Optimal education subsidy and taxes in an endogenous growth model with human capital*

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Abstract

This paper considers a three-overlapping-generations model of endogeneous growth wherein human capital is the engine of growth. It first contrasts the *laissez-faire* and the optimality solutions. Then it discusses alternative sets of tax-transfer instruments that allow for decentralization of the social optimum.

Keywords: endogenous growth, education policy, intergenerational transfers.

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1 Introduction

In most industrialized societies, one observes that the bulk of public transfers benefits the young (through education) and the elderly (through pensions) and that these transfers are financed by a tax on labor income. This pattern is often justified by arguments of myopia (for pensions), liquidity constraints (for education) and redistribution (for both transfers). There are other reasons for such an intervention of the government particularly when we adopt the dynamic setting of an endogenous growth model wherein human capital is the engine of growth. The purpose of this paper is to abstract from considerations of myopia, liquidity constraint and redistribution and to study a three-overlapping-generations model of endogenous growth in order to verify if the above stylized facts results from our theoretical analysis.

In a model of endogenous growth with human capital there is a double need for public intervention. First, educational investment tends to be insufficient because its private value is lower than its social value by the positive externality it exerts on future generations. A subsidy is called for. Second, for a given social rate of time preference, it is likely that individual saving decisions don't generate the appropriate amount of capital accumulation. Thus some transfers between generations are desirable. In this paper we first study and compare the laissez-faire and social optimality solutions. Then we turn to the issue of decentralization. First, we show the rather striking result that social optimum decentralization is possible with a lump sum tax-transfer on retirees and two distortionary tools, a tax on earning and a subsidy on education. Second, instead of a subsidy on education and a tax on labor income, we can have a wage subsidy with a tax on educational spending. Third, there is no reason for the government to provide retired workers with public pension benefits. Taxing the retirees can make sense to finance educational subsidy and achieve the right amount of capital accumulation. Our analysis will be conducted in a first-best setting. This implies that the government is assumed to have at its disposal the appropriate instruments. In a companion paper we are studying the second-best problem which occurs when the right instruments are not available.

Quite surprisingly there is little work in this area. Our paper can be viewed as an extension of Michel (1993) who studies an overlapping generations growth model where agents live for three periods. In the first, they incur some education cost which is financed by borrowing on the financial market. In the second period, they work with their labor supply being fixed and their wage depending on their educational investment and also on the overall level of human capital. Their earnings are used to finance current consumption, refund the amount borrowed for their education and save for retirement. In the third period, they are retired. We generalize Michel (1993) and also de la Croix and Michel (2002) by using more general production and utility functions and also a less restrictive human capital function. We also develop the decentralization analysis more completely. Azariadis and Drazen (1990) develop a model in the same vein but with human capital produced out of forgone labor. Finally Docquier and Michel (1999) study a model wherein human capital is produced out of financial investment and of labor time. Because the setting of this last paper is quite complex it has to resort to numerical simulations.

We begin our analysis with the presentation of the model in Section 2. Then in the next two sections we characterize the *laissez-faire* equilibrium and the first-best optimum. We investigate in Section 5 under which conditions the first-best can be decentralized. We start with what we consider to be the standard instruments of decentralization: a subsidy on education and lump-sum transfers in both the second and the third period. Then we consider alternative set of instruments allowing for decentralization. The paper concludes with a short summary of the main findings.

2 The model

Individuals live for three periods: they invest in education in the first period, work in the second one and retire in the third one. The generation at work in period t, which is thus born in period $t-1$, is indexed by t. The size of generation t is given by $N_t = (1+n)N_{t-1}$, where n is the constant rate of population growth. In period t the population size is thus $N_{t-1} + N_t + N_{t+1}$.

An individual of generation t invests in education an amount e_{t-1} in period $t-1$. This results in a level of (per capita) human capital or effective labor supply that is given by $h_t = \Phi(e_{t-1}, h_{t-1})$ where h_{t-1} is the level of human capital inherited from parents. Function Φ is assumed to be homogeneous of degree 1 in its two arguments. Therefore, we can write it in intensive form as:

$$h_t = h_{t-1}\varphi(\overline{e}_{t-1}) \quad \text{with } \overline{e}_{t-1} = \frac{e_{t-1}}{h_{t-1}} \tag{1}$$

where $\varphi(\overline{e})$ is positive, increasing and strictly concave, and satisfies the Inada conditions: $\varphi'(0) = \infty$ and $\varphi'(\infty) = 0$.

With c_t and d_{t+1} denoting consumption in the first and second periods respectively, the preferences of generation-t individuals are represented by the following utility function:

$$u_t = u(c_t, d_{t+1}). \tag{2}$$

This function is strictly increasing, strictly concave and homogeneous of degree $b < 1$ in its two arguments (indifference curves are therefore homothetic in the (c, d)-space). It also verifies the following Inada conditions: $u_c(0, d) = \infty$ and $u_d(c, 0) = \infty$.

The production side is represented by an aggregate production function relating output Y_t in period t to the physical capital, K_t, and the aggregate human capital or effective labor supply, $H_t = N_t h_t$, that are available in period t. This function is given by $Y_t = F(K_t, H_t)$, which is assumed to be homogeneous of degree 1. Therefore, we can also write:

$$Y_t = H_t f(\overline{k}_t) \quad \text{with } \overline{k}_t \equiv \frac{K_t}{H_t} \tag{3}$$

where $f(\overline{k})$ is assumed to be positive, strictly increasing and strictly concave in $\overline{k}$, i.e. the ratio of physical capital to human capital.[1]

3 Laissez-faire equilibrium

We assume a perfect credit market. Individuals of generation t borrow e_{t-1} when they get educated in $t-1$ and reimburse $R_t e_{t-1}$ when they work in t, with $R_t = 1 + r_t$ being the interest factor. With w_t denoting the wage rate per unit of effective labor, their net income in period t is therefore:

$$W_t = w_t h_t - R_t e_{t-1} = h_{t-1}(w_t \varphi(\overline{e}_{t-1}) - R_t \overline{e}_{t-1}). \tag{4}$$

Individuals use this net income for current consumption c_t and saving s_t so that when retired they consume

$$d_{t+1} = R_{t+1} s_t. \tag{5}$$

They choose $\overline{e}_{t-1}$ and s_t so as to maximize their utility:

$$u(c_t, d_{t+1}) = u(h_{t-1}(w_t \varphi(\overline{e}_{t-1}) - R_t \overline{e}_{t-1}) - s_t, R_{t+1} s_t).$$

[1] In Michel (1993) the utility function is loglinear and both production and human capital functions are Cobb-Douglas.

The optimal choice of education by an individual of generation t is thus given by:

$$\varphi'(\overline{e}_{t-1}) = \frac{R_t}{w_t} \tag{6}$$

while his optimal choice of saving satisfies:

$$u_c(c_t, d_{t+1}) = R_{t+1} u_d(c_t, d_{t+1}). \tag{7}$$

The above two conditions together yield the demand function for education and first- and second- period consumptions: $e_{t-1}(R_t, w_t, h_{t-1})$, $c_t(R_t, R_{t+1}, w_t, h_{t-1})$ and $d_{t+1}(R_t, R_{t+1}, w_t, h_{t-1})$ in the laissez-faire economy. Then, using (6) the net income of an individual of generation t is:

$$W_t = h_{t-1} w_t [\varphi(\overline{e}_{t-1}) - \varphi'(\overline{e}_{t-1})\overline{e}_{t-1}], \tag{8}$$

in which the expression in brackets is positive since the strict concavity of $\varphi(\overline{e})$ implies that $\varphi(\overline{e}) - \overline{e}\varphi'(\overline{e}) > \varphi(0) \geq 0$. Using (1), relation (8) can also be written as:

$$W_t = h_t w_t [1 - \lambda(\varphi(\overline{e}_{t-1}))] \tag{9}$$

with λ denoting the elasticity defined as:

$$\lambda(\varphi(\overline{e})) = \frac{\overline{e}\varphi'(\overline{e})}{\varphi(\overline{e})} < 1.$$

In competitive markets the remunerations of the production factors are equal to their marginal products:

$$R_t = f'(\overline{k}_t) \equiv R(\overline{k}_t), \tag{10}$$

$$w_t = f(\overline{k}_t) - \overline{k}_t f'(\overline{k}_t) \equiv w(\overline{k}_t), \tag{11}$$

where $R(\overline{k}_t)$ is decreasing and $w(\overline{k}_t)$ is increasing in $\overline{k}_t$. Plugging (10) and (11) into (6) yields:

$$\overline{e}_{t-1} = \varphi'^{-1}\left(\frac{R(\overline{k}_t)}{w(\overline{k}_t)}\right) \equiv \overline{e}(\overline{k}_t) \tag{12}$$

with $\overline{e}(\overline{k}_t)$ increasing in $\overline{k}_t$. Using this result the net income W_t in (9) can be written as:

$$W_t = h_t \psi(\overline{k}_t) \tag{13}$$

where

$$\psi(\overline{k}_t) \equiv w(\overline{k}_t)[1 - \lambda(\varphi(\overline{e}(\overline{k}_t))] > 0.$$

Our assumption that the utility function is homogeneous in c_t and d_{t+1} (i.e. indifference curves are homothetic) makes saving proportional to net income. In other words the saving rate that we denote by σ depends only on the interest factor R_t:

$$s_t = \sigma(R_{t+1})W_t = \sigma(R_{t+1}(\overline{k}_{t+1}))h_t\psi(\overline{k}_t). \tag{14}$$

The clearing condition in the financial market, $N_t s_t = K_{t+1} + N_{t+1}e_t$, can be written as:

$$s_t = (1+n)\overline{k}_{t+1}h_{t+1} + (1+n)h_t\overline{e}_t.$$

Using (1), (12) and (14), this market-clearing condition is equivalent to:

$$\psi(\overline{k}_t) = \chi(\overline{k}_{t+1}) \tag{15}$$

with

$$\chi(\overline{k}_{t+1}) \equiv \frac{1+n}{\sigma(R(\overline{k}_{t+1}))}\left[\overline{k}_{t+1}\varphi(\overline{e}(\overline{k}_{t+1})) + \overline{e}(\overline{k}_{t+1})\right].$$

If ψ and χ are both increasing functions, the dynamic path of $\overline{k}_t$ as defined by (15) is monotonic, and $\overline{k}_t$ converges towards a steady-state value[2] we denote by $\overline{k}^{LF}$. One then obtains a balanced growth path along which the variables chosen by individuals (s_t, e_{t-1}, c_t and d_{t+1}) grow at the same rate as individual human capital: $1+g^{LF} = h_{t+1}/h_t = \varphi(\overline{e}^{LF})$ with $\overline{e}^{LF} = \overline{e}(\overline{k}^{LF})$.

4 Social optimum

To characterize the first-best social optimum we use as objective of the social planner the sum of lifetime utilities over generations discounted by a factor γ ($0 < \gamma < 1$) reflecting social time preferences. This maximization is subject to human capital equation (1) and to the following resource constraint

$$h_t f(k_t/h_t) = c_t + \frac{d_t}{1+n} + (1+n)e_t + (1+n)k_{t+1} \tag{16}$$

[2] The monotonic sequence $\overline{k}_t$ is bounded above because the limit of $\psi(\overline{k})/\chi(\overline{k})$ when $\overline{k}$ tends to ∞ is equal to zero.

where $k_t = K_t/N_t$ is the physical capital per worker. Notice that $\overline{k}_t = k_t/h_t$.

The Lagrangean problem can now be written as:

$$\max \sum_{t=0}^{\infty} \gamma^t \Big\{ u(c_t, d_{t+1}) + q_t[h_t f(k_t/h_t) - c_t - \frac{d_t}{1+n} \\ - (1+n)(e_t + k_{t+1})] + p_t[h_t \varphi(e_t/h_t) - h_{t+1}] \Big\} \quad (17)$$

where $\gamma^t q_t$ and $\gamma^t p_t$ are the multipliers associated with the resource constraint and the human capital equation respectively. Maximizing the above Lagrangean with respect to c_t, d_t, e_t, k_{t+1} and h_{t+1} yields:

$$u_c(c_t, d_{t+1}) = q_t \quad \text{and} \quad u_d(c_{t-1}, d_t) = \frac{\gamma q_t}{1+n},$$

$$p_t \varphi'(\overline{e}_t) = q_t(1+n),$$

$$q_t(1+n) = \gamma q_{t+1} f'(\overline{k}_{t+1}),$$

and

$$p_t = \gamma q_{t+1}[f(\overline{k}_{t+1}) - \overline{k}_{t+1} f'(\overline{k}_{t+1})] + \gamma p_{t+1}[\varphi(\overline{e}_{t+1}) - \overline{e}_{t+1}\varphi'(\overline{e}_{t+1})].$$

Together with the transversality condition:

$$\lim_{t \to \infty} \gamma^t (q_t \overline{k}_{t+1} + p_t) h_{t+1} = 0,$$

those conditions are sufficient since the problem is concave.

Eliminating the multipliers from the above conditions first yields:

$$\frac{u_c(c_t, d_{t+1})}{u_d(c_t, d_{t+1})} = f'(\overline{k}_{t+1}), \quad (18)$$

which is satisfied in the competitive laissez-faire equilibrium as it can be inferred from (7) and (10). Next we obtain:

$$\frac{u_d(c_{t-1}, d_t)}{u_d(c_t, d_{t+1})} = \frac{\gamma f'(\overline{k}_{t+1})}{1+n}. \quad (19)$$

This condition determines the optimal accumulation of physical capital.

One can also infer from the first-order conditions for a maximum of (17):

$$w(\overline{k}_{t+1})\ \varphi'(\overline{e}_t) = f'(\overline{k}_{t+1}) - (1+n)\varphi'(\overline{e}_t)\frac{\varphi(\overline{e}_{t+1}) - \overline{e}_{t+1}\varphi'(\overline{e}_{t+1})}{\varphi'(\overline{e}_{t+1})}, \tag{20}$$

which yields the optimal accumulation of human capital. It is straightforward to infer from (6), (10) and (11) that this condition is not satisfied in the competitive laissez-faire equilibrium. When choosing their investment on education, individuals ignore the intergenerational externality represented by the second term on the *rhs* of the above condition. Their investment does not only affect their own income but also that of their children through the inherited human capital. Accordingly the investment on education is socially too low at the laissez-faire equilibrium, which is a well known feature of this sort of model. Another way to express this point is to rewrite (20) as:

$$w(\overline{k}_{t+1})\varphi'(\overline{e}_t) = f'(\overline{k}_{t+1})\,(1 - \Delta_{t+1}) \tag{21}$$

where the educational externality is expressed as some fraction of the education cost, denoted by Δ_{t+1}. In the next section where policies for decentralizing the social optimum will be investigated, this fraction will define the government's implicit subsidy to education.

Finally, the transversality condition can be written as:

$$\lim_{t\to\infty} \gamma^t u_c(c_t, d_{t+1})h_{t+1}\left(\overline{k}_{t+1} + \frac{1+n}{\varphi'(\overline{e}_t)}\right) = 0. \tag{22}$$

Along a balanced growth path, the per-unit-of-effective-labor variables are constant, $\overline{e}_t = \overline{e}$ and $\overline{k}_t = \overline{k}$, while the per-capita variables c_t, d_t and h_t are growing at the constant rate g defined by $1 + g = \varphi(\overline{e})$ (as it can be inferred from (1)).

Since $u(c_t, d_{t+1})$ is assumed to be homogeneous of degree $b < 1$, $u_c(c_t, d_{t+1})$ and $u_d(c_t, d_{t+1})$ are homogeneous of degree $b-1$, which implies that $u_d(c_{t-1}, d_t)$ $/u_d(c_t, d_{t+1}) = [\varphi(\overline{e})]^{1-b}$. From (19) we then have:

$$\frac{\gamma f'(\overline{k})}{1+n} = [\varphi(\overline{e})]^{1-b} = (1+g)^{1-b} \tag{23}$$

along the balanced-growth path. This is the analogue of Diamond (1965)'s modified golden rule in our endogenous growth model. It is important to note that along the balanced-growth path the growth factor of the expression on the ℓhs of (22) is $\gamma[\varphi(\overline{e})]^b$, hence the transversality condition, given in (22), is equivalent to $\gamma[\varphi(\overline{e})]^b < 1$.

In Appendix A we prove the following proposition that states how the socially optimal values of $\overline{k}$ and $\overline{e}$ are related to the generational discount factor, γ. In this proposition, $\gamma_{\max}$ denotes the upper bound of γ for which the transversality condition holds.

Proposition 1 *There exists an upper bound $\gamma_{\max}$ such that the transversality condition is satisfied if and only if $\gamma < \gamma_{\max}$. The long-run socially optimal values of $\overline{k}$ and $\overline{e}$ are increasing with γ on the interval $(0, \gamma_{\max})$ and both tend to 0 when γ tends to 0.*

In the rest of this section we investigate how the laissez-faire equilibrium compares with the social optimum according to γ. Let us first recall that in Diamond (1965)'s model of exogenous growth when underaccumulation prevails in the laissez-faire equilibrium, the capital-labour ratio corresponds to its socially optimal level for some γ (i.e. the modified golden rule is verified). The present context is more complex since both the levels of human and physical capital are involved in the comparison. It is however immediate that along a balanced-growth path we cannot expect a result equivalent to that obtained in Diamond's model. For a given $\overline{k}$, the presence of the intergenerational externality mentioned above will always make the laissez-faire level of $\overline{e}$ differ from the social optimum.

In Appendix B we prove the following proposition in which we index variables with LF and $\star$ to distinguish their laissez-faire and socially optimal values. Let $\overline{k}^*_{\max}$ denote the limit of $\overline{k}^*(\gamma)$ when γ tends to $\gamma_{\max}$.

Proposition 2 *Consider a balanced-growth path in the laissez-faire economy which satisfies $\overline{k}^{LF} < \overline{k}^*_{\max}$. Then, there are two discount factors $\tilde{\gamma}$ and $\hat{\gamma}$, $0 < \tilde{\gamma} < \hat{\gamma} < \gamma_{\max}$ such that $\overline{k}^*(\hat{\gamma}) = \overline{k}^{LF}$ and $\overline{e}^*(\tilde{\gamma}) = \overline{e}^{LF}$. For $\gamma < \tilde{\gamma}$, both the levels of $\overline{k}^*(\gamma)$ and $\overline{e}^*(\gamma)$ are lower than $\overline{k}^{LF}$ and $\overline{e}^{LF}$ respectively; for $\gamma > \hat{\gamma}$, they are both higher; and for $\tilde{\gamma} < \gamma < \hat{\gamma}$, we have $\overline{e}^*(\gamma) > \overline{e}^{LF}$ and $\overline{k}^*(\gamma) < \overline{k}^{LF}$.*

The results of these Propositions 1 and 2 are depicted in the following figure where the curve is the locus of the socially optimal combinations of $\overline{k}$ and $\overline{e}$ for values of γ in the interval $(0, \gamma_{\max})$ and point LF represents the combination of $\overline{k}$ and $\overline{e}$ at the laissez-faire equilibrium. By definition the socially optimal combination for $\gamma = \hat{\gamma}$ (and for $\gamma = \tilde{\gamma}$) is located at the vertical (and at the horizontal) of LF on the curve. For $0 \le \gamma \le \tilde{\gamma}$ both $\overline{k}^*$ and $\overline{e}^*$ are below $\overline{k}^{LF}$ and $\overline{e}^{LF}$ while for $\hat{\gamma} < \gamma \le \gamma_{\max}$ they are both above. For intermediary values of γ ($\tilde{\gamma} < \gamma < \hat{\gamma}$), physical capital accumulation at the laissez-faire equilibrium is higher than at the social optimum while the opposite is true for human capital accumulation. It ought to be stressed that if for a given γ either $\overline{k}$ or $\overline{e}$ is set at a value different from its optimal value, it is not desirable to fix the other variable at its optimal value on the curve. This is an application of the standard second-best argument.

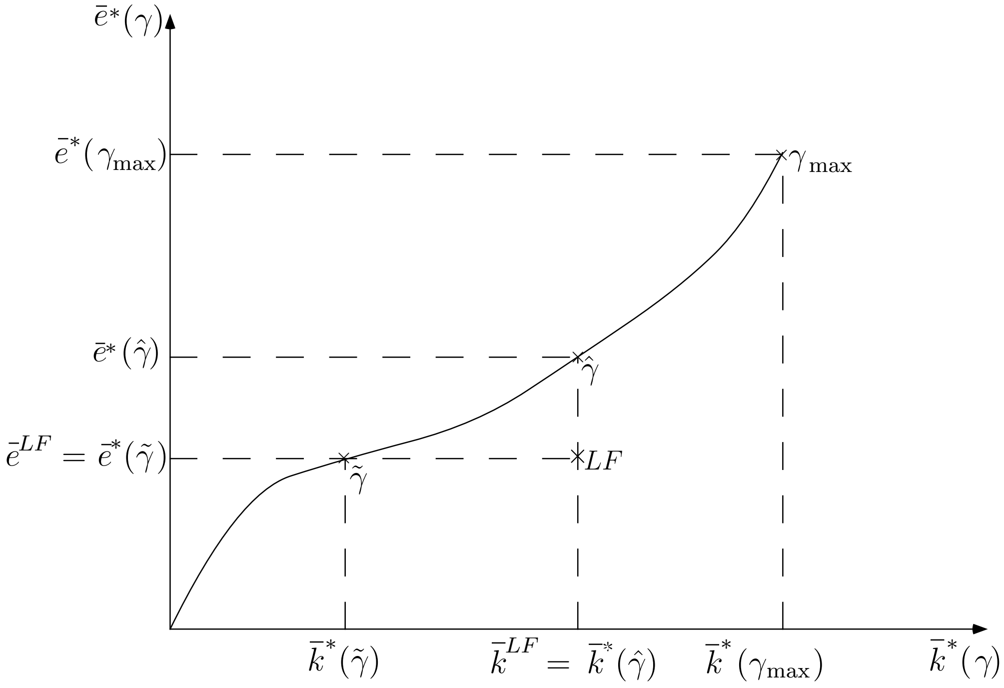

Figure 1: Optimal levels of human and physical capital as functions of γ

5 Decentralization of social optimum

In the exogenous-growth model the social optimum can be achieved by means of appropriate transfers between generations. These insure that the accumulation of physical capital satisfying the modified golden rule is reached. The transfer is from the young to the old when the per-worker physical capital in the laissez-faire equilibrium is larger than that satisfying this rule, and in the opposite direction in the other case. In our endogenous-growth setting we need an additional instrument, namely an education subsidy or alternatively an earning subsidy. In the following we consider alternative sets of tax instruments.

An individual born at time $t-1$ may face five taxes or transfers during his life span. Let σ_{t-1} denote the rate of subsidy on educational spending, θ_t the rate of subsidy on the reimbursement of the educational loan, τ_t the ad valorem tax on labor income, T_t^1 and T_{t+1}^2 the lump-sum taxes in the periods of work and of retirement. With these instruments, the budget constraints of the individual are:

$$(1-\tau_t)\, w_t h_t - (1-\theta_t)\, R_t\, (1-\sigma_{t-1})\, e_{t-1} - T_t^1 = c_t + s_t \tag{24}$$

and

$$d_{t+1} = R_{t+1} s_t - T_{t+1}^2. \tag{25}$$

Subject to these budget constraints where h_t is substituted from $h_t = h_{t-1}\varphi(e_{t-1}/h_{t-1})$ the utility function $u(c_t, d_{t+1})$ is maximized with respect to e_{t-1} and s_t, which yields the following first order conditions:

$$(1-\tau_t)\, w_t\, \varphi'\left(\overline{e}_{t-1}\right) = (1-\theta_t)\, R_t\, (1-\sigma_{t-1}) \tag{26}$$

and

$$u_c\left(c_t, d_{t+1}\right) = R_{t+1}\, u_d\left(c_t, d_{t+1}\right). \tag{27}$$

It is important to note that while the tax τ_t and the subsidy θ_t occur in the period the individual works, the subsidy σ_{t-1} occurs one period earlier when the individual gets educated. It implies that the latter subsidy directly affects the accumulation of physical capital. At time t, the market-clearing condition for capital is

$$(1+n)\, k_{t+1} = s_t - (1+n)\,(1-\sigma_t)\, e_t. \tag{28}$$

The government revenue constraint including all tax instruments can be written as:

$$\tau_t w_t h_t + T_t^1 + T_t^2/\,(1+n) = \theta_t R_t\,(1-\sigma_{t-1})\, e_{t-1} + (1+n)\,\sigma_t e_t. \tag{29}$$

Using the individual budget constraints and the Euler equation for the homogeneous production function, this can be shown to be equivalent to the resource constraint

$$F\left(k_t, h_t\right) = c_t + d_t/\,(1+n) + (1+n)\, k_{t+1} + (1+n)\, e_t. \tag{30}$$

Our purpose in the rest of this section is to investigate which combinations of policy instruments should be used in order to achieve a social optimum in a market economy. To this end, let us denote by $c_t^*, d_t^*, e_t^*, h_t^*$ and k_t^* the optimal values of the key variables at time t ($t = 0, 1, 2, \cdots$) for a given value of γ. These optimal values satisfy the following conditions:

$$h_t^* = h_{t-1}^* \varphi(e_{t-1}^*/h_{t-1}^*), \tag{1}$$

$$h_t^* f(k_t^*/h_t^*) = c_t^* + \frac{d_t^*}{1+n} + (1+n)e_t^* + (1+n)k_{t+1}^*, \tag{16}$$

$$u_c(c_t^*, d_{t+1}^*) = f'(k_{t+1}^*/h_{t+1}^*) u_d(c_t^*, d_{t+1}^*), \tag{18}$$

$$u_d(c_{t-1}^*, d_t^*) = \frac{\gamma f'(k_{t+1}^*)/h_{t+1}^*)}{1+n} u_d(c_t^*, d_{t+1}^*), \tag{19}$$

and

$$\begin{aligned} w(k_t^*/h_t^*)\varphi'(e_{t-1}^*/h_{t-1}^*) &= f'(k_t^*/h_t^*) \\ &-(1+n)\varphi(e_{t-1}^*/h_{t-1}^*) \frac{\varphi(e_t^*/h_t^*) - (e_t^*/h_t^*)\varphi'(e_t^*/h_t^*)}{\varphi'(e_t^*/h_t^*)}. \end{aligned} \tag{20}$$

At a market equilibrium, where conditions (10) and (11) – according to which the prices of factors are equated to their marginal productivities – hold, conditions (1), (16) and (18) are automatically satisfied. The first of these conditions is taken into account in the individual's optimization problem, the second one is equivalent to the government budget constraint, and the third one is equivalent to (27).

Therefore, it remains to satisfy conditions (19) and (20) by some proper choice of the policy instruments. Condition (19) has to do with the accumulation of physical capital (k_{t+1}^*). Let us denote by $s_t^*(\sigma_t)$ the saving required at time t to achieve this optimal accumulation:

$$s_t^*(\sigma_t) = (1+n)k_{t+1}^* + (1+n)(1-\sigma_t)e_t^*. \tag{31}$$

As to condition (20), it concerns the optimal choice of education spending (e_{t-1}^*). Accordingly, from (24), (25) and (26), the policy instruments need to be set so as to satisfy the three following conditions for $t = 1, 2, \cdots$:

$$(A_t) \quad T_t^1 = (1-\tau_t)\, w_t^*\; h_t^* - (1-\theta_t)\, R_t^*\, (1-\sigma_{t-1})\, e_{t-1}^* - c_t^* - s_t^*\,(\sigma_t) \;\; ;$$

$$(B_t) \quad T_{t+1}^2 = R_{t+1}^*\; s_t^*\,(\sigma_t) - d_{t+1}^* \; ;$$

$$(C_t) \quad (1-\tau_t)\,(1-\Delta_t^*) = (1-\theta_t)\,(1-\sigma_{t-1})\,.$$

where from (21), Δ_t^* is defined by:

$$(1-\Delta_t^*) f'(k_t^*/h_t^*) = w(k_t^*/h_t^*)\varphi'(e_{t-1}^*/h_{t-1}^*).$$

In $t = 0$, given e_{-1}, s_{-1}, k_0, h_0 and hence w_0 and R_0, one has:

$$(A_0) \quad T_0^1 = (1-\tau_0)\, w_0\; h_0 - (1-\theta_0)\, R_0\; e_{-1} - c_0^* - s_0^*\,(\sigma_0)\,,$$

$$(B_0) \quad T_0^2 = R_0\; s_{-1} - d_0^*.$$

This leads us to our next proposition, which is one of decentralization.

Proposition 3 *If the tax policy verifies conditions (A_t) and (B_t) for $t \geqslant 0$ and (C_t) for $t \geqslant 1$ then the social optimum is a market equilibrium.*

We now consider alternative sets of tax instruments which allow for such a decentralization of the social optimum.

6 Alternative sets of tax instruments

6.1 The standard package

Traditionally for such a problem one expects a subsidy on education and some intergenerational transfers to achieve the desirable capital accumulation. Formally, on sees that lump-sum taxes T_t^1 and T_{t+1}^2 if available can be used to fulfill (A_t) and (B_t) i.e. to achieve the optimal accumulation of physical capital. Then, to obtain condition (C_t) one can use either one of the three instruments τ_t, σ_{t-1} and θ_t, or a combination of them as long as they verify this condition. If one of these instruments is used alone, the three alternative sets are (i) $(\sigma_{t-1}, \theta_t, \tau_t) = (\Delta_t^*, 0, 0)$, (ii) $(\sigma_{t-1}, \theta_t, \tau_t) = (0, \Delta_t^*, 0)$ and (iii) $(\sigma_{t-1}, \theta_t, \tau_t) = (0, 0, -\Delta_t^*(1-\Delta_t^*)^{-1})$. Given that Δ_t^* represents the fraction of educational cost that corresponds to the human capital externality, it is not surprising that it is equal to the rate of subsidy on education spending or on the educational loan if one of these two instruments is used alone. As to alternative (iii), a negative tax on earnings is an indirect way of fostering education.

In Appendix C we study an analytical illustration of our model in which the following specification of our key functions is used: $u(c_t, d_{t+1}) = \ln c_t + \beta \ln d_{t+1}$, $\varphi(e_t) = \beta e_t^\lambda$ and $f(k_t) = Ak_t^\alpha$, with $\beta > 0$, $B > 0$, $0 < \lambda < 1$, $A > 0$ and $0 < \alpha < 1$. Our main purpose in this appendix is to investigate the properties of the optimal policies along balanced growth paths. Let us focus on the properties of the optimal policy that, in addition to T^1 and T^2, uses the education subsidy as sole instrument to achieve the optimal accumulation of human capital i.e. $(\sigma, \theta, t) = (\Delta^*, 0, 0)$. With the above specification, this policy is characterized using (C.6) in Appendix C by $\sigma = \gamma(1 - \lambda)$, showing that the subsidy rate on education spending rises with the discount factor γ, which is in agreement with the discussion in Section 4. As to the second-period lump-sum tax, it can be inferred from Appendix C using (C.15) and (C.16) where we substitute $\sigma = \gamma(1 - \lambda)$ from (C.6) that $\overline{T}_2 = T_t^2/h_t$ rises with γ. This is as expected by analogy to the exogenous-growth model: more concern given to future generations increases the required level of physical accumulation and therefore saving, which is achieved by increasing T^2 and decreasing T^1. However, in our endogenous-growth model there is no clear-cut result about the way $\overline{T}^1 \equiv T_t^1/h_t$ changes as γ rises (see (C.19) in Appendix C). It is because workers are subject to two fiscal instruments, σ and T^1. Therefore, what matters is the aggregate tax liability of workers. It is also possible to show that with the above specification, there exists a critical value of γ, say $\overline{\overline{\gamma}}$, with $0 < \overline{\overline{\gamma}} < 1$, such that $\overline{T}^2$ is negative for $\gamma < \overline{\overline{\gamma}}$ and positive for $\gamma > \overline{\overline{\gamma}}$. So when $\gamma < \overline{\overline{\gamma}}$, T^2 can be interpreted as a pension benefit given to retirees.

6.2 More realistic packages

As mentioned in the introduction, in the real world education is subsidized, elderly people receive some benefits and both transfers are financed by a tax on labor income. We accordingly assume that $T_t^1 = 0$.

6.2.1 Package $(\theta_t, \tau_t, T_{t+1}^2)$ with $T_t^1 = \sigma_{t-1} = 0$

We first consider the use of policy instruments θ_t, τ_t and T_{t+1}^2. At first sight one could believe that the two instruments θ_t and τ_t are equivalent and so do not allow to achieve the social optimum. This turns out to be wrong. Lump-sum tax T_{t+1}^2 is defined by (B_t), and from (A_t) and (C_t) we have:

$$(1 - \tau_t) w_t^* h_{t-1}^* [\varphi(\overline{e}_{t-1}^*) - \overline{e}_{t-1}^* \varphi'(\overline{e}_{t-1}^*)] - c_t^* - s_t^*(0) = 0,$$

and

$$(1-\tau_t)(1-\Delta_t^*) = 1-\theta_t,$$

which shows that τ_t and θ_t are not substitutable instruments. As a matter of fact there is an infinity of pairs (θ_t, τ_t) that satisfy the latter condition and so allow to achieve the optimal accumulation of human capital. Moving from one such pair to another leads to increase (or decrease) simultaneously the tax rate, τ_t, and the subsidy rate, θ_t, that both affect the first-period income available to workers. Therefore, by choosing the appropriate pair (θ_t, τ_t) and the appropriate level of T_{t+1}^2 it is possible to induce workers to save the amount that is needed to reach the optimal accumulation of physical capital.

With the specification of the analytical illustration in Appendix C, there again exists a critical value of γ between 0 and 1 such that along the balanced growth path, $\overline{T}^2$ is negative and positive for values of γ below and above this critical value respectively; furthermore, $\overline{T}^2$ is monotonically increasing with γ. It is also the case that the tax rate τ monotonically decreases with γ. For some parameter values of the key functions as specified in Appendix C, it is quite possible that both θ and τ are negative.

6.2.2 Package $(\sigma_t, \tau_t, T_{t+1}^2)$ with $\theta_t = T_t^1 = 0$

Instead of θ_t we now use σ_t as policy instrument. As before, T_t^2 is defined by (B_t) and we infer from (A_t) and (C_t):

$$1-\sigma_{t-1} = \frac{(1-\Delta_t)^*(c_t^* + s_t^*(\sigma_t))}{w_t^* h_{t-1}^* [\varphi(\overline{e}_{t-1}^*) - \overline{e}_{t-1}^* \varphi'(\overline{e}_{t-1}^*)]}$$

and

$$(1-\tau_t)(1-\Delta_t^*) = 1-\sigma_{t-1}.$$

Using σ_t instead of θ_t makes the problem more complicated since we now have a dynamic equation in σ_t. This is because contrary to θ_t, σ_{t-1} directly affects the market-clearing condition for capital.

7 Conclusions

In OLG models of exogenous growth the long-run level of capital accumulation at the laissez-faire equilibrium is easily compared to the one at the social optimum. If underaccumulation prevails in the former, capital-labor ratio k^{LF} is too low with respect to the modified golden rule. This occurs when $\gamma > \tilde{\gamma}$ where $\tilde{\gamma} < 1$ is such that the optimal capital-labor ratio, $k^*(\tilde{\gamma})$, is equal to k^{LF}. On the other hand, if overaccumulation prevails at the laissez-faire long-run equilibrium, that is $k^{LF} > k^*(\gamma)$, this equilibrium is dynamically inefficient, which happens when $\gamma < \tilde{\gamma}$. In both cases, the social optimum can be achieved by lump-sum intergenerational transfers.

However in our endogenous growth model with education the comparison between the laissez-faire equilibrium and the social optimum has to do with two variables, namely the capital-labor ratio $\left(\overline{k}\right)$ and the education-human capital ratio $(\overline{e})$. In the absence of altruism there is always underinvestment in education at the laissez-faire equilibrium because the impact on future generations of this investment is not internalized by individual decision makers.

In this paper we have focused on situations in which the capita-labor ratio at the long-run laissez-faire equilibrium is such that $\overline{k}^{LF} < \overline{k}^*(\gamma_{\max})$, where $\gamma_{\max}$ is the upperbound of the values of γ for which a long-run social optimum exists. Quite intuitively, in the long run the optimal levels of both ratios, $\overline{k}^*(\gamma)$ and $\overline{e}^*(\gamma)$, increase in γ. Furthermore, for γ low enough there is overaccumulation of both physical and human capitals at the long-run laissez-faire equilibrium ($\overline{k}^{LF} > \overline{k}^*(\gamma)$ and $\overline{e}^{LF} > \overline{e}^*(\gamma)$) while for γ high enough there is underaccumulation of both capitals. For intermediate values of γ, at the long-run laissez-faire equilibrium there is overinvestment in physical capital ($\overline{k}^{LF} > \overline{k}^*(\gamma)$) and underinvestment in education ($\overline{e}^{LF} < \overline{e}^*(\gamma)$).

For given γ, decentralizing the social optimal now requires, in addition to lump-sum intergenerational transfers (for adjusting $\overline{k}$), some subsidy to education (for adjusting $\overline{e}$). This subsidy internalizes the positive externality that an education investment exercises on future generations. Alternatively a subsidy to labor earnings can be used. We have also considered the case where the available instruments were a tax-transfer to the retirees, a tax on earnings and a subsidy on education. With these intruments a first-best can be achieved but possibly with a tax on the retirees, a subsidy on wage income and a tax on educational spending. In this paper we have only considered instruments allowing for first-best decentralization. In a companion paper we plan to study the second-best problem which arises when there are not enough appropriate instruments to achieve the social optimum.

In the introductory paragraph we question the rationale for the observed pattern of public transfers and taxes; the working generation financing the education of the young and the pension of the old. Within the limits of our model we have shown that quite different patterns can result even in a first-best framework.

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Appendix A: Proof of Proposition 1

From (23) we can write:

$$\gamma[\varphi(\overline{e})]^b = (1+n)\varphi(\overline{e})/f'(\overline{k}) \tag{A.1}$$

and hence the transversality condition, $\gamma[\varphi(\overline{e})]^b < 1$, is equivalent to:

$$f'(\overline{k}) - (1+n)\varphi(\overline{e}) > 0. \tag{A.2}$$

We also have from (20):

$$f'(\overline{k}) - (1+n)\varphi(\overline{e}) = \varphi'(\overline{e})(f(\overline{k}) - \overline{k}f'(\overline{k}) - (1+n)\overline{e}) \tag{A.3}$$

and thus the transversality condition is also equivalent to:

$$f(\overline{k}) - \overline{k}f'(\overline{k}) - (1+n)\overline{e} > 0. \tag{A.4}$$

Total differentiation of (A.3) with respect to $\overline{k}$ and $\overline{e}$ yields:

$$\frac{d\overline{k}}{d\overline{e}} = \frac{\varphi''(\overline{e})[f(\overline{k}) - \overline{k}f'(\overline{k}) - (1+n)\overline{e}]}{f''(\overline{k})[\overline{k}\varphi'(\overline{e}) + 1]}, \tag{A.5}$$

which is positive when the transversality condition is satisfied. This means that $\overline{k}(\gamma)$ and $\overline{e}(\gamma)$ vary in the same direction. On the other hand, differentiating (23) yields:

$$f'(\overline{k})d\gamma + \gamma f''(\overline{k})d\overline{k} - \mu d\overline{e} = 0 \tag{A.6}$$

with $\mu = (1+n)(1-b)[\varphi(\overline{e})]^{-b}\varphi'(\overline{e}) > 0$, and hence:

$$\frac{d\overline{e}}{d\gamma} = \frac{f'(\overline{k})}{\mu - \gamma f''(\overline{k})\dfrac{d\overline{k}}{d\overline{e}}} \tag{A.7}$$

is positive. Therefore, both $\overline{k}(\gamma)$ and $\overline{e}(\gamma)$ are increasing in γ. From this, one infers that the ℓhs of (A.2) is decreasing in γ. So if the transversality condition holds for $\gamma = \gamma_1$ it does also for $\gamma < \gamma_1$. Also, the transversality condition is satisfied for $\gamma \in (0, \gamma_{\max})$, where the upperbound of the interval is the value of γ for which the left-hand side of (A.2) or equivalently of (A.4) becomes nil. The values of $\overline{k}(\gamma_{\max})$ and $\overline{e}(\gamma_{\max})$ are obtained by solving the system of two equations (A.2) and (A.4) where the inequalities are replaced by equalities.

We now show by contradiction that $\lim_{\gamma\to 0} \overline{k}(\gamma) = 0$. Suppose that it were positive. Then, since $f'(\overline{k}) > 0$ for $\overline{k} > 0$, relation (23) would imply that $\lim_{\gamma\to 0} \varphi(\overline{e}(\gamma)) = 0$, which in turn implies that $\overline{e}(\gamma)$ and so $\varphi(\overline{e}(\gamma)) - \overline{e}(\gamma)\varphi'(\overline{e}(\gamma))$ have their limits equal to zero. However, from relation (20) written along the balanced-growth path we infer that $\varphi'(\overline{e}(0)) = f'(\overline{k})[f(\overline{k}) - \overline{k}f'(\overline{k})]^{-1}$ would thus be finite. This contradicts our assumption that $\lim_{\overline{e}\to 0} \varphi'(\overline{e}) = \infty$. Consequently, $\lim_{\gamma\to 0} \overline{k}(\gamma) = 0$. The same reasoning applies to show that $\lim_{\gamma\to 0} \overline{e}(\gamma) = 0$. If this limit were positive, the limit of $f'(\overline{k}(\gamma))$ would be infinite from (23) while it would be finite from (20).

Appendix B: Proof of Proposition 2

Given Proposition 1, since $\overline{k}^{LF} < \overline{k}_{\max}$ by assumption, there is a value $\hat{\gamma} < \gamma_{\max}$ such that $\overline{k}^*(\hat{\gamma}) = \overline{k}^{LF}$; moreover, for $\gamma < \hat{\gamma}$, $\overline{k}^*(\gamma) < \overline{k}^{LF}$ and for $\gamma > \hat{\gamma}$, $\overline{k}^*(\gamma) > \overline{k}^{LF}$.

From (20), we have

$$\varphi'(\overline{e}^*(\hat{\gamma})) < \frac{f'(\overline{k}^*(\hat{\gamma}))}{f(\overline{k}^*(\hat{\gamma})) - \overline{k}^*(\hat{\gamma})f'(\overline{k}^*(\hat{\gamma}))} = \varphi'(\overline{e}^{LF})$$

where the equality comes from (10), (11) and $\overline{k}^*(\hat{\gamma}) = \overline{k}^{LF}$ by definition of $\hat{\gamma}$. Therefore, we have: $\overline{e}^{LF} < \overline{e}^*(\hat{\gamma})$. Since Proposition 1 states that $\overline{e}^*$ rises with γ, it implies that there exists a $\tilde{\gamma} < \hat{\gamma}$ such that $\overline{e}^*(\tilde{\gamma}) = \overline{e}^{LF}$. Furthermore, for $\gamma < \tilde{\gamma}$, we have $\overline{e}^*(\gamma) < \overline{e}^{LF}$, and for $\gamma > \tilde{\gamma}$, $\overline{e}^*(\gamma) > \overline{e}^{LF}$.

Appendix C: Analytical illustration of Sections 5 and 6

In this analytical example we use the following specification of the key functions:

$$u(c_t, d_{t+1}) = \ln c_t + \beta \ln d_{t+1}, \tag{C.1}$$
$$\varphi(e_t) = Be_t^{\lambda}, \tag{C.2}$$
$$f(k_t) = Ak_t^{\alpha}, \tag{C.3}$$

with $\beta > 0$, $B > 0$, $0 < \lambda < 1$, $A > 0$ and $0 < \alpha < 1$.

Optimality condition (19) along the balanced growth path can then be written as:

$$\gamma \frac{f'(\overline{k}^*)}{1+n} = \varphi(\overline{e}^*) \tag{C.4}$$

since (C.1) implies that $u_d(c_{t-1}^*, d_t^*)/u_d(c_t^*, d_{t+1}) = d_{t+1}^*/d_t^* = \varphi(\overline{e})$ along the balanced growth path. As to optimality condition (20) it yields using (C.4) and (C.2):

$$w(\overline{k}^*)\varphi'(\overline{e}^*) = f'(\overline{k}^*)\left[1 - \frac{1+n}{f'(\overline{k}^*)}(\varphi(\overline{e}) - \overline{e}\varphi'(\overline{e}))\right] = f'(\overline{k}^*)[1-\Delta^*] \tag{C.5}$$

where

$$\Delta^* = \gamma(1-\delta). \tag{C.6}$$

From (10), (11) and (C.3) we infer

$$\frac{w(\overline{k}^*)}{f'(\overline{k}^*)} = \frac{1-\alpha}{\alpha\overline{k}^*}, \tag{C.7}$$

which we substitute in (C.5) to obtain

$$\overline{k}^* = \frac{\alpha}{1-\alpha}\frac{1-\Delta^*}{\varphi'(\overline{e}^*)} = \frac{\alpha}{1-\alpha}\frac{1-\Delta^*}{\lambda B}(\overline{e}^*)^{1-\lambda}$$

where we have also used (C.2). From this expression we obtain by means of (C.4) and (C.3):

$$\begin{aligned}
\overline{e}^* &= \frac{1-\alpha}{\alpha}\frac{\lambda B}{1-\Delta^*}\overline{k}^*(\overline{e}^*)^{\lambda} \\
&= \frac{1-\alpha}{\alpha}\frac{\lambda}{1-\Delta^*}\overline{k}^*\varphi(\overline{e}^*) \qquad \text{using (C.2)} \\
&= (1-\alpha)\frac{\lambda}{1-\Delta^*}\frac{\gamma}{1+n}A(\overline{k}^*)^{\alpha} \text{ using (C.4) and (C.3).}
\end{aligned} \tag{C.8}$$

Let us denote by v^* the fraction of production used for education along the balanced growth path, that is:

$$v^* \equiv \frac{(1+n)\overline{e}^*}{A(\overline{k}^*)^{\alpha}}. \tag{C.9}$$

We then infer from (C.8):

$$v^* = \gamma(1-\alpha)\frac{\lambda}{1-\Delta^*}. \tag{C.10}$$

Let us denote by $\overline{C}^*$ the aggregate consumption per worker of h_t and unit of h_t along the balanced growth path:

$$\begin{aligned}\overline{C}^* &\equiv \frac{1}{h_t}\left(c_t^* + \frac{d_t^*}{1+n}\right) \\ &= f(\overline{k}^*) - (1+n)\overline{e}^* - (1+n)\frac{h_{t+1}^*}{h_t^*}\overline{k}^*.\end{aligned}$$

Since $h_{t+1}^*/h_t^* = \varphi(\overline{e}) = \overline{k}^*\gamma f'(\overline{k}^*)(1+n)^{-1}$ from (C.4), we then obtain using (C.3) and (C.9):

$$\overline{C}^* = A(\overline{k}^*)^\alpha(1-v-\gamma\alpha). \tag{C.11}$$

To determine how this aggregate consumption is shared between first- and second- period consumptions, let us first remark that along the balanced-growth path, optimality condition (18) yields using (C.1):

$$f'(\overline{k}^*) = \frac{u_c(c_t, d_{t+1})}{u_d(c_t, d_{t+1})} = \frac{d_{t+1}^*}{\beta c_t^*} = \frac{\varphi(\overline{e}^*)\overline{d}^*}{\beta\overline{c}^*}, \tag{C.12}$$

with $\overline{c}^* \equiv c_t^*/h_t^*$ and $\overline{d}^* \equiv d_t/h_t^*$. Using (C.4) allows to infer from (C.11):

$$\frac{\overline{d}^*}{\overline{c}^*} = \frac{\beta}{\gamma}(1+n) \tag{C.13}$$

and therefore

$$\overline{c}^* = \frac{\gamma}{\beta+\gamma}\overline{C}^* \text{ and } \overline{d}^* = \frac{\beta(1+n)}{\beta+\gamma}\overline{C}^*. \tag{C.14}$$

We also have using (C.4) and (C.9):

$$\begin{aligned}\overline{s}(\sigma) \equiv \frac{s_t}{h_t} &= (1+n)\varphi(\overline{e})\overline{k}^* + (1+n)(1-\sigma)\overline{e}^* \\ &= [\gamma\alpha + (1-\sigma)v]A(\overline{k}^*)^\alpha. \end{aligned} \tag{C.15}$$

The above results will enable us to obtain an expression for the value of $\overline{T}^2$, i.e. the lump-sum tax T_{t+1}^2 divided by h_t that must be levied on retirees for decentralizing the social optimum along the balanced growth path.

From condition (B_t) we have

$$\overline{T}^2(\gamma,\sigma) = T^2_{t+1}/h^*_t = R^*\overline{s}^*(\sigma) - \varphi(\overline{e}^*)\overline{d}^*,$$

which, using the above results in (C.10), (C.13) and (C.14), can be rewritten as:

$$\overline{T}^2(\gamma,\sigma) = R^*A(\overline{k}^*)^\alpha\gamma\psi(\gamma,\sigma) \tag{C.16}$$

with

$$\psi(\gamma,\sigma) \equiv \alpha + \frac{(1-\sigma)(1-\alpha)\lambda}{1-\gamma(1-\lambda)} - \frac{\beta}{\beta+\gamma} + \frac{\beta\gamma}{\beta+\gamma}\left(\alpha + \frac{(1-\alpha)\lambda}{1-\gamma(1-\lambda)}\right). \tag{C.17}$$

It is easily verified that $d\psi/d\gamma > 0$, $0 \leq \gamma \leq 1$. We also have $\psi(0,\sigma) = \alpha + (1-\sigma)(1-\alpha)\lambda - 1 < 0$ iff $\sigma > -(1-\lambda)\lambda^{-1}$ and $\psi(1,\sigma) = \alpha + (1-\sigma)(1-\alpha) > 0$. Therefore, iff $\sigma > -(1-\lambda)\lambda^{-1}$, there exists a unique value $0 < \overline{\overline{\gamma}}(\sigma) < 1$ such that $\overline{T}^2(\gamma,\sigma) < 0$ for $\gamma < \overline{\overline{\gamma}}(\sigma)$ and $\overline{T}^2(\sigma) > 0$ for $\gamma > \overline{\overline{\gamma}}(\sigma)$; otherwise $\overline{T}^2(\gamma,\sigma)$ is positive for anly $0 < \gamma < 1$. On the other hand, $\overline{T}^2$ falls when σ rises.

Likewise, an expression for $\overline{T}^1$ can be found. Using condition (C_t) we infer from condition (A_t):

$$\overline{T}^1 \equiv \frac{T^1_t}{h_t} = (1-\tau)[w^* - R^*\frac{\overline{e}^*}{\varphi(\overline{e}^*)}(1-\Delta^*)] - \overline{c}^* - \overline{s}^*(\sigma).$$

Substituting from (C.7), (C.13) and (C.14) this expression can then be written as:

$$\begin{aligned}\overline{T}^1(\gamma,\tau,\sigma) = \{&(1-\tau)(1-\alpha)(1-\lambda) \\ &-\frac{\gamma}{\beta+\gamma)}[1-\sigma v + \beta(\gamma\alpha + (1-\sigma)v)]\}A(\overline{k}^*)^\alpha.\end{aligned} \tag{C.18}$$

The derivative of $\overline{T}^1$ w.r.t. γ is of ambiguous sign while those w.r.t. τ and σ are negative and positive respectively.

We can finally find an expression for the value of $(1-\tau)$ that must be chosen to decentralize the social optimum. From condition (A_t) we obtain:

$$\begin{aligned}\overline{c}^* + \overline{s}(\sigma) + \overline{T}^1 &= (1-\tau)w^*\left[1 - \frac{(1-\theta)R^*(1-\sigma)}{(1-\tau)w^*}\frac{\overline{e}}{\varphi(\overline{e})}\right] \\ &= (1-\tau)w^*(1-\lambda)\end{aligned}$$

where we have used (26) and (C.2). From this expression we then derive by means of (C.14), (C.15) and (C.3):

$$1-\tau = \frac{1}{(1-\alpha)(1-\lambda)(\beta+\gamma)}\left\{\gamma(1+\alpha\beta)+v[\beta-\sigma(\beta+\gamma)]\right\}+\frac{\overline{T}^{1}}{w^{*}(1-\lambda)}. \tag{C.19}$$