jump in the capital price which would occur in the case this agent j were to become entrepreneur. This increase may more than outweigh the effects of the differences between capital endowments. By a similar argument agent s may have no incentive to become rentier at an equilibrium in which richer individuals are rentiers, simply because this would induce a downward jump in the capital price. We show however that non-natural equilibria are destroyed when the number of agents is sufficiently large.

entrepreneurs (rentiers), would be undetermined.

Then we concentrate on the efficiency properties of the equilibrium. In our simple framework, efficiency only requires that there should be no replication of fixed costs, i.e. that there is no more than one firm at equilibrium. More generally the extent of inefficiency is an increasing function of the number of entrepreneurs. We show that, if the economy is replicated by multiplying the number of agents of each type by the same strictly positive scalar, the proportion of entrepreneurs in the total population of agents decreases: larger economies are thus more efficient. Finally, we investigate the effect, on the natural equilibrium, of a redistribution of the initial capital endowments among the agents, which is realized by taxing the endowments at a constant rate and redistributing the tax proceeds in a lump-sum uniform fashion. Such a redistribution is shown to increase the number of firms at the resulting natural equilibrium whenever the least endowed agent, among those who have decided to become entrepreneurs, has an initial endowment of capital which does not exceed the average wealth in the economy. In that case, wealth redistribution thus increases the extent of inefficiency of equilibrium by increasing fixed costs' replication.

2 The model

The economy we consider consists of two commodities – a consumption good and a capital good – and n agents ranked in $N = \{1, ..., n\}$ by order of increasing capital endowments $k(j)$, namely $k(j+1) \geq k(j)$, $i = 1, ..., n-1$. Each agent can use his/her capital endowment in two different ways. Either agent j decides to become an *entrepreneur*: then he (she) invests his (her) endowment $k(j)$ and, possibly, other capital units borrowed from the capital market, into a productive process transforming capital into consumption good units. This transformation operates according to the production function F defined by

$$F(K(j)) = Max\{\alpha K(j) - Z, 0\}$$

$\alpha > 0$, $Z > 0$, where $K(j)$ denotes the amount of capital invested by entrepreneur j, including the amount borrowed from the capital market. We denote by $y(j)$ the amount of the good produced that this entrepreneur sells on the capital market in exchange of the amount of capital he (she) borrows from this market, with $y(j) \leq F(K(j))$. Alternatively, agent j decides to become a *rentier*, then lending part of, or the whole capital he is endowed with, at a unit price p, measured in units of the consumption good. We denote by $s(j)$, $s(j) \leq k(j)$, the amount of the capital good lent by rentier j. Also, in the following, we shall denote by E the set of agents who have decided to be entrepreneurs and by e the cardinality of this set. Similarly, the set R denotes the set of those agents who have decided to be rentiers. We shall assume that the price p clears the capital market so that the equality

$$p \sum_{i \in R} s(i) = \sum_{i \in E} y(i) \tag{1}$$

must hold, or

$$p = \frac{\sum_{i \in E} y(i)}{\sum_{i \in R} s(i)}.$$

We analyse the equilibria of the following two-stage game. In the first stage, the agents decide whether to be a rentier or an entrepreneur. In the second stage, taking as given the resulting partition of N into the sets E and R, agents in R decide about the amount of capital they lend, whereas agents in E decide about the amount of the consumption good produced they sell. In this second stage game, all agents take into account the incidence of their individual decisions on the price p, and, accordingly, on the resulting amounts of consumption good they obtain as payoffs. Now in the first stage, in which each agent has two strategies, to be a rentier or an entrepreneur, an equilibrium is defined as a partition of N into two non-empty sets, E and R, such that no entrepreneur in E wishes to become a rentier, and no rentier in R wishes to become an entrepreneur[3].

3 Equilibrium Analysis

Assume that a partition of N into two subsets E and R has been selected in the first stage of the game and consider some rentier $j \in R$. In the second stage, his (her) strategy corresponds to sending to the capital market any amount of capital $s(j)$ which does not exceed his (her) initial endowment $k(j)$. Similarly, for an entrepreneur $j \in E$, a strategy consists in sending to the capital market an amount $y(j)$ of the produced good which does not exceed the production obtained from using his (her) initial capital endowment $k(j)$, augmented by the amount of capital borrowed from the market place at price $p = \frac{\sum_{i \in E} y(i)}{\sum_{i \in R} s(i)}$. Nevertheless, we shall assume that there is a minimal indivisible amount ε of the consumption good, $\varepsilon > 0$, which an entrepreneur must necessarily supply if he (she) is willing to exchange the consumption good against units of capital. We call this amount ε the *market entry cost.* In other words, the strategy of entrepreneur j is either equal to zero if he (she) decides to refrain from exchange, or belongs to the interval $[\varepsilon, F(K(j))]$ if he (she) chooses to operate in the capital market.

Consider now some n-tuple of strategies $\{k(j)_{j \in R}; y(j)_{j \in E}\}$ corresponding to the starting partition $\{E, R\}$. For all agents, the payoff of the second-stage game is defined as the amount of the consumption good he obtains at the end of the second-stage game as a consequence of the above strategy profile.

[3]This equilibrium notion is reminiscent of the notion of internal and external stability introduced by d'Aspremont and *alii* (1983).

The payoff of rentier j at this vector of strategies is defined by

$$\frac{\sum_{i\in E} y(i)}{\sum_{i\in R} s(i)} s(j) - \delta\left[k(j) - s(j)\right] \tag{2}$$

when $\sum_{i\in R} s(i) > 0, \delta > 0$. In (2), the number δ can be interpreted as an inventory cost which has to be borne by a rentier should he decide to keep his or her capital at home; this *inventory cost* can be chosen arbitrarly small, but its existence is needed in order to rule out equilibria at which no capital market would exist (see Proposition 1 below). When, on the contrary, $\sum_{i\in R} s(i) = 0$, the payoff of rentier j at this vector of strategies is now defined by

$$-\delta k(j). \tag{3}$$

As for an entrepreneur j, when $\sum_{i\in E} y(i) > 0$, his payoff obtains as

$$\alpha\left[k(j) + y(j)\frac{\sum_{i\in R} s(i)}{\sum_{i\in E} y(i)}\right] - Z - y(j). \tag{4}$$

and as

$$\alpha\left[k(j)\right] - Z$$

otherwise. In the first case ($\sum_{i\in E} y(i) > 0$), entrepreneur j operates on the capital market where he exchanges a quantity $y(j)$ of the consumption good against a quantity $\frac{y(j)}{p} = \frac{\sum_{i\in R} s(i)}{\sum_{i\in E} y(i)} y(j)$ of capital. In the second case ($\sum_{i\in E} y(i) = 0$), entrepreneur j acts in autarky.

3.1 The second-stage game

In order to identify the second-stage game equilibrium corresponding to a given partition of N into subsets E and R, we now derive the best reply function of each agent in this second-stage game. Let $\{s(j)_{j\in R}; y(j)_{j\in E}\}$ be a given n-tuple of strategies, and consider some rentier $j \in R$. His (her) best reply obtains as the solution to the problem

$$\max_{s(j)} \frac{\sum_{i\in E} y(i)}{\sum_{i\in R} s(i)} s(j) - \delta\left[k(j) - s(j)\right]$$

which is given by

$$s^0(j) = k(j) \tag{5}$$

with a strictly positive inventory cost, it is always a dominant strategy for any $j \in R$ to offer the whole of his (her) capital endowment on the capital market[4].

As for entrepreneur j, his or her best reply is obtained by maximising his/her payoff with respect to $y(j)$, subject to $y(j) \in \{0\} U [\varepsilon, F(K(j))]$. In order to identify precisely the solution to this problem for all possible partitions of N and all vectors of strategies in the corresponding second-stage game, we shall consider successively the following different possibilities.

(i) First, consider the case in which the partition satisfies the inequalities $n > e \geq 2$, and the vector of strategies satisfies $\sum_{i \in R} s(i) > 0$ and $\sum_{i \in E, i \neq j} y(i) > 0$. Then the best reply of entrepreneur j obtains as the solution to the problem

$$Max_{y(j)} \alpha \left[k(j) + y(j) \frac{\sum_{i \in R} s(i)}{\sum_{i \in E} y(i)} \right] - Z - y(j).$$

For all $j \in E$, the first-order necessary and sufficient condition for a maximum writes as

$$\alpha \left[1 - \frac{y(j)}{\sum_{i \in E} y(i)} \right] - p = 0, \tag{6}$$

with $p = \frac{\sum_{i \in R} s(i)}{\sum_{i \in E} y(i)}$. Summing over these e equations, we obtain

$$p = \alpha \frac{e-1}{e}. \tag{7}$$

Since, at equilibrium, we have $s^0(j) = k(j)$ for all $j \in R$, (see (5)), and since the equality $p \sum_{i \in R} s(i) = \sum_{i \in E} y(i)$ must hold by (1), equation (6) can be

[4]Without a strictly positive inventory cost, the best reply of rentier j in the case where $\sum_{i \in E} y(i) = 0$ would be any $s(j) \in [0, k(j)]$. When $\sum_{i \in R, i \neq j} s(i) = 0$, it would be again any $s(j) \in [0, k(j)]$.

rewritten as

$$\alpha\left[1-\frac{y(j)}{\alpha\frac{e-1}{e}\sum_{i\in R}k(i)}\right]-\alpha\left(\frac{e-1}{e}\right)=0 \tag{8}$$

where p has been replaced by its equilibrium value obtained in (7). Solving for $y(j)$ in (8), we get, for all $e\in\{2,n-1\}$,

$$y^0(j)=\frac{\alpha(e-1)}{e^2}\sum_{i\in R}k(i) \tag{9}$$

for all $j\in E$.[5] Substituting this equilibrium value in (4), we obtain, for the case $\sum_{i\in R}s(i)>0$, $\sum_{i\in E,i\neq j}y(i)>0$ and $e>2$, the second-stage equilibrium payoff $V_E(j)$ for each agent $j\in E$, namely,

$$V_E(j)=\alpha\left[k(j)+\frac{\sum_{i\in R}k(i)}{e^2}\right]-Z. \tag{10}$$

Now, for the sake of completeness, note that the best reply of entrepreneur j to a n-tuple of strategies for which $\sum_{i\in R}k(i)=0$ is to set $y^0(j)=0$, which is a corner solution to entrepreneur j problem. Since we have shown above that $s^0(j)=k(j)>0$, this can never occur at equilibrium, except in the special case where R is empty, which implies that nobody has chosen the strategy of being a rentier in the first stage of the game. Also notice that the best reply of entrepreneur j in the case $\sum_{i\in R}s(i)>0$ and $\sum_{i\in E,i\neq j}y(i)=0$ is obviously $y^0(j)=\varepsilon$ since it yields a payoff $\alpha\left[k(j)+\sum_{i\in R}k(i)\right]-Z-\varepsilon$ which is larger than the autarkic payoff $\alpha\left[k(j)\right]-Z$ for ε sufficiently small. In equilibrium this can occur only in the monopsony case with a single entrepreneur. Then the case $\sum_{i\in R}s(i)>0$ and $\sum_{i\in E,i\neq j}y(i)>0$ occurs in equilibrium if, and only if, the inequalities $2\leq e<n$ hold. As for the equilibrium payoff $V_R(j)$ of a rentier $j\in R$ when $e\in[2,n-1]$, we get it by substituting the equilibrium value for p resulting from (7) in the payoff of rentier j as defined by (2),

[5] Note that, if one assumes, for $n>2$, that the inequality $0<\varepsilon<\frac{\alpha(n-2)}{(n-1)^2}k(1)$ is satisfied, we are guaranteed that (9) always satisfies the constraint $y^0(j)\geq\varepsilon$.

namely

$$V_R(j) = \alpha(\frac{e-1}{e})k(j). \tag{11}$$

(ii) Now let us consider the case of a partition for which $e = n$. Then, obviously there are no rentiers and for all $j \in E = N$, $y^0(j) = 0$ so that $V_E(j) = \alpha\,[k(j)] - Z$.

(iii) Finally, it remains to examine the case in which $e = 1$, i.e. the case in which a single entrepreneur is a monopsonist on the capital market. Then the price of capital is zero (see (7)) but, due to the assumption that the entrepreneur must supply a minimum quantity $\varepsilon > 0$ in order to operate on the capital market, his best reply is then $y^0(j) = \varepsilon$ so that $V_E(j) = \alpha\,[k(j) + \sum_{i\in R} k(i)] - Z - \varepsilon$. This payoff tends toward the RHS of (10) as ε tends toward 0. As for the equilibrium payoff of rentier j when $e = 1$, taking into account the fact that, in this case, $y^0(j)$ is equal to the market entry cost ε, it follows from (1) that

$$p = \frac{\varepsilon}{\sum_{i\in R} k(i)}.$$

We conclude that, when $e = 1$, rentier j obtains as equilibrium payoff

$$V_R(j) = \frac{\varepsilon k(j)}{\sum_{i\in R} k(i).}$$

This expression tends toward the value given by (11) when the entry cost ε tends to 0.

3.2 The first-stage game

Now let us consider an equilibrium in the first stage game. Remember that, in this subgame, each agent has two strategies: to be a rentier or an entrepreneur. An equilibrium is then defined as a partition of N into two non-empty sets, E and R, such that no entrepreneur in E wishes to be a rentier, and no rentier in R wishes to be an entrepreneur, when taking into account the payoffs in the second stage game obtained from the new partition which would obtain from their unilateral move. More precisely, a partition of N into two non-empty sets E and R is an equilibrium if, for each agent $j \in E$,

$$V_E(j) - V_{RU\{j\}}(j) \geq 0$$

and, for each agent $j \in R$,

$$V_{EU\{j\}}(j) - V_R(j) \leq 0.$$

The first (resp. second) inequality guarantees that a unilateral move of any entrepreneur (resp. rentier) is unprofitable. When $e \in [2, n-1]$, using (10) and (11), the two last inequalities rewrite as

$$V_E(j) - V_{RU\{j\}}(j) = \alpha \left[\frac{k(j)}{e-1} + \frac{\sum_{i \in R} k(i)}{e^2} \right] - Z \geq O \qquad (12)$$

for all $j \in E$,and

$$V_{EU\{j\}}(j) - V_R(j) = \alpha \left[\frac{k(j)}{e} + \frac{\sum_{i \in R} k(i) - k(j)}{(e+1)^2} \right] - Z \leq 0. \qquad (13)$$

for all $j \in R$. When $e = n$, there is no rentier and only condition (12) has to hold. Now consider the case $e = 1$. Then conditions (12) and (13) defining a first-stage equilibrium have to be modified for the following two reasons. First, the second-stage equilibrium payoff of the sole entrepreneur j is then equal to $\alpha [k(j) + \sum_{i \in R} k(i)] - Z - \varepsilon$, which slightly differs from (10) and, on the other hand, $V_{RU\{j\}}(j)$ is not given by (11) (since, would agent j decide to become a rentier, there would be no entrepreneur and, hence, no market!), but by $V_{RU\{j\}}(j) = -\delta k(j)$. Hence, the condition for a non-profitable unilateral deviation in the first stage game for the unique entrepreneur has now to be written as[6]

$$\alpha \left[k(j) + \sum_{i \in R} k(i) \right] - Z - \varepsilon + \delta k(j) \geq 0.$$

On the other hand, when $e = 1$, condition (13) has to be modified as well since now we have $V_R(j) = \frac{\varepsilon k(j)}{\sum_{i \in R} k(i).}$ (see above). Hence, when $e = 1$, the condition for a non-profitable unilateral deviation in the first stage game for the $n-1$ rentiers has now to be written as

$$\alpha \left[k(j) + \frac{\sum_{i \in R} k(i) - k(j)}{4} \right] - Z - \frac{\varepsilon k(j)}{\sum_{i \in R} k(i)} \leq 0 \qquad (14)$$

for all $j \in R$.

In order to prove the existence of an equilibrium, we introduce the following assumptions:

Assumption 1 $\alpha \left[\sum_{i \in N} k(i) \right] - Z > \varepsilon$

Assumption 2 $\alpha k(1) - Z < 0.$

Assumption 1 simply says that one can produce an amount of the consumption good which strictly exceeds the market entry cost by using the whole amount of capital existing in the economy. Assumption 2 means that the "fixed cost" Z is high enough to prevent the poorest capital endowed agent to produce a positive amount of the consumption good in an autarkic way.

Proposition 1 *Under Assumptions 1-2, there exists at least one rentier and one entrepreneur at equilibrium.*

[6]Notice that, if $\alpha \left[k(j) + \sum_{i \in R} k(i) \right] - Z > 0$, there always exists a value $\varepsilon^{\circ}, \varepsilon^{\circ} > 0$,such that the above condition is automatically fulfilled for all $\varepsilon, 0 < \varepsilon < \varepsilon^{\circ}$.

Proof: Suppose that $e = n$ (no rentier); agent 1 should then prefer to be an entrepreneur than to become a rentier. According to (12) it must then be that

$$\frac{\alpha k(1)}{n-1} \geq Z;$$

but this contradicts Assumption 2. Now suppose that $e = 0$ (no entrepreneur); agent n should then prefer to be a rentier than to become an entrepreneur. In the first case, his (her) payoff is $-\delta k(n)$ while in the second it would be $\alpha \left[\sum_{i \in N} k(i)\right] - Z - \varepsilon$. Consequently, we need

$$\alpha \left[\sum_{i \in N} k(i)\right] - Z - \varepsilon + \delta k(n) < 0,$$

which contradicts Assumption 1. Q.E.D.

In the following we define a *natural equilibrium* as an equilibrium such that, for all $j \varepsilon E$ and $j^{'} \varepsilon R$, $k(j) > k(j^{'})$. Thus, a natural equilibrium has the property that all agents who decide to become entrepreneurs initially own larger amounts of capital than do rentiers.

Proposition 2 *Under Assumptions 1-2, for all* $n \geq 2$, *there exists a natural equilibrium.*

Proof: For $n = 2$, the proof follows immediately from Proposition 1. Now suppose that $n > 2$. For all $j \in N$, and all $x \in [j, j+1]$, define the function $k(x)$ by

$$k(x) = k(j) + \left[k(j+1) - k(j)\right](x - j)$$

and denote by $H(x)$ the function defined by

$$H(x) = \alpha \left[\frac{k(x)}{n-x} + \frac{\sum_{i=1}^{x-1} k(i)}{(n-x+1)^2}\right] - Z.$$

The function $H(x)$ coincides with $V_E(j) - V_{RU\{j\}}(j)$ when $x = j$, $j \in \{1, n-1\}$ or, equivalently, with $V_{EU\{j\}}(j) - V_R(j)$ when $x = j$, $j \in \{1, n-2\}$. This is a continuous piecewise linear increasing function on $[0, n]$ which, by Assumption 2, is negative for $x = j = 1$ .When $x = n$, we know from above that

$$V_E(n) - V_{RU\{n\}}(n) = \alpha \left[\sum_{i \in N} k(i) \right] - Z - \varepsilon + \delta k(n) > 0, \tag{15}$$

where the last inequality follows from Assumption 1. When $x = n - 1$, we already know from (14) that, for $j = n - 1$, we have

$$\begin{aligned} & V_{EU\{n-1\}}(n-1) - V_R(n-1) \\ = \;& \alpha \left[k(n-1) + \frac{\sum_{i=1}^{n-1} k(i)}{4} \right] - Z - \frac{\varepsilon k(n-1)}{\sum_{i=1}^{n-1} k(i)} \\ < \;& H(n-1). \end{aligned} \tag{16}$$

There are only two possible cases . Either there exists $x^0 \in \{2, n-1\}$ such that $H(x^\circ - 1) \leq 0 \leq H(x^\circ)$ and the partition $R = \{1, ..., x^0\}$ and $E = \{x^0 + 1, ..., n\}$ satisfies all the conditions required by Proposition 2. Or $H(x) < 0$ for all $x \in \{1, .., n-1\}$ and, more specifically,

$$H(n-1) = \alpha \left[k(n-1) + \frac{\sum_{i=1}^{n-1} k(i)}{4} \right] - Z < 0.$$

Using the inequalities (15) and (16), we can now conclude that the partition $R = \{1, .., n-1\}$ and $E = \{n\}$ is an equilibrium partition satisfying all the conditions required by Proposition 2. Q.E.D.

Proposition 1 is necessary in order to prepare the proof of Proposition 2: this proposition guarantees the existence of a capital market, discarding thereby the trivial autarkic equilibrium as the unique solution of the game. The price to be paid for this is the assumption that the poorest agent cannot manufacture a positive amount of the consumption good in an autarkic way.The following example shows that, without this assumption, a capital market could fail to exist. Let $n = 4, \alpha = 1, Z = \frac{1}{4}, k(i) = i, i = 1, .., 4$. Assume that agent 1 would accept to lend his capital endowment. The most favourable case for agent 1 would be a capital market with three entrepreneurs on the demand side since the price $\frac{\alpha(e-1)}{e}$ is increasing with the number e of entrepreneurs. Then he would obtain an amount of output equal to $\frac{\alpha(e-1)}{e} = \frac{2}{3}$, which is smaller than the amount of output $\alpha k(1) - Z = \frac{3}{4}$ that he can produce in an autarkic way. Of course autarkic production by agent 1 violates assumption 2.

Proposition 2 shows the existence of a *natural equilibrium*. The example below shows however that there may exist other equilibria than the natural ones. Let $n = 5, \alpha = 1, Z = 4.5, k(i) = i$. At any equilibrium there are two entrepreneurs and three rentiers, and the price of capital is equal to $\frac{1}{2}$. There is indeed a natural equilibrium with $E = \{4,5\}$ and $R = \{1,2,3\}$, but $E = \{3,5\}$ and $R = \{1,2,4\}$, or $E = \{3,4\}$ and $R = \{1,2,5\}$, are equilibria as well. In the next section, we show that such equilibria are destroyed when the number of agents in the economy is sufficiently large.

4 The Equilibrium Number of Firms

First, it is worthwhile to notice that, given the definition of $H(x)$ in proposition 2, the number of entrepreneurs corresponding to the natural equilibrium – denote it by $e(n)$ – is a stepwise decreasing function of $\frac{Z}{\alpha}$ and a stepwise increasing function of n. Similarly, the number of rentiers at equilibrium is an increasing function of n. Given the cost function parameters (Z, α), for any n, consider an economy including n agents with increasing capital endowments $k(i)$, namely $k(i+1) \geq k(i)$, $i = 1, ..., n-1$, with $e(n)$ denoting the number of entrepreneurs at its natural equilibrium. The proof of the next proposition relies on the following lemma which is proved in appendix. Given n, define by $\Omega(e(n))$ the set of partitions of $\{1, ..., n\}$ into two subsets E and R such that (i) the cardinal of E ($Card\Omega(e(n))$) is equal to $e(n)$ and (ii) the partition corresponds to an equilibrium in the economy with cost parameters (Z, α) and increasing capital endowments $k(i)$, namely $k(i+1) \geq k(i)$, $i = 1, ..., n-1$.

Lemma 1 *$card\Omega(e(n)) > 1$, then*

$$\frac{k(n-e(n))}{k(n-e(n)+1)} > \frac{(e(n)-1)(3e(n)^2 + e(n)^3 + 2e(n)(n-1) - n)}{(e(n)+1)^2 e(n)^2}.$$

Now we are in position to prove the following

Proposition 3 *There exists an integer n° such that, for all economies including n agents with $n \geq n^\circ$ and sharing the cost function parameters (Z, α), the only equilibrium is the natural one.*

Proof. First remember that the number $e(n)$ of entrepreneurs and the number $n - e(n)$ of rentiers are both stepwise increasing functions of n. Furthermore, since the function $e(n)$ is stepwise increasing, the sequence $e(n)$ must converge either to a finite value or to $+\infty$ when n tends to $+\infty$. Also notice that, due to the fact that the sequence of capital endowments $k(i)$ is increasing in i for any n, the LHS of the inequality in the statement of lemma 1 is always smaller than 1. Denote by $\varphi(e(n), n)$ the RHS of the same inequality. First assume that $e(n)$ tends to a finite value when $n \to +\infty$. Then $\varphi(e(n), n)$ tends to $+\infty$ and, since the LHS is bounded by 1, the inequality in lemma 1 must be reversed for some value n° and all values of n larger than n°. Accordingly, it follows from lemma 1 that, for all $n \geq n^\circ$, $card\Omega(e(n)) = 1$ and the only remaining equilibrium is the natural one. Now assume that $e(n)$ tends to $+\infty$ when $n \to +\infty$. Then we notice that $\varphi(e(n), n) > \varphi(e(n), e(n))$ since $e(n) > 1$. Accordingly, if $e(n)$ tends to $+\infty$ when $n \to +\infty$, $\varphi(e(n), n)$ tends to a number strictly larger than $\lim_{n \to \infty} \varphi(e(n), e(n)) = 1$. Since the LHS in the inequality of the lemma is bounded by 1, this inequality must be reversed for some value n° and all values of n larger than n° , which completes the proof of the proposition. Q.E.D.

The following examples give, for specific capital endowment distributions and specific values of the fixed cost parameter Z, the number of agents n° which is needed to guarantee that the natural equilibrium remains the only equilibrium for all values of n exceeding n°. These values are computed from the condition stated in Lemma 1 above.

Example 1 *Assume* $Z = 5.1876510, k(i) = (1.05)i$ *; then* $n^\circ = 964$.

Example 2 *Assume* $Z = 0.435076, k(i) = i$*; then* $n^\circ = 1002$.

5 The efficiency of equilibrium

Due to its simplicity, our model presents the advantage of allowing a very simple Pareto- ranking of possible outcomes: the larger the number e of entrepreneurs, the larger the number of times that the fixed cost Z has to be borne by the society as a whole, and the smaller the resulting welfare. The above results do not give too many insights concerning the number of entrepreneurs at equilibrium. The only interesting insight is related to the equilibrium price (7): the smaller the number of entrepreneurs, the smaller the price of capital and the weaker the retribution of a rentier! Accordingly, a small number of entrepreneurs, which is a good feature in terms of efficiency, may give each of these entrepreneurs too much market power and cancel thereby the willingness of rentiers to lend their capital. At equilibrium these two forces balance each other: the price of capital is sufficiently low to induce entrepreneurs to be net demanders of capital, and sufficiently high to induce rentiers to be net suppliers. But this market balance can possibly require a large number of entrepreneurs, at the expense of efficiency. This is the reason why it could be interesting to explore more in depth the efficiency properties of the equilibrium.

5.1 Efficiency and the size of the economy

First of all we study how this number of entrepreneurs is related to the size of the economy. To this end, starting with an economy with n agents as defined above, we define the rth-replica of this economy as the economy obtained by replicating r-times the n initial agents, with r agents of *type* $i, i = 1, .., n$, all agents of type i being endowed with the same amount of capital as agent i in the starting economy, i.e. $k(ih) = k(i), h = 1, .., r$. All agents in the rth-replica have access to the same productive technology as in starting one. In the following we shall assume that the equilibrium partition satisfies the inequalities $2 < e < n$.

Proposition 4 *The proportion $\frac{e}{n}$ of entrepreneurs at equilibrium decreases with the number of replicas.*

Proof. Let E and R be an equilibrium partition in T_1 and e the cardinality of E. We know that the inequality (12) must hold for all $j \in E$. Now consider the partition of the set T_r into the sets E_r and R_r with E_r (resp. R_r) consisting of r agents of type i, $i \in E$ (resp. $i \in R$). If the sets E_r and R_r would be an equilibrium in the economy T_r, it would require that the inequality (12) be satisfied for any agent $i \in E_r$ or

$$\alpha \left[\frac{k(j)}{re-1} + \frac{\sum_{ih \in R_r} k(ih)}{(re)^2} \right] - Z = \alpha \left[\frac{k(j)}{re-1} + \frac{r \sum k(i)}{(re)^2} \right] - Z \geq 0,$$

which, clearly, cannot be possible for all r. Consequently, there exists a value of r which is sufficiently large for violating equation (12) so that the proportion of entrepreneurs at equilibrium must be smaller, the larger the number of replicas. Q.E.D.

Thus, Proposition 3 shows that an increase by replication of the size of the economy tends to reduce the proportion of entrepreneurs and, accordingly, the inefficiencies related to the replication of fixed costs. However this is not surprising since the "size" of increasing returns, Z, relative to the total quantity $r \sum_{i=1}^{n} k(i)$ of capital existing in the economy, tends to zero when r becomes arbitrarily large. In other words, the economy tends to be closer and closer to a constant returns to scale economy, in which production efficiency is independent of the number of firms.

5.2 Efficiency and redistribution

Since the equilibrium number of firms depends on the distribution of initial endowments, it is also natural to examine how redistributing these endowments can possibly affect this number and, thereby, the efficiency of the natural equilibrium. To this end, let us consider a redistributive scheme defined by

$$\kappa(i) = (1-t)k(i) + t\frac{\sum_{j=1}^{n} k(j)}{n}$$

with $\kappa(i)$ denoting the after-tax and transfer wealth of agent i (lump-sum transfers). No redistribution corresponds to the case: $t = 0$, and an increase of t implies more redistribution. Define the function $H(e,t)$ by

$$H(e,t) = \alpha \left[\frac{(1-t)k(n-e+1) + t\frac{\sum_{j=1}^{n} k(j)}{n}}{e-1} + \frac{\sum_{j=1}^{n-e} k(j)(1-t) + t\frac{(n-e)\sum_{j=1}^{n} k(j)}{n}}{e^2} \right] - Z.$$

Notice that the integer part of the real number e which solves the equation $H(e,t) = 0$ corresponds to the equilibrium number of firms after the redistribution t (by the no-advantageous deviation condition, we must have $H(e,t) \geq 0$ for the "marginal" entrepreneur $n-e+1$). Clearly the function

$H(e,t)$ is increasing in e. Differentiating $H(e,t)$ with respect to t, we obtain

$$\frac{\partial H(e,t)}{\partial t} = \frac{\frac{\sum_{j=1}^{n} k(j)}{n} - k(n-e+1)}{e-1} + \frac{(n-e)\frac{\sum_{j=1}^{n} k(j)}{n} - \sum_{j=1}^{n-e} k(j)}{e^2}. \quad (17)$$

The first term of this derivative is positive (resp. negative) whenever the average wealth over the whole population is larger (resp. smaller) than the "marginal" entrepreneur's wealth. As for the second term, it is obviously always positive since the average wealth of the whole economy is always larger than the average rentiers' wealth. Thus, we can conclude that, whenever the average wealth over the whole population is larger than the "marginal" entrepreneur's wealth, *wealth redistribution will increase the equilibrium number of firms, and thereby decrease efficiency!* However, notice that the expression (22) tends toward $-\infty$ as e tends to 1. Consequently, when the equilibrium number of firms is below some critical value, wealth redistribution is likely to reduce the equilibrium number e of firms. Of course, this number is an endogenous variable in the model, which depends on the ratio Z/α. It is straightforward that e is a decreasing function of this ratio. So we can conclude that, for values of Z/α which are sufficiently small, the equilibrium number of firms increases with t: wealth redistribution entails an inefficient replication of fixed costs. To illustrate, consider the following example. Suppose that $k(i) = i$. It is easy to show that e increases with t whenever the ratio Z/α is smaller than $\frac{(n+1)(n-\sqrt{n}+1)}{2(n-\sqrt{n})}$, a number which tends toward $\frac{n+1}{2}$ as n tends to infinity. In this example, any wealth redistribution is inefficient when the richest agent holds approximately more than twice the amount of fixed costs required to operate a firm[7].

[7] A similar decrease of welfare due to income redistribution was put in evidence in the context of a vertical product differentiation model; see Gabszewicz and Thisse (1979).

6 Conclusion

In this paper we have studied a simple bilateral oligopoly model in which economic agents, initially endowed with capital, decide sequentially (i) whether they want to act as producers (entrepreneurs) or as capital lenders (rentiers) and, then, (ii) which quantity of the consumption good they would like to produce for the former, and which quantity of capital they would like to lend for the latter, through exchange of capital against the produced good. Production takes place under increasing returns to scale. We have shown the existence of a natural equilibrium at which the wealthier capital owners become entrepreneurs while the remaining ones decide to be rentiers. We have also studied the efficiency of equilibria which was shown to increase by replication of the economy, but sometimes to decrease as a consequence of wealth redistribution.

The model we have considered can be viewed as analogous to the bilateral oligopoly model introduced in the framework of pure exchange economies (see Gabszewicz and Michel (1997) and Bloch and Ferrer (2000)). The main difference with this exchange model lies in the fact that, while traders in the latter are assigned to the buying or selling side of the market according to the commodity they are initially owning, the agents in the former decide themselves on which side of the market they want to operate. The concept of equilibrium used in the present approach is reminiscent of the notion utilised by d'Aspremont and *alii* (1983) in which a sequential game representing an industry is considered. The game consists in selecting sequentially in the first stage a partition of the agents between a cartel and a competitive fringe while, in the second stage, the cartel decides about the market price and the competitive fringe about the quantities its members supply at that price.

Our analysis calls for being generalized in several directions. First of all, it would be natural to introduce labor as a second input. This would certainly complicate the analysis but, most probably, it would not change fundamentally the results we have obtained. Also the presence of uncertainty could be considered, which would submit the production delivery by the entrepreneurs to a nonnull probability of default. In the present formulation, which does not include any uncertainty, entrepreneurs cannot commit themselves to supply amounts of the consumption good they will not be able to deliver. This would no longer be the case if the productive activity would be contingent on random events,which are beyond the control of the agents. Also we could have opted for an alternative causality to explain why wealthier people are willing to be entrepreneurs. We have in mind that better endowed agents have easier access to the capital market because lenders put more confidence on these agents when they evaluate the probability of reimbursement default. Finally, a more general approach could be conceived, inspired from the second stage-game considered above. In this extended model, some agents would be factor owners while the remaining ones would own the technological knowledge necessary to produce the consumption goods. Factor owners, as well as "technology owners", would act strategically to manipulate the equilibrium prices on the various input and output markets via the quantities they send to these markets, as in the market games *à la Shapley-Shubik*. Studying the Nash equilibria of the resulting game is certainly a challenging research project.

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Appendix: Proof of Lemma 1

Let us consider a non natural partition of N such that (i) $cardE = e$, (ii) m_E is the marginal entrepreneur, namely $m_E = \min\{j \in E\}$, and (iii) m_R is the marginal rentier, i.e. $m_R = \max\{j \in R\}$ and $m_R > m_E$. This partition must satisfy $m_R > n-e$ (the number of entrepreneurs who hold more capital than the richest rentier is lower than the number of firms) and $m_E < n-e+1$ (the number of rentiers holding less capital than the poorest entrepreneur is strictly lower than the number of rentiers in the economy). For such a partition to be an equilibrium one we already know from the no profitable deviation conditions that it must satisfy

$$\frac{k(m_R)}{e} + \frac{\sum_{j\in R} k(j) - k(m_R)}{(e+1)^2} < \frac{Z}{\alpha} \leq \frac{k(m_E)}{e-1} + \frac{\sum_{j\in R} k(j)}{e^2}.$$

Rearranging the elements of these inequalities, if this partition is an equilibrium for some interval of values of Z/α, it must satisfy

$$\frac{e-1}{e^2}k(m_R) - \left[\sum_{j\in R} k(j) - k(m_R)\right]\frac{1+2e}{(e+1)^2 e^2} < \frac{k(m_E)}{e-1}. \tag{A.1}$$

Now we notice that

$$\sum_{j\in R} k(j) - k(m_R) < (n-e-1)k(m_R),$$

so that

$$\left[\frac{e-1}{e^2} - (n-e-1)\frac{1+2e}{(1+e)^2 e^2}\right] k(m_R)$$
$$< \frac{e-1}{e^2}k(m_R) - \left[\sum_{j\in R} k(j) - k(m_R)\right]\frac{1+2e}{(1+e)^2 e^2}.$$

Then, for the inequality A.1 to hold, it must be that

$$\frac{(e-1)(3e^2+e^3+2e(n-1)-n)}{(e+1)^2 e^2} < \frac{k(m_E)}{k(m_R)}.$$

Finally, observe that over the set of *non natural partitions* of N such that $cardE = e$, the ratio $\frac{k(m_E)}{k(m_R)}$ is maximum when $m_E = n-e$ and $m_R = n-e+1$. Consequently, we obtain that $\Omega(e) > 1 \Rightarrow$

$$\frac{k(n-e)}{k(n-e+1)} > \frac{e-1)(3e^2+e^3+2e(n-1)-n}{(e+1)^2e^2},$$

which completes the proof of Lemma 1.