Using Column Generation to Solve an Industrial Mixing Problem.

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Abstract

The problem considered in this paper is a real world problem. It concerns the management of the deliveries of coal to the several plants of a coke industry firm in order to meet the demand of several clients at minimal purchasing, transportation and production cost. The problem is solved using a column generation technique. At the lower level, a mix of one ton satisfying all the client quality requirements is determined for each plant at each period. At the upper level, the deliveries of coal and the level of use of the mixes are determined in order to meet the demands of the clients.

1 Introduction

The manager of a coke production firm wanted to optimize the operation of the five plants of the firm. This implies mixes composed of several coals with intrinsic characteristics (e.g. sulfur rate, alkali rate). The objective of the management is to determine the optimal coal purchase plan, the optimal transportation plan to the five plants, the optimal combination of mixes for each period and each plant and the optimal demand satisfaction plan.

There are several quality constraints on the product which, for some of them, may depend on the clients. These constraints are all linear. But there are also, for technical reasons, a maximum number of coals in the mix, this number depending on the number of gates of the plant. There is also a minimal proportion for each coal present in the mix. As we shall see in Section 3, the modelling of these two conditions (See Williams [7]) introduces integer variables and non linear constraints.

The industrial problem tackled in this paper is more difficult than the coal blending problem of the OR literature (See Sarker and Gunn [4] and Greenberg [2]) because of the presence of binary variables. These binary variables are required to model the minimum and/or maximum level when a coal is used. Sarker and Gunn [4] solve the tactical planning/coal blending problem. This leads to a nonlinear nonconvex problem for which solutions are computed using simple Successive Linear Programming. They used the Lasdon SLP implementation based on the XLP code of Marsten (See Lasdon [3]). Greenberg [2] considers the same pooling problem that arises in blending materials but considers a different application, namely the blending of crude or refined petroleum. He presents a new method based upon computational geometry which provides exact answers to questions of sensitivity analysis for this non linear non convex problem.

We solve the more complicated problem through the following column generation technique (See Vanderbeck and Wolsey [5] for an exact algorithm for IP column generation or Vanderbeck [6] for the branch-and-price algorithm). For each time period, and each plant, fixing the coal delivery prices at the entry of the plant, we determine the optimal coal mix for one unit of production. This constitutes the lower level of our problem. Then, at the upper level, we determine the level of use of these mixes. At this upper level, we also determine the coal delivery plan to the plants and the coke delivery plan to the clients. As we shall see in Section 4, the solution of the upper level program leads to reviewing the delivery prices at the entry of the plant for each coal and a new mix based on these new prices is generated at the lower level. This generates new columns for the upper level problem. There are also integer variables at the upper level since only a limited number of different mixes can be considered at each time period.

The paper is organized as follows. Section 2 gives a description of the industrial mixing problem. The mathematical formulation of the problem is given in Section 3. Section 4 describes the solution method based on column generation. Section 5 presents the numerical results. In Section 6, our solution method is compared to an alternative column generation method and to a reformulation technique.

2 Problem Description

The manager of the coke production firm wanted to determine the operation schedule for the next 3 months. We use t as index for the time periods. There are three time periods corresponding to the months January to March. We note $days_t$ the number of days for month t. The demand is assumed to be known with accuracy for the first three months of the year.

2.1 Plants description

The five plants are geographically located in different regions of the country (Belgium). Table 1 gives the daily capacities in tons of coal entering the plant. We note by cap_k the daily capacity of plant k. For a technical reason (the plant can never be stopped), one also has a minimal utilization rate for each plant, noted $minuse_k$ for plant k. These rates are also given in Table 1.

To input the coals into the oven, there is only a limited number of entry gates, noted $gates_k$ for plant k. This means that the number of coals in the mix is limited. There are also a minimal and a maximal proportion for each coal in the mix, noted respectively $mincoal_k$ and $maxcoal_k$ for plant k. These data are given in Table 2.

Plants also differ by their unit production cost. These are given in Table 3 in EUR per ton of coal entering the oven. We note this information $prodcost_{kt}$ for the unit production cost of plant k at time period t.

Plant (k)	Capacity (cap_k)	Minimal utilization rate ($minuse_k$)
1	2 300	60 %
2	2 850	60 %
3	1 250	75 %
4	3 500	60 %
5	3 500	60 %

Table 1: Plant characteristics

Plant (k)	Gates ($gates_k$)	minimal percentage ($mincoal_k$)	maximal percentage ($maxcoal_k$)
1	8	5 %	100 %
2	4	15 %	35 %
3	8	10 %	100 %
4	8	10 %	100 %
5	8	10 %	100 %

Table 2: Characteristics of the mix

$prodcost_{kt}$	$t = 1$	$t = 2$	$t = 3$
$k = 1$	11.450	11.400	11.350
$k = 2$	18.175	18.100	18.025
$k = 3$	38.150	38.000	37.850
$k = 4$	24.125	24.025	23.925
$k = 5$	11.300	11.250	11.200

Table 3: Unit production cost (EUR per ton of coal)

2.2 coal characteristics

The firm can use 16 coals which differ by their characteristics such as the ash rate (noted ash_c for coal c), the sulfur rate (noted $sulf_c$ for coal c), the alkali rate (noted alk_c for coal c), the volatile part (noted vol_c for coal c) and the wet part (noted wet_c for coal c). These data are given in Table 4. One also distinguishes three classes among the coals: the "High Volume", noted HV, the "Mid Volume", noted MV, and the "Low Volume", noted LV. These classes are also given by Table 4.

coal c	Ash (%) ash_c	Sulfur (%) $sulf_c$	Alkali (%) alk_c	Volatile (%) vol_c	Wet (%) wet_c	Type
1	4.99	.85	.12	17.89	6.63	LV
2	5.53	.70	.09	17.48	7.77	LV
3	7.72	.94	.31	22.71	10.15	LV
4	8.30	.67	.14	21.00	8.00	LV
5	8.07	.70	.16	23.60	9.47	MV
6	4.83	.88	.16	30.19	6.80	HV
7	6.86	1.15	.24	29.90	7.54	HV
8	6.14	.84	.22	31.39	8.66	HV
9	6.01	.82	.19	32.75	6.29	HV
10	6.45	.88	.17	33.08	7.20	HV
11	7.44	.66	.24	27.43	6.45	HV
12	7.70	.71	.23	32.09	6.81	HV
13	7.00	.98	.32	25.10	8.00	MV
14	7.60	.57	.18	19.30	10.14	LV
15	5.79	.82	.18	24.39	8.99	MV
16	5.30	.72	.13	33.50	7.90	HV

Table 4: coal characteristics

2.3 Availability and delivery of the coals

Some of the coals come by boat. They mainly come from North America and from Australia. There are two possible landing harbours that we index by h. There is an initial coal inventory at the beginning of the year in the two harbours. These data, noted $initstock_{ch}$ for the initial stock of coal c in harbour h, are given in tons in Table 5 for all the coals coming by boat.

$initstock_{ch}$	harbour 1	harbour 1
coal 1	558	7 620
coal 2		
coal 4		36 655
coal 5	42 760	22 570
coal 10		
coal 11	5 950	
coal 12		
coal 14	43 505	
coal 16		6 450

Table 5: Initial inventory of coal in the two harbours

Deliveries corresponding to already ordered quantities are expected for some of these coals. The landing harbour is not yet decided. Typical delivery data are given in Table 6. We note these quantities $expq_{ct}$. Additional quantities may be ordered for all the coals coming by boat for the 3 time periods.

$expq_{ct}$	$t = 1$	$t = 2$	$t = 3$
coal 1	70 000	70 000	100 000
coal 5	40 000		60 000
coal 14	40 000		60 000
coal 16	25 000	25 000	

Table 6: Deliveries of coals expected by boat

The other coals are delivered by rail. They mainly come from Germany and from Eastern-Europe. There is no storage capacity for these coals coming by rail. Deliveries are expected for these coals. The final destination of these quantities is not yet decided. As for the coals coming by boat, we note $expq_{ct}$ the expected quantity of coals coming by rail c at time period t. Typical data are given by Table 7.

$expq_{ct}$	$t = 1$	$t = 2$	$t = 3$
Coal 3	26 000	26 000	26 000
Coal 6	49 600	49 600	49 600
Coal 7	14 800	14 800	14 800
Coal 8	10 000	10 000	10 000
Coal 9	20 000	20 000	20 000
Coal 13	16 000	16 000	16 000
Coal 15	42 400	42 400	42 400

Table 7: Expected quantities by rail

2.4 Coal prices

Some of the coals are paid in US dollar (USD), others in Euro (EUR). Typical expected exchange rate for the next three months are given by Table 8.

Period	$t = 1$	$t = 2$	$t =3$
USD	0.975	0.925	0.875
EUR	1	1	1

Table 8: Currency rate

The USD is the reference currency for the coals coming by boat. The EUR is used for the other coals. Prices are given at the harbour or station of departure (depending on the transportation mode) in table 9 where the relevant currency is also given for each coal. Multiplying the price in foreign currency by the expected exchange rate, we obtain the price of coal c at time period t in EUR, noted $price_{ct}$.

coal	currency	$t = 1$	$t = 2$	$t = 3$
1	USD	46.80	46.80	46.80
2	USD	46.75	46.75	46.75
4	USD	37.75	37.75	37.75
3	EUR	55.65	55.65	55,65
5	USD	45.75	45.75	45.75
6	EUR	42.225	42.225	42.225
7	EUR	42.225	42.225	42.225
8	EUR	41.775	41.775	41.775
9	EUR	43.575	43.575	43.575
10	USD	46.65	46.65	46.65
11	USD	49.25	49.25	49.25
12	USD	44.10	44.10	44.10
13	EUR	55.125	55.125	55.125
14	USD	40.00	40.00	40.00
15	EUR	44.95	44.95	44.95
16	USD	46.80	46.80	46.80

Table 9: Coal prices at the harbour or at the station of departure

2.5 The transportation costs

The transportation costs by boat to the harbour (noted $boatcost_{ch}$) are given by Table 10 in USD per ton of coal. There is no difference for the transportation by boat within the two arrival harbours.

Typical handling costs at the harbour (noted $dockcost_h$) are given by Table 11 in EUR per ton. Note that they are the same at the two harbours due to the competition between the harbours.

$boatcost_{ch}$	harbour 1	harbour 2
Coal 1	5.10	5.10
Coal 2	4.50	4.50
Coal 4	10.25	10.25
Coal 5	8.15	8.15
Coal 10	3.75	3.75
Coal 11	3.30	3.30
Coal 12	0.	0.
Coal 14	7.50	7.50
Coal 16	5.10	5.10

Table 10: Boat transportation costs (USD per ton of coal)

harbour	Dock cost
1	3.3875
2	3.3875

Table 11: Dock cost at the arrival harbour (EUR per ton of coal)

Typical transportation costs from harbours to plants (noted $transpcost_{hk}$) are given in EUR per ton by Table 12. Note that there is no transportation cost between harbour 2 and plant 3, the plant being located in the harbour area. Note also that plant 3 is only supplied from harbour 2. There is no gain to deliver the coal at harbour 1 and then to pay an additional rail transportation cost.

$transpcost_{hk}$	plant 1	plant 2	plant 3	plant 4	plant 5
Harbour 1	4.4675	2.6375	$+\infty$	3.86	2.460
Harbour 2	4.4675	3.25	0	3.86	4.105

Table 12: Transportation cost between the harbour and the plants (EUR per ton)

The four other plants are also supplied by rail. The transportation costs by rail between the departure station and the plants (noted $railcost_{ck}$) are given in EUR per ton by Table 13 for all coals that are coming directly from the production site to the plant by rail.

$railcost_{ck}$	plant 1	plant 2	plant 4	plant 5
Coal 3	9.155	7.39	8.29	7.7475
Coal 6	4.67	4.2625	6.08	2.31
Coal 7	4.67	4.2625	6.08	2.31
Coal 8	4.67	4.2625	6.08	2.31
Coal 9	9.155	7.39	8.29	7.7475
Coal 13	9.155	7.39	8.29	7.7475
Coal 15	4.67	4.2625	6.08	2.31

Table 13: Transportation cost by rail (EUR per ton)

2.6 Characteristics of the demand

The firm has 13 clients. We used a as index for the client. Their demand (noted dem_{at}) for the next three months is given in tons of coke in Table 14 which also gives the plant requested by the clients. Some of them can be delivered by more than one plant.

dem_{at}	$t = 1$	$t = 2$	t 3	Plant required
Client 1	11500	12600	12600	plant 3
Client 2		4000	4000	plant 1
Client 3	45000	45000	45000	plant 1
Client 4	7000	8000	8000	plant 2 or plant 3
Client 5	2000	2000	2000	plant 3
Client 6	16500	24800	14400	plant 1 or plant 2
Client 7	26700	19400	14800	plant 2
Client 8		3500		plant 2
Client 9	6000	6000	5000	plant 1
Client 10	16000	16000	14000	plant 2
Client 11	12000	12000	12000	plant 3
Client 12	58452	49002	51644	plant 4
Client 13	68516	68132	71162	plant 5

Table 14: Demand for the next three months

2.7 Mix specifications

We enumerate here the specifications the coal mix must meet.

The first one concerns the **volatiles rate**. The volatile rate of the mix must be between 24 % and 26 %. We note $minvol$ and $maxvol$ these two quantities.

The second one concerns the **ash rate**. The mix has a maximal ash rate depending on the client. We note $maxash_a$ this quantity for client a. For all minimal and/or maximal characteristic rate that depend on the client, we shall consider as specification for the mix of the plant the specification of the most restrictive client that the plant has to serve.

The third specification concerns the **sulfur rate**. There is a lower and upper limit which also depends on the client. We note $minsulf_a$ and $maxsulf_a$

these quantities for client a.

The fourth specification concerns the **alkali rate**. There is an upper limit which also depends on the client. We note $maxalk_a$ this quantity for client a.

The fifth specification concerns the **Low Volume rate**. Here, there is a lower and an upper limit which also depends on the client. We note these quantities $minlv_a$ and $maxlv_a$ for client a.

Typical minimal or maximal rate for ash, sulfur, alkali and Low Volume coal are given in Table 15.

Client	$maxash_a$	$minsulf_a$	$maxsulf_a$	$maxalk_a$	$minlv_a$	$maxlv_a$
1	10.		1.	.30	30.	100.
2	10.		1.	.30	30.	100.
3	10.		1.	.30	30.	100.
4	10.		1.	.30	30.	100.
5	10.		1.	.30	30.	100.
6	9.5	.7	.9	.30	30.	100.
7	9.5	.7	.9	.30	30.	100.
8	10.		1.	.30	40.	50.
9	10.		1.	.30	30.	100.
10	10.		1.	.30	30.	100.
11	10.		1.	.30	30.	100.
12	9.5	.7	.9	.30	30.	100.
13	9.5	.7	.9	.30	30.	100.

Table 15: Specifications which depend on the client.

Furthermore, for three characteristics (ash, sulfur, alkali), there is a multiplicative coefficient from the mix to the coke: they are noted respectively $multash$, $multsulf$ and $multalk$. These coefficients are given by Table 16.

The sixth specification concerns the **rate in "Mid Volume"**. Here there are lower and upper bounds which do not depend on the client. They are noted $minmv$ and $maxmv$ respectively.

	Multiplicative coefficient
Ash	1.32
Sulfur	.92
Alkali	1.32

Table 16: Multiplicative coefficient from the mix to the coke.

The seventh specification concerns the **soft rate**. Here, there is an upper bound, noted $maxsoft$, which does not depend on the client. The only soft coal is coal 12.

The eighth specification concerns the **australian coal rate.** Here, there is an upper bound, noted $maxaus$, which does not depend on the client. The only australian coal is coal 4.

Table 17 gives the lower and upper bound on the mix for the last three characteristics.

Parameter	Minimal rate	maximal rate
Soft		.10
Mid Volume	.25	.25
Australian		.30

Table 17: Minimal and maximal rate for Soft and Mid Volume

Finally, one must account for the fact that the number of mixes that can be used for each period and each plant is limited. This maximum number of different mixes, noted $maxmix_{kt}$, is given for each plant in Table 18.

3 Problem formulation

3.1 choice of indices

The six main indices that we have already introduced are the following. We indicate the several coals by c, the several plants by k, the time periods by t, the several mixes by m, the harbours by h and finally the clients by a.

$maxmix_{kt}$	$t = 1$	$t = 2$	$t = 3$
Plant 1	2	2	2
Plant 2	2	2	2
Plant 3	2	2	2
Plant 4	2	2	2
Plant 5	2	2	2

Table 18: Maximal number of mixes per period.

We also use subindices such as b for the coals that are coming by boat and r for the coals that are coming by rail.

3.2 choice of decision variables

The principal decision variables of our problem are the following (See Table 19). We note by $COAL_{cktm}$ the quantity of coal c used in plant k at the

$COAL_{cktm}$	quantity of coal c used in plant k at time period t in mix m,
SUM_{ktm}	total coal quantity for plant k at time period t in the mix m,
$COALPRES_{cktm}$	coal c presence in mix m for plant k at time t,
$MIXPRES_{ktm}$	mix m is used for plant k at time period t,
$XBOAT_{bth}$	quantity of coal b sent to harbour h at time t,
$XRAIL_{rkt}$	quantity of coal r that sent to plant k at time t,
$ORDQ_{ct}$	quantity of coal coming c ordered at time period t,
$STOCK_{bth}$	inventory of coal delivered by boat b at the end of time period t in harbour h,
$COALDEL_{bkth}$	deliveries of coal coming by boat b from harbour h to plant k at time period t,
$PROD_{kta}$	coke produced in plant k at time period t for client a.

Table 19: Decision variables

time period t in the mix m. We also note by SUM_{ktm} the total quantity of coals for plant k in the mix m at the time period t. These two quantities are given in tons. We must also use binary variables, noted $COALPRES_{cktm}$ to indicate the fact that coal c is present in the mix m for plant k at time period t and the binary variables $MIXPRES_{ktm}$ to indicate the fact that mix m is used for plant k at time period t.

Other variables are used to manage the deliveries and inventories of coals (See Figure 1). We note by $XBOAT_{bth}$ the quantity of coal coming by boat

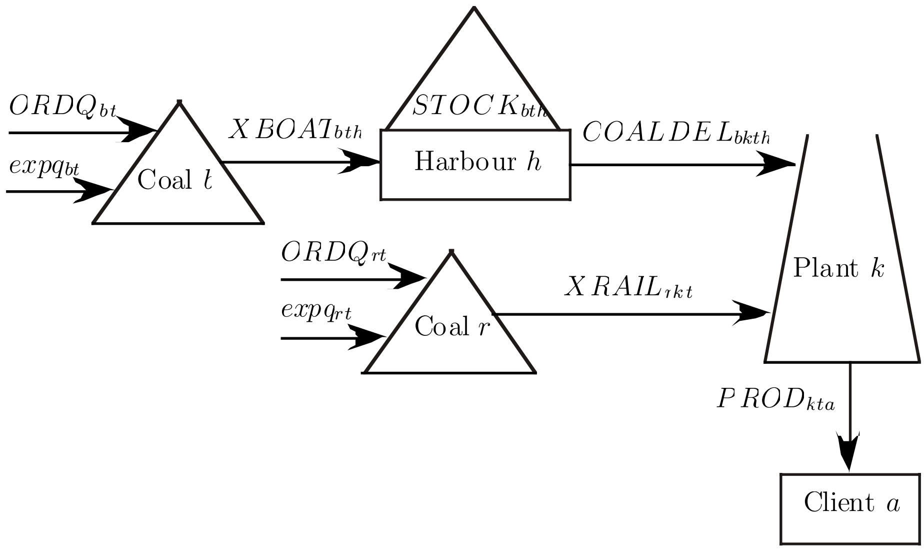

Figure 1: Choice for the decision variables

b that is sent to harbour h at time period t and by $XRAIL_{rkt}$ the quantity of coal coming by rail r that is sent to coke plant k at time period t. We note by $ORDQ_{bt}$ the quantity of coal coming by boat b that is ordered at time period t. This quantity comes in addition to the expected quantities. We note by $STOCK_{bth}$ the inventory of coal delivered by boat b at the end of time period t in harbour h. We also note by $ORDQ_{rt}$ the quantity of coal delivered by rail r that is ordered at time period t. This quantity comes in addition to the expected quantities at time period t, $expq_{rt}$. We note by $COALDEL_{bkth}$ the coal b deliveries from harbour h to plant k at time period t. We note by $PROD_{kta}$ the tons of coke produced in plant k at time period t for client a. These variables are necessary since some clients can be supplied by several plants.

3.3 Mathematical expression of the objective

The objective function is the sum of six following terms:

- the purchasing cost of the several coals:

$$\sum_c \sum_t price_{ct}(expq_{ct} + ORDQ_{ct})$$

  where $price_{ct}$ is the purchasing price of coal c at time period t,

- the boat transportation cost and the handling cost at the harbours:

$$\sum_b \sum_t \sum_h (boatcost_{bh} + dockcost_h) XBOAT_{bth}$$

  where $boatcost_b$ is the unit transportation cost of coal coming by boat b to harbour h and $dockcost_h$ is the unit handling cost at harbour h,

- the holding inventory cost:

$$\sum_b \sum_t \sum_h rate \cdot delprice_{b,t,h} STOCK_{bth}$$

  where $rate$ is the monthly opportunity rate (we here take a monthly $rate$ of 0,5 %) and $delprice_{b,t,h}$ is the delivery price at period t for coal coming by boat b at the harbour h (sum of the purchasing cost, the boat transportation cost and the handling cost at the harbour):

$$delprice_{b,t,h} = price_{bt} + boatcostbh + dockcost_h$$

- transportation cost from harbour to plants:

$$\sum_b \sum_k \sum_t \sum_h transpcost_{hk} COALDEL_{bkth}$$

  where $transpcost_{hk}$ is the unit transportation cost from harbour h to plant k

- transportation cost by rail:

$$\sum_r \sum_k \sum_t railcost_{rk} XRAIL_{rkt}$$

  where $railcost_{rk}$ is the unit transportation cost by rail for coal r to plant k,

- the production costs:

$$\sum_k \sum_t \sum_m prodcost_{kt} SUM_{ktm}$$

where $prodcost_{kt}$ if the unit production cost for plant k at time period t.

3.4 Mathematical expression of the constraints

- Non negativity of the variables:

$$\begin{aligned} COAL_{cktm}, SUM_{ktm}, COALDEL_{bkth}, PROD_{kta}, & \\ XBOAT_{bth}, XRAIL_{rkt}, ORDQ_{bt}, STOCK_{bth} & \geq 0 \end{aligned}$$

- Binary variables:

$$COALPRES_{cktm}, MIXPRES_{ktm} \in \{0, 1\}$$

- Sum of coals in the several coals:

$$SUM_{ktm} = \sum_c COAL_{cktm}$$

- Definition of the binary variables:

$$\begin{aligned} COAL_{cktm} &\leq COALPRES_{cktm} cap_k days_t \\ SUM_{ktm} &\leq MIXPRES_{ktm} cap_k days_t \end{aligned}$$

where cap_k is the daily coal entry capacity for plant k and $days_t$ is the number of days in time period t. This number of days is needed since capacity is a daily capacity and the variables are monthly quantities of entering coal.

- minimal utilization rate and capacities of plants:

$$minuse_k cap_k days_t \leq \sum_m SUM_{ktm} \leq cap_k days_t$$

where $minuse_k$ is the minimal utilization of plant k.

- The coal delivered by boat must be allocated between the two harbours:

$$expq_{bt} + ORDQ_{bt} = \sum_h XBOAT_{bth}$$

where $expq_{bt}$ is the expected quantity of coal delivered by boat b at time period t.

- The coal delivered by rail must be allocated between the plants:

$$expq_{rt} + ORDQ_{rt} = \sum_k XRAIL_{rkt}$$

where $expq_{rt}$ is the expected quantity of coal delivered by rail r at time period t.

- Balance for each coal coming by boat at each plant:

$$\sum_h COALDEL_{bkth} = \sum_m COAL_{bktm} \quad (1)$$

- Balance for each coal delivered by rail:

$$XRAIL_{rkt} = \sum_m COAL_{rktm} \quad (2)$$

- Maximal number of mixes for each plant and each time period:

$$\sum_m MIXPRES_{ktm} \leq maxmix_{kt}$$

where $maxmix_{kt}$ is the maximal number of mixes for plant k at time period t.

- Conservation of coal delivered by boat at each inventory:

$$\begin{aligned} STOCK_{b,t,h} &= STOCK_{b,t-1,h} + XBOAT_{bth} \\ &\quad - \sum_k COALDEL_{bkth} \end{aligned}$$

- Balance at the exit of the plants:

$$\sum_m \sum_c (1 - wet_c) * COAL_{cktm} = \sum_{a \text{ delivered by } k} PROD_{kta}$$

- Demand satisfaction:

$$\sum_{k \text{ delivering}_a} PROD_{kta} \geq dem_{at}$$

where dem_{ta} is the coke demand of client a at time period t.

- Maximal number of coals in the mix:

$$\sum_c COALPRES_{cktm} \leq gates_k$$

where $gates_k$ is the number of entering gates for the coals in plant$k$.

- Minimal use of each coal in a mix:

$$mincoal_k COALPRES_{cktm} \leq \frac{COAL_{cktm}}{SUM_{ktm}} \tag{3}$$

where $mincoal_k$ is the minimal use of a coal in the mix for plant k. Note that this parameter needs to be multiplied by the $COALPRES$ variable, otherwise, all the coals would be present in the mix.

- Maximal use of each coal in a mix:

$$\frac{COAL_{cktm}}{SUM_{ktm}} \leq maxcoal_k COALPRES_{cktm} \tag{4}$$

where $maxcoal_k$ is the maximal rate of a coal in the mix for plant k. Note that the two last constraints (3) and (4) are non linear.

All the other quality constraints on the mix are easy to write. They are listed in appendix A.

4 Solution Method

Except for constraints (3) and (4), all the constraints of the problem are purely linear. As already mentioned in Section 3, the left hand side of constraint (3) must be multiplied by the variable COALPRES, otherwise all the coals would be present in the mix.

The solution technique followed by Sarker and Gunn [4] is to define for each ratio $COAL_{cktm}/SUM_{ktm}$ a variable, say f_{cktm} introducing for each c, k, t and m the constraint:

$$f_{cktm}SUM_{ktm} = COAL_{cktm}.$$

They obtain a problem with a nonlinear part of bilinear type (product of variables). Heuristics for solving such problems have been proposed in the OR literature. They are related to techniques for solving fractional linear programs and resort to successive linear programming (SLP).

But in our problem, integer variables remain since constraints (3) and (4) can be written:

$$mincoal_k COALPRES_{cktm} \leq f_{cktm} \leq maxcoal_k COALPRES_{cktm}.$$

We can obtain linear constraints by using the following column generation technique. For each plant, and for each time period, fixing the delivery price at the entry of the plant, we seek a mix of **one ton** that satisfies all the quality constraints on the mix. This gives a column of coal use rates for each plant for each time period.

At an upper level, we shall determine the level of utilization of these mixes in order to meet the demand of the several clients at minimal production, transportation and coal purchasing costs. Multiplying the level of use by the column of the coal rates of the mix for a particular plant, we obtain the coal quantities needed for the plant at the time period. The upper level also determines how to deliver these quantities to the plant in order to minimize the coal purchase and transportation costs.

To update the delivery price at the entry of the plant, we consider the dual price of the balance at the entry of the plant constraints (1) or (2) depending on the transportation mode. This dual variable gives the marginal effect on the objective function of increasing by one unit the delivery of this coal at this period for the plant.

In fact, initially, we suppose, at the lower level, that all the coals where bought from the producer to the plant to compute the initial coal delivery price at the entry of the plant. But there are initial inventories at the harbours and there are also planned quantities of several coals to be delivered. Thus, for example, the utilization of a coal already arrived at the harbours is less expensive than ordering a new quantity from the producer.

At the end of the first main iteration (one main iteration is the solution of the 15 subproblems and of one main problem), we update the initial delivery price at the entry of the plant by substituting the dual price of constraints (1) or (2).

4.1 The sub programs

The subproblems correspond to the following task: find, for each time period and each plant, the optimal mix for one unit of coal mix. The constraints encompass all the mix quality constraints, the maximal number of coal constraint and the minimal and maximal rate for each coal in the mix constraints. The only variables are variables $COAL_c$ and $PRESCOAL_c$. The objective function encompasses at the first iteration the coal purchasing cost, transportation cost, handling cost and the production cost. At the following iterations, we only consider the dual variables and the production cost.

We need thus to solve as many problems as there are plants and time periods. This can be done by using two inner loops in GAMS/OSL (See Brooke et al[1]): the first on time periods and the second on plants. These problems are easy to solve: they have only 16 linear variables and 16 binary variables.

4.2 The main program

The main program corresponds to the determination of the delivery planning of coals to the plants, the allocation of planned quantities to the harbours, the determination of the level of use of the mixes generated by the sub-problems for each plant and each time period and finally the demand satisfaction planning.

This gives a linear program with integer variables $MIXPRES$ since a maximal number of different mixes for each plant at each time period must be imposed. This program can also be solved by GAMS/OSL. At the first iteration, there are 481 rows and 556 columns. Each iteration adds 15 rows and 30 columns to the main problem (See table 24).

4.3 The global process

The global process includes 3 levels of inner loops in GAMS/OSL: the first loop on the main iterations, the second loop on the time periods and the third loop on the plants. It can be summarized by the following procedure:

```
Initialize P(c,k,t) to the delivery price of coal c at plant k
   for period t assuming that the coal must be bought ;
For m = 1, 2, etc..
   {
    For t = January, February, March
       {
        For k= 1 to 5
           {Determine  the optimal mix for plant k, time t
               when using P(c,k,t) in the objective;
            Let COAL(c,k,t,m) the solution obtained
               by the branch and bound procedure;
           }
        }
    Determine the optimal coal purchases, mix use level
       and demand satisfaction planning using branch & bound;
    Update P(c,k,t) to the dual variables of the balance
       equation for coal c at plant k for time period t;
   }
```

The stopping criterion is the following: stop when there is no new column generated for the subproblems. The numerical results are presented in the following section.

Before analyzing these results, let us stress the fact that the procedure we propose is heuristic. In fact, with an alternative organization of the master problem and subproblem hierarchy, we obtain a better objective function value (See Section 6).

Let us conclude this section with a few words on the dual prices used in the column generation. In fact, our master problem is a Mixed Integer Problem due to the MIXPRESS binary variables. We have solved this MIP problem using GAMS/OSL and we have used the dual information given at the optimum. The duals variables are taken by GAMS from the optimal (integer) solution. After the solver has found the optimal solution, GAMS fixes the levels of the integer variables and reruns the model as an LP to obtain the duals.

One could also use the dual prices of the LP relaxation of the whole MIP problem (with the last bunch of columns added). We refer to Section 6 for the comparison of the results of the two possible choices for the dual information. The reserve on the use of the dual information of a MIP problem can explain why there is an important improvement of the objective function during the two first iterations: the binary variables do not play any role in these first two iterations (one can allow 2 mixes per plant and per period).

5 Numerical results

At the first iteration, we obtain the mixes given in table 20. It can be seen that plants 4 and 5 use the same mix and this for the first two time periods. This can be explained by the fact that they have a single client which has similar mix quality constraints and by the fact that the two plants have the same characteristics for the number of coals and the minimal and maximal rate of each coal in the mix. The only difference is the fact that in period 3, plant 4 used 0.10 of coal 12 and 0.10 of coal 14 reducing the use of coals 4 and 6. This can also be explained by the fact that the delivery price at the entry of the plant for coal 12 is lower than for coals 4 and 6 at this time period. Since for plant 5, coals 4 and 6 remain cheaper than coal 12, coal 12 is not preferred to coal 4 and 6. The fact that the use of coal 12 is limited to 10 % can easily be explained: coal 12 is the only soft coal and the maximal rate of soft coal is 10 %.

For plant 2, the mix is the same at period 1 and 3, the mix at period 2 is different due to the fact that client 8 appears for the first and last time in period 2 and this client is more restrictive for the Low Volume rate of the mix. For the main program, the optimal cost function has a value of

$$z_1^* = 91\ 569\ 578.20 \text{ EUR}$$

We do not give the full solution to the main problem but we explain why the current solution is not optimal. For example, the inventory of coal 16 increases at harbour 1 during the first two periods: in fact, no mix uses this coal and deliveries for this coal are expected during the first two time periods. When considering the dual price of constraint (1), this coal with a high level

Period 1					
Coal	Plant 1	Plant 2	Plant 3	Plant 4	Plant 5
4	0.300	0.300	0.251	0.300	0.300
5			0.250		
6	0.450	0.300		0.450	0.450
7		0.150			
10			0.284		
12			0.100		
14			0.115		
15	0.250	0.250		0.250	0.250
Period 2					
Coal	Plant 1	Plant 2	Plant 3	Plant 4	Plant 5
1			0.100		
4	0.300	0.250	0.300	0.300	0.300
5			0.250		
6	0.450	0.350		0.450	0.450
10			0.250		
12			0.100		
14		0.150			
15	0.250	0.250		0.250	0.250
Period 3					
Coal	Plant 1	Plant 2	Plant 3	Plant 4	Plant 5
1			0.100		
4	0.300	0.300	0.300	0.200	0.300
5			0.250		
6	0.450	0.300		0.350	0.450
7		0.150			
10			0.250		
12			0.100	0.100	
14				0.100	
15	0.250	0.250		0.250	0.250

Table 20: First iteration: solutions to the subproblems

of stock will receive a negative price and will be used during the following iteration.

At the **second iteration**, the new generated mixes are presented in table 21. Note that coal 16 is now used by plant 3 since it has benefitted from

Period 1					
Coal	Plant 1	Plant 2	Plant 3	Plant 4	Plant 5
1	0.406	0.300	0.450	0.406	0.406
5	0.250	0.250	0.250	0.250	0.250
7		0.158			
11	0.114	0.292		0.114	0.114
16	0.230		0.300	0.230	0.230
Period 2					
Coal	Plant 1	Plant 2	Plant 3	Plant 4	Plant 5
1	0.333	0.288	0.300	0.333	0.333
2		0.150			
5	0.250	0.250	0.250	0.250	0.250
10	0.125	0.312		0.125	0.125
11	0.292		0.350	0.292	0.292
16			0.100		
Period 3					
Coal	Plant 1	Plant 2	Plant 3	Plant 4	Plant 5
1	0.333	0.305	0.300	0.333	0.333
5	0.250	0.250	0.250	0.250	0.250
10	0.125	0.150		0.125	0.125
11	0.292	0.295	0.350	0.292	0.292
16			0.100		

Table 21: Second iteration: solutions to the subproblems

price reduction. Also note that coal 14 and coal 12 are replaced by other coals since their dual price has increased. For the main program, the optimal cost function has a value of

$$z_2^* = 79\ 432\ 349.83 \text{ EUR}$$

There is thus an important global cost reduction between the first two iterations.

At the **third iteration**, the new generated mixes are presented in table 22.

Period 1					
Coal	Plant 1	Plant 2	Plant 3	Plant 4	Plant 5
1	0.360	0.269		0.403	0.360
5	0.250	0.250	0.250	0.250	0.250
7	0.390				0.390
8		0.331		0.347	
10			0.170		
12			0.100		
14		0.150	0.480		
Period 2					
Coal	Plant 1	Plant 2	Plant 3	Plant 4	Plant 5
1	0.450	0.326	0.450	0.450	0.450
5	0.250		0.250	0.250	0.250
14		0.150			
15		0.250			
16	0.300	0.274	0.300	0.300	0.300
Period 3					
Coal	Plant 1	Plant 2	Plant 3	Plant 4	Plant 5
1	0.450	0.350	0.450	0.450	0.450
5	0.250	0.250	0.250	0.250	0.250
7		0.150			
16	0.300	0.250	0.300	0.300	0.300

Table 22: Third iteration: solutions to the subproblems

Plant 1 has the same mix for the first two time periods. For plant 3, the mix is the same for the last two time periods. Plant 4 and 5 have the same mix for the last two time periods. For the main program, the optimal cost function has a value of

$$z_3^* = 75\ 029\ 945,98\ \text{EUR}$$

For the main program, since there are three possible mixes for each plant at each period, we must impose the $MIXPRES$ variables and the constraint limiting the number of different mixes to 2. The use of the mixes in the solution of the main problem at the end of the third iteration is given in Table 23. It can be seen that most of the time, plant used two of the three available mixes simultaneously.

Period 1			
	Mix 1	Mix 2	Mix 3
Plant 1	used		
Plant 2	used		used
Plant 3	used		used
Plant 4		used	used
Plant 5	used		used
Period 2			
	Mix 1	Mix 2	Mix 3
Plant 1		used	used
Plant 2	used		
Plant 3	used		used
Plant 4	used		used
Plant 5	used		used
Period 3			
	Mix 1	Mix 2	Mix 3
Plant 1	used		used
Plant 2	used		used
Plant 3	used		used
Plant 4	used		used
Plant 5	used	used	

Table 23: Third iteration: mixes used in the solution

Table 24 presents the successive objective function for the main program, the execution time for GAMS/OSL and the size of the main program. Two

Iteration	Objective value (EUR)	Execution time (sec)	Number of rows	Number of columns
1	91 569 578.20	3	481	556
2	79 432 349.83	4	496	586
3	75 029 945.98	11	511	616
4	71 579 163.48	48	526	646
5	69 789 373.33	49	541	676
6	68 615 626.73	522	556	706
7	68 514 039.00	52	571	736
8	68 513 503.25	409	586	766
9	68 428 567.03	119	601	796
10	68 393 561.65	4151	616	826
11	68 388 314.23	664	631	856
12	68 361 242.08	4403	646	886
13	68 358 137.05	494	661	916
14	68 349 665.80	2412	676	946
15	68 347 547.93	1865	691	976
16	68 346 480.40	3184	706	1006
17	68 346 480.40	386	721	1036
18	68 346 480.40	238	736	1066
19	68 346 480.40	10802	751	1096
20	68 346 480.40	10803	766	1126

Table 24: Evolution of the objective function of the MP

remarks can be made. The first one is the important objective function decrease (about of 2 500 000 EUR per iteration) during the first five iterations. This global cost reduction continues at a lower rate during the following iterations. Secondly, remark that the process converges in only 16 iterations.

6 Other solution methods

To conclude this paper let us say a few words about two other solution methods that were suggested to us to solve the problem.

The first one is a variant of our column generation method that was suggested to us by professor Yves Pochet of CORE, Université Catholique de Louvain, Belgium . Since the only difficulty in the main program is the fact that there are binary variables in a product with another variable, one can, for the main program fix the coals present in the mix instead of fixing their proportion in the mix. This implies, in our method that we send from the lower to the upper level not the $COAL$ variables but only the $COALPRES$ variables. This also implies that we must impose, in the main program, all the quality constraints on the mix listed in appendix A. We have also tried this solution method. Table 25 presents the evolution of the objective function value, the execution time for GAMS/OSL and the size of the main program.

A slightly better solution (68 341 879.48 EUR instead of 68 346 480.40 EUR) was obtained after 14 main iterations but it took much longer (14 hours 28 min instead of 5 hours 6 minutes for our method). This increase in the execution time is due to the fact that the size of the main problem is increased (compare the last two columns of table 24 and 25). From this example, one conclude that the second method is not competitive since it gives an analogous solution in much more time.

Another solution method was suggested to us by Xavier Delorme, LAMIH ROI, Université de Valenciennes, France. Instead of using a column generation method solve directly the original formulation with constraint (3) replace by its big m reformulation:

$$-mincoal_k(1 - COALPRES_{cktm})M + mincoal_k SUM_{ktm} \leq COAL_{cktm}$$

where M is an upper bound on the variable SUM_{ktm}. We have used the capacity of the plant as upper bound :

$$M = cap_k days_t$$

But, unfortunately, the direct resolution of this formulation does not give any integer solution by GAMS/OSL in a reasonable execution time. This

Iteration	Objective value (EUR)	Execution time (sec)	Number of rows	Number of columns
1	89 378 751.35	3	1124	796
2	76 709 252.58	23	1782	1066
3	75 052 581.23	18	2440	1336
4	70 284 765.43	215	3098	1606
5	68 484 730.45	49	541	676
6	68 458 216.33	1497	4414	2146
7	68 408 431.40	543	5072	2416
8	68 374 826.85	1010	5730	2686
9	68 362 017.08	599	6388	2956
10	68 353 850.75	8165	7046	3226
11	68 348 399.20	10803	8362	3766
12	68 345 582.43	10803	646	886
13	68 342 647.50	10803	9020	4036
14	68 341 879.48	7575	9678	4306
15	68 341 879.48	4625	10336	4576
16	68 341 879.48	9339	10994	4846

Table 25: Evolution of the objective function of the MP for the second method

can be explained by the fact that the big M is not a very tight upper bound on the SUM_{ktm} variable.

A third heuristic can by used. The only difference with the first method is the choice of the dual information to update the coal delivery prices for the subproblem. Instead of using the dual information given at the optimal solution of the MIP version of the master problem, we solved the relaxed version of this master problem and used the dual information of this RMIP problem. Table 26 presents the evolution of the objective function value, the execution time for GAMS/OSL for the RMIP and for the MIP version of the master program.

Iteration	Objective value (EUR)	Execution time RMIP (sec)	Execution time MIP (sec)
1	91 569 578.20	2	2
2	79 432 349.83	2	4
3	75 029 945.98	2	19
4	72 105 817.88	2	24
5	68 835 693.58	3	1071
6	68 503 629.70	2	468
7	68 433 685.88	3	1 243
8	68 400 568.13	2	75
9	68 379 477.03	2	624
10	68 358 511.35	2	270
11	68 349 689.18	2	390
12	68 349 689.18	2	385
13	68 348 287.88	2	2458
14	68 346 804.38	2	555
15	68 346 804.38	2	2876

Table 26: Evolution of the objective function of the MP for the third method

A slightly expansive solution (68 346 804.38 EUR instead of 68 346 480,40 EUR) was obtained after 14 main iterations but in less time (2 hours 10 minutes instead of 5 hours 6 minutes with our method).

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A Quality constraints on the mix

- maximal ash rate of the mix:

$$\sum_c ash_c multash COAL_{cktm} \leq \min_{a \text{ delivered by } k} maxash_a SUM_{ktm}$$

where ash_c is the ash rate of coal c, $multash$ is the multiplicative factor for ash and $maxash_a$ is the maximal ash rate for client a. Note that we impose to the mix the condition of the most critical client.

- minimal sulfur rate of the mix:

$$\sum_c sulf_c multsulf COAL_{cktm} \geq \max_{a \text{ delivered by } k} minsulf_a SUM_{ktm}$$

where $sulf_c$ is the sulfur rate of coal c, $multsulf$ is the multiplicative factor for sulfur and $minsulf_a$ is the minimal sulfur rate for client a.

- maximal sulfur rate of the mix:

$$\sum_c sulf_c multsulf COAL_{cktm} \leq \min_{a \text{ delivered by } k} maxsulf_a SUM_{ktm}$$

where $maxsulf_a$ is the maximal sulfur rate for client a.

- maximal alkali rate of the mix:

$$\sum_c alk_c multalk COAL_{cktm} \leq \min_{a \text{ delivered by } k} maxalk_a SUM_{ktm}$$

where alk_c is the alkali rate of coal c, $multalk$ is the multiplicative factor for alkali, and $maxalk_a$ is the maximal alkali rate for client a.

- minimal and maximal volatiles rate of the mix:

$$minvol SUM_{ktm} \leq \sum_c vol_c COAL_{cktm} \leq maxvol SUM_{ktm}$$

where $minvol$ and $maxvol$ are respectively the minimal and maximal volatile rate and vol_c is the volatile rate for coal c. Note that the maximal and minimal volatile rates do not depend on the clients nor on the plant.

- minimal Low Volume rate of the mix:

$$\sum_{c \text{ is a Low Volume}} COAL_{cktm} \geq \max_{a \text{ delivered by } k} minlv_a SUM_{ktm}$$

where $minlv_a$ is the minimal low volume rate for client a.

- maximal Low Volume rate of the mix:

$$\sum_{c \text{ is a Low Volume}} COAL_{cktm} \leq \min_{a \text{ delivered by } k} maxlv_a SUM_{ktm}$$

where $maxlv_a$ is the maximal Low Volume rate for client a.

- minimal and maximal Mid Volume rate of the mix:

$$minmvSUM_{ktm} \leq \sum_{c \text{ is a Mid Volume}} COAL_{cktm} \leq maxmvSUM_{ktm}$$

where $minmv$ and $maxmv$ are respectively the minimal and maximal Mid Volume rate. Note that the maximal and minimal Mid Volume rates do not depend on the client nor on the plant.

- maximal rate for Soft coal in the mix:

$$\sum_{c \text{ is a Soft Coal}} COAL_{cktm} \leq maxsoftSUM_{ktm}$$

where $maxsoft$ is the maximal rate for soft coal in the mix. Note that the maximal rate for soft coal does not depend on the client nor on the plant.

- maximal rate for Australian coal in the mix:

$$\sum_{c \text{ is an australian Coal}} COAL_{cktm} \leq maxausSUM_{ktm}$$

where $maxaus$ is the maximal rate for australian coal in the mix. Note that the maximal rate for australian coal is not depending on the client nor on the plant.