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BUBBLES AND LONG–RANGE DEPENDENCE IN ASSET PRICES VOLATILITIES

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Abstract

A model for a financial asset is constructed with two types of agents. The agents differ in terms of their beliefs. The proportions of the two types change over time according to a stochastic process which models the interaction between the agents. Thus, unlike other models, agents do not persist in holding "wrong" beliefs. Bubble-like phenomena in the asset price occur. We consider several tests for detecting long range dependence and change–points in the conditional variance process. Although the model seems to generate long–memory properties of the volatility series, we show that this is due to the switching of regimes which are detected by the tests we propose.

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1 Introduction

Price dynamics are at the heart of economics. Aggregate movements do not correspond to those that might be generated by the maximising behaviour of a single individual. The problem of explaining such dynamics is one to which Claus Weddepohl has made substantial contributions. One way of explaining the complex dynamics is to suggest that the economy or market is trying to adjust to the equilibrium and that whilst it has not converged out of equilibrium phenomena such as rationing play an important role. This is the route that Claus has taken. We propose to take another route here and to follow an approach which has been also developed by Cars Hommes and others. We suggest that what we are observing in the price behaviour are bubbles created by the direct interaction between the agents at the micro level.

It is surprising that the idea of testing for the existence of "price bubbles" in markets has occurred so recently since, although particularly associated with the markets for financial assets, such phenomena have been documented for a wide variety of markets over a considerable period of time. One of the earliest bubbles was that in the price of red mullet in the first century A.D. The red mullet fever is documented by Cicero, Horace, Juvenal and Martial. A survey of other historical bubbles, such as the Tulip, South Sea and Mississippi bubbles, may be found in Garber (2000). More recently there has been a substantial literature on the theoretical basic for and testing of bubbles, see for example Blanchard and Watson (1982), Flood and Garber (1980), Meese (1986), Tirole (1985), West (1988), Woo (1987), Stiglitz (1990), Flood and Hodrick (1990), Avery and Zemsky (1998) and Shiller (2000).

There are two basic problems involved in the discussion of bubbles, on the one hand their definition, and on the other their detection and identification. This paper, after a brief discussion of the literature, will present an economic model which produces price bubbles and will then examine how various of the tests proposed for detecting bubbles perform on the data generated by the model. The latter is based on a simple stochastic process developed by the first author with Hans Föllmer. It is this process which produces the bubble-like phenomena resulting from agents changing their forecasts and which was used as the basis for the analysis in Kirman (1991, 1993). In introducing bubbles we follow Evans (1991), with two differences. Firstly, instead of simply testing data from a stochastic process with bubble-like characteristics, we use data from a model of economic behaviour with interacting agents. Secondly, since this model is characterized by switches from one type of stochastic process to another, we apply several procedures developed by Kokoszka and Leipus (2000), Horváth, Kokoszka and Teyssière (2001) and Kokoszka and Teyssière (2001) for detecting changes in regime in the conditional second moments.

In Kirman and Teyssière (2002), we have shown that this class of models is able to replicate the empirical long–memory properties of asset prices volatilies: while asset prices returns are uncorrelated, their power transformation display strong dependence, the degree of which is common to several financial time series. Interested readers are referred to Beran and Ocker (2001), Cheung (1993), Granger and Ding (1995, 1996), Mandelbrot (1997), Taylor (1986), Teyssière (1997, 1998) for references on these empirical properties. In the model by Kirman and Teyssière (2002) this common degree of long–memory was linked to the swing in opinions of interacting agents. This is in line with the works of Mikosch and Stărică (1999) who claimed that the strong dependence in asset prices volatility is the outcome of some form of non stationarity and changing regimes in the conditional variance process. Thus, it is of interest to test whether the long range dependence generated by our model is spurious, and is the outcome of a change–

point process.

When discussing markets in general, it is difficult to separate significant swings in prices into their two components, those due to speculation and those due to swings in fundamental characteristics of the economy, some of which may be in principle measurable such as technological change, and others of which such as tastes may be unmeasurable. However, when considering the market for financial assets the situation is somewhat simpler. Theory suggests that the price of a share should, for example, reflect the discounted expected value of future dividends (the fundamentals). Thus, any prolonged departure from the underlying fundamentals could be defined as a bubble as Froot and Obstfeld (1989), for example, suggest.

We will require two features of bubbles which go beyond the simple departure from fundamentals. Firstly, they should "burst" at some time and not be perpetually explosive, see Diba and Grossman (1988), and secondly that they should be endogenous, i.e., not directly produced by exogenous shocks.

The earlier part of the recent literature on bubbles, such as the contributions of Le Roy and Porter (1981), Shiller (1981) and Blanchard and Watson (1982), all came to the view that asset prices were too volatile to be explained by fundamentals alone. Thus it was argued that there was "excess volatility". Meese and Rogoff (1983) came to the same conclusion for exchange rates.

The debate then, however, swung somewhat in the opposite direction. In part it has been suggested that the econometric analysis in the papers mentioned was faulty and in part that the process governing the fundamentals had been misspecified, a good idea of the main issues in this discussion can be obtained from Campbell and Shiller (1987), Flavin (1983), Kleidon (1986), Mankiw, Romer and Shapiro (1985), Marsh and Merten (1986), West (1987, 1988), and Flood and Hodrick (1990), or even that some unobserved fundamentals might have been omitted, see Hamilton and Whiteman (1985).

As Campbell *et al.* (1997) point out much of the heat has now gone out of this debate since it is now recognised that a rejection of a model which discounts future dividends at a constant rate is not a rejection of the efficient market hypothesis and partly because economists have become convinced that expected asset returns are time–varying. However, this does not in any way answer the basic question of this paper, which is simply, can there be bubbles in the data on asset returns and how might they be detected?

Diba and Grossman (1988) claimed that the data for stock prices do not have the explosive characteristics one would expect if bubbles existed. Perron (1989) however, suggested that the unit roots tests commonly used may fail to reject the presence of unit roots, when in fact the underlying process is one with a "broken trend" or a shift in regime; interested readers are referred to Stock (1994) for a survey. Indeed Evans (1991) has found, by testing data from simulating a stochastic process known to contain bubbles, that in general the unit roots hypothesis was not rejected. Thus, whether or not there are bubbles in asset price data is still an open question.

The real question seems, however, not to be quite as simple as that discussed in the general debate on the issue. If asset prices do in general follow fundamentals, but periodically depart from them, then the situation is rather complicated. If fundamentals follow a random walk as theory might suggest in the case of stock prices for example, then for some, maybe substantial, part of the time this is the process that will be realised. The process that is followed at other periods has, of course, to be specified. If the process always returns, in the long run, to the fundamentals, then this is rather different from the situation envisaged by Perron (1989) for example. He argues that there are occasional large exogenous shocks that cause a permanent change in the system but that between these shocks, stock prices are trend stationary.

The sort of model which we are suggesting will be permanently affected by shocks, but these shocks are endogenous. However, its characteristics are determined by two things. The relative time it spends in the fundamental phase and the particular characteristics of the process when it is not in that phase. Unfortunately, the natural characterisation of the non fundamental phase following the literature in finance is one which has a class of processes and not a single one. Thus, Hamilton's (1989) suggestion that the market could be thought of as switching from one stationary process to another is not really applicable.

In particular, it should be clear that the basic point at issue here is not does or does not the asset price process have unit roots but how far and for how long does it deviate from that process and how can one separate out these deviations? In a certain sense, in the long run, as indeed the word suggests, bubbles do not matter, but their impact in the short run may be very significant.

In the foreign exchange market which we take as an example, roughly two thirds of all turnover consists of spot transactions, see Suvanto (1993), McKinnon (1979) and Kouri (1983). Since dealers have very short horizons, many have to have a closed position at the end of the day, and their customers are sensitive to price changes, it is clear that episodes in which extrapolatory behaviour can take the market away from fundamentals can be very important. Yet most dealers argue that, "in the long run fundamentals matter", see Barrow (1994). The way in which fundamentals eventually pull prices back is through underlying customer flows, see Kouri (1983), but since these in turn are affected by the evolution of current prices the magnitude and duration of deviations are difficult to calculate.

All of this suggests that the testing strategy used up to now to detect the presence of bubble-like episodes is not appropriate, as will become clear.

We shall now present a simple model which has the basic characteristics just outlined.

2 A simple theoretical model

This simple economic model, developed in Kirman (1991) and originally based on experimental evidence on the behaviour of ants, Kirman (1993), captures four essential features which one would like financial markets with bubbles to have. Firstly, there are underlying fundamentals which the price process follows for certain periods. Secondly, agents are heterogeneous in the sense that over time different opinions prevail in the population. Thirdly, unlike most of the literature in which there are several groups of agents, see for example the papers on "Noise Traders" by De Long *et al.* (1989, 1990a, 1990b, 1991) on "Bulls, bears and market sheep" by Day and Huang (1990) and on "herding" by Sharfstein and Stein (1990), none of the agents in this model can be classified as "irrational", in the sense that they cling to beliefs which are revealed to be "wrong" by their observations. Indeed, all the agents in the model behave rationally in the sense that each agent plays her best response given the common knowledge about the structure of the beliefs and behaviour in the model. Bubbles, in the standard sense, occur in the price process as prices depart from fundamentals and then return to them. Finally, bubbles can be both negative and positive. Models which are close in spirit to this sort of model are those of Brock and Hommes (1999) and Föllmer and Schweizer (1993).

We now give a brief description of the model which was developed with Hans Föllmer. The model is then simulated to generate the data to be used for evaluating the standard testing procedures for bubbles and change point or their absence.

2.1 The model

Agents are faced with a price process P_t for a financial asset and form expectations about tomorrow's prices. There are two different ways[1] of forming expectations and each agent uses one of them. However, the expectations of the individual agents are influenced by random meetings with other agents. Call the two methods of forming expectations the two "opinions" in the model, then if there are N agents, we say that

> The state of the system is defined by the number k of agents holding opinion one, i.e., $k \in \{0, 1, \ldots, N\}$.

The stochastic process governing the state evolves as follows. Two agents meet at random and the first is converted to the second's view with probability $(1-\delta)$. If the meeting is considered as a drawing from an urn with balls of two different colours, it is obvious that which agents is the "first" and which is the "second" is of no importance since the symmetric event occurs with the same probability. There is also a small probability ε that the first agent will change her opinion independently of whom she meets. This is a technical necessity to prevent the process from being "absorbed" into one of the two states O or N, but can be allowed to go to zero as N becomes large.[2] Indeed in what follows we shall require for the basic results that ε be small.

The process then evolves as follows:

$$k \longrightarrow \begin{cases} k+1 \text{ with probability } p(k,k+1) = \left(1-\frac{k}{N}\right)\left(\varepsilon + (1-\delta)\frac{k}{N-1}\right) \\ k \text{ with probability } p(k,k) = 1 - p(k,k+1) - p(k,k-1) \\ k-1 \text{ with probability } p(k,k-1) = \frac{k}{N}\left(\varepsilon + (1-\delta)\frac{N-k}{N-1}\right) \end{cases} \tag{2.1}$$

The first problem is to look at the equilibrium distribution $\mu(k)$, $k = 0, \ldots, N$ of the Markov Chain defined by (2.1).

This is important in the economic model since it describes the proportion of time that the system will spend in each state. The distribution is given by

$$\mu(k) = \sum_{l=0}^{N} \mu(l) p(l,k) \tag{2.2}$$

[1] There could be any finite number of ways of forming expectations and this would not change the nature of the results. Franz Palm suggested that agents might choose a convex combination of the two processes but this could not be handled within the framework developed here.

[2] This ε can be thought of as the replacement of some old agents in each new period by agents who may hold either opinion (see for example the evolutionary model of Young and Foster (1991)) or by some external shock which influences some people's expectations.

but given that the process is symmetric and reversible then it follows that

$$\mu(l)p(l,k) = \mu(k)p(k,l) \tag{2.3}$$

From this expression one obtains

$$\frac{\mu(k+1)}{\mu(k)} = \frac{p(k,k+1)}{p(k+1,k)} = \frac{\left(1-\frac{k}{N}\right)\left(\varepsilon+(1-\delta)\frac{k}{N-1}\right)}{\frac{k+1}{N}\left(\varepsilon+(1-\delta)\left(1-\frac{k}{N-1}\right)\right)} \tag{2.4}$$

since it is clear from (2.3) that

$$\mu(k) = \frac{\frac{\mu(1)}{\mu(0)}\cdots\frac{\mu(k)}{\mu(k-1)}}{1+\sum_{l=1}^{N}\frac{\mu(1)}{\mu(0)}\cdots\frac{\mu(l)}{\mu(l-1)}} \tag{2.5}$$

Now the form of $\mu(k)$ will depend, naturally, on the values of ε and δ. It is easy to see that if $\varepsilon < (1-\delta)/(N-1)$ then $\mu(k)$ will indeed be convex. Thus this case, in which the process spends most of its time in the extremes corresponds to the case in which the probability of "self conversion" is small relative to the probability of being converted by the person one meets. Although this probability of conversion is independent of the numbers in each group, which type will actually meet which type depends on the relative numbers in each type at any moment, i.e. on the state of the system. Thus, when one type is in the minority conversion of any individual is much less likely than when the numbers of the two types are fairly equal.

The ε might be considered as being simply a technical artefact, therefore it is worth looking at what happens to the process when N becomes large and ε goes to zero. Consider the asymptotic form of $\mu(k)$ when we choose ε for each N so that $\varepsilon < (1-\delta)/N$. When N becomes large redefine μ as $\mu\ (k/N)$ and consider the limit distribution as $N \to \infty$. Call this limit distribution, which will be continuous, f. Then one can prove the following[3]

PROPOSITION 2.1 *f is the density of a symmetric Beta distribution i.e.* $f(x) = Cx^{\alpha-1}(1-x)^{\alpha-1}$, *where C is a constant.*

This stochastic model of shifts of opinion given here is related to the urn models of Arthur *et al.* (1985) and also to models which have been developed for shifts in voter opinion (see the examples given by Weidlich, cited in Haken (1977), where a similar bimodal distribution is derived). The latter model could also have been taken as the basis for the conversion from one opinion to another here.

2.2 The market for a financial asset

Consider two types of individuals who forecast the value of an asset or, as in the model developed by Frankel and Froot (1986), the value of the exchange rate.

"Fundamentalists" believe that the exchange rate P_t at time t is related to some underlying fundamental $\bar{P}_t$ which might be a constant $\bar{P}$, some long run equilibrium, or might be governed

[3]Clearly for any given value of k, $\mu(k/N)$ increases proportionately with N. This proposition was proved by Hans Föllmer and the proof is given in Kirman (1993).

by some dynamic deterministic or stochastic process. Their forecast for the value at the next period, conditional on the information set I_t available at time t, is given by

$$E^f\left(P_{t+1}|I_t\right) = \bar{P}_t + \sum_{j=1}^{M_f} \nu_j (P_{t-j+1} - \bar{P}_{t-j}) \tag{2.6}$$

where ν_j, $j = 1, \ldots, M_f$ are positive constants, M_f is the memory of the fundamentalists. We assume that the fundamentals $\bar{P}_t$ follow a random walk, i.e.,

$$\bar{P}_t = \bar{P}_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \tag{2.7}$$

Chartists, on the other hand, forecast by simple extrapolation of the past history of prices [4] and hence predict that the next period exchange rate will be given by

$$E^c\left(P_{t+1}|I_t\right) = \sum_{j=0}^{M_c} h_j P_{t-j} \quad \text{with} \quad \sum_{j=0}^{M_c} h_j = 1 \tag{2.8}$$

where h_j, $j = 0, \ldots, M_c$ are constants, M_c is the memory of the chartists.

The market view or forecast is given by a weighted average of the two forecasts, i.e.,

$$E^m(P_{t+1}|I_t) = k_t E^f(P_{t+1}|I_t) + (1 - k_t)E^c(P_{t+1}|I_t) \tag{2.9}$$

The weights are determined in Frankel and Froot's model by portfolio managers who effectively choose the weights in such a way as to make the actual outcome constant with the market forecast.

In the model here the weights k_t are determined endogenously, and they will depend on the number of agents who act according to each view. To see this consider the process as taking place in two steps. Firstly, individuals meet each other and form an opinion after these meetings about how prices will change. "Meeting" of course does not mean meeting in the literal physical sense, a better word might be "contact". A foreign exchange dealer is, for example, faced with several screens full of general information and indicative quotes given by other dealers. In addition he has loudspeakers from which he can hear brokers' quotes, and furthermore he is in telephone communication with other dealers and with his clients. Each of these individual quotes or "contacts" may cause him to shift his expectations and thus modify his action.

In addition agents also observe the market as a whole and decide what is the prevailing opinion. This adheres to Keynes's beauty queen idea. If I am interested in forecasting who will win the contest what matters is not what I think but what the others think. Again, pursuing the example of a foreign exchange dealing room, dealers maintain that they focus on specific pieces of information, this corresponds to the "meetings" in the model, but that they are also aware of all the information with which they are faced and this they somehow process to obtain an idea of "market sentiment". Having made their specific and general observations, agents forecast and then act accordingly.

Thus the process can be described as follows:

[4] It would, of course, be interesting to try other more sophisticated forms of extrapolation. Although this would change the dynamics of the model it would not change the basic stochastic alternation between regimes.

i) Agents meet each other at random and are converted to each others' opinions as defined in the process described above. Consider w_t as the number of individuals at time t who are "fundamentalists" and the remaining $N - w_t$ as "chartists". Allow some fixed number M of meetings to take place at each time.

ii) Defining $q_t = w_t/N$ each agent now makes an observation of q_t i.e., tries to assess which opinion is in the majority. She observes q_t with some noise. Thus, the signal she receives is

$$q_{i,t} = q_t + \varepsilon_{i,t} \quad \varepsilon_{i,t} \sim N(0, \sigma_\vartheta^2) \tag{2.10}$$

If now agent i receives a signal $q_{i,t} \geq 1/2$, then she will make a fundamentalist forecast since the majority is doing so. Conversely, if $q_{i,t} < 1/2$ she will forecast as a chartist and act accordingly. The number and proportion of agents who base their demand on fundamental forecasts are therefore given by

$$k_t = \frac{1}{N}\#\left\{i : q_{i,t} \geq \frac{1}{2}\right\} \tag{2.11}$$

The natural question here is whether it is rational for agents to act in this way. One might think that an agent should act according to her own beliefs. However, if she knows the structure of the model and knows herself to be in a minority it is not rational for her to base her demand on her own forecast rather than that of the majority since it is the latter that determines the price. Indeed it is easy to see that, if everybody knows that everybody else is following the majority, the best response is to do so oneself. Thus, a Nash equilibrium will occur if everybody follows majority behaviour.[5] It should be noted that the relationship between q_t and k_t depends crucially on the variance of the noise in the signal about the majority. If this variance is large the process will spend more time close to the equal proportion situation than does the k_t process. It seems reasonable to consider the variance as small since otherwise agents should give more weight to their private information. Having now determined the proportion of agents who forecast as fundamentalists the market forecast is given by

$$E^m(P_{t+1}|I_t) = k_t E^f(P_{t+1}|I_t) + (1 - k_t)E^c(P_{t+1}|I_t) \tag{2.12}$$

The price on the market, i.e., the market exchange rate if one is considering the market for foreign exchange, is given by

$$P_t = c\Delta P_{t+1}^m | I_t + Z_t \tag{2.13}$$

where c is a constant and Z_t is an index of a vector of fundamental variables according to Frankel and Froot (1986). Economists will require, of course, that this model be justified by some underlying model of individual behaviour. Consider the following simple:[6]

Agent i has a utility function given by

$$U^i(W_{t+1}^i) = E(W_{t+1}^i) - \lambda V(W_{t+1}^i) \tag{2.14}$$

[5]The logic is inescapable but there is a problem, of course, of indeterminacy. For a discussion of similar problems see Ellison and Fudenberg (1993).

[6]This example was suggested by Michael Woodford.

where λ denotes the risk aversion coefficient, $E(.)$ and $V(.)$ denote the expectation and variance operators, and W_{t+1}^i, her wealth at time $t+1$, is given by

$$W_{t+1}^i = (1+\rho_{t+1})P_{t+1}d_t^i + (W_t^i - P_t d_t^i)(1+r) \tag{2.15}$$

The variables are defined as follows:

- ρ_{t+1} is the dividend in foreign currency paid on one unit of foreign currency;
- P_{t+1} is the exchange rate at time $t+1$;
- d_t^i is the demand by the i^{th} individual for foreign currency;
- r is the interest rate on holdings of domestic currency.

P_{t+1} the exchange rate at time $t+1$ and ρ_{t+1} the foreign dividend are both considered by agents to be random variables. The first two moments of the distribution of P_{t+1}, from the point of view of individual i, are given by[7]

$$E(P_{t+1}) = \Delta P_{t+1}^i + P_t \qquad V(P_{t+1}) = \sigma_P^2 \tag{2.16}$$

and for ρ_{t+1} by

$$E(\rho_{t+1}) = \rho \qquad V(\rho_{t+1}) = \sigma_\rho^2 \tag{2.17}$$

Furthermore, assume ρ_{t+1} and P_{t+1} to be independent. From these assumptions

$$E(W_{t+1}^i|I_t) = (1+\rho)E^i(P_{t+1}|I_t)d_t^i + (W_t^i - P_t d_t^i)(1+r) \tag{2.18}$$

and

$$V(W_{t+1}^i|I_t) = (d_t^i)^2\zeta_t \quad \text{with} \quad \zeta_t = V\left(P_{t+1}(1+\rho_{t+1})\right) \tag{2.19}$$

Demand d_t^i is found by maximising utility and writing the first order condition

$$(1+\rho)E^i(P_{t+1}|I_t) - (1+r)P_t - 2\zeta_t\lambda d_t^i = 0 \tag{2.20}$$

i.e.,

$$d_t^i = \frac{(1+\rho)E^i(P_{t+1}|I_t) - (1+r)P_t}{2\zeta_t\lambda} \tag{2.21}$$

where $E^i(.|I_t)$ denotes the expectation of an agent of type i. Let k_t be the proportion of fundamentalists at time t, the market demand is then given by:

$$d_t = \frac{(1+\rho)\left(k_t E^f(P_{t+1}|I_t) + (1-k_t)E^c(P_{t+1}|I_t)\right) - (1+r)P_t}{2\zeta_t\lambda} \tag{2.22}$$

Now consider the supply of foreign exchange X_t and recall that agents only differ in their forecasts as to the value of the future exchange rate, then the market is in equilibrium if $X_t = d_t$, which gives

$$(1+r)P_t = (1+\rho)\left(k_t E^f(P_{t+1}|I_t) + (1-k_t)E^c(P_{t+1}|I_t)\right) - 2\zeta_t\lambda X_t \tag{2.23}$$

[7] Of course, in fact, the variance of P_{t+1} is time dependent, but we can pardon the agents for making this mistake in the second moment if we accept "noise traders" who fail even to observe the correct mean!

In Gaunersdorfer and Hommes (2000) X_t is set to zero. In that case, we refer to d_t as excess demand. We suppose here that $2\zeta_t \lambda X_t/(1+\rho) = \gamma \bar{P}_t$. If we assume that $M_f = M_c = 1$, then the equilibrium price is given by

$$P_t = \frac{k_t - \gamma}{A}\bar{P}_t - \frac{k_t \nu_1}{A}\bar{P}_{t-1} + \frac{(1-k_t)h_1}{A}P_{t-1} \tag{2.24}$$

where

$$A = \frac{1+r}{1+\rho} - (1-k_t)h_0 - k_t \nu_1 \tag{2.25}$$

Thus, when k_t switches from 0 to 1 and vice–versa, equation (2.24) defines a change point process in the conditional mean. It is then of interest to test whether this process generates changes in regime in the conditional variance.

3 Simulations and testing

The series P_t generated by the underlying Markov process described above have the following features:

i) time varying coefficients,

ii) unit roots,

iii) heteroskedastic errors,[8] the conditional heteroskedastic structure having varying coefficients,

iv) long-range dependence in the power transformation of the returns $|R_t|^\delta$, where $R_t = \Delta \ln(P_t)$ and $\delta > 0$.

In fact, the series $|R_t|^\delta$ share the second moment properties of long-memory processes, i.e., a slow hyperbolic rate of decay of the autocorrelation function, henceforth ACF, $\rho(k) = O(k^{2d-1})$, where $d \in (0, 1/2)$ is the parameter which governs the slow rate of decay of the ACF and then parsimoniously summarizes the degree of long-range dependence of the series. Interested readers are referred to Beran (1994) and Robinson (1994) for a complete reference on long-memory processes. These empirical features are well captured by the class of long-memory ARCH, ARCH(∞), processes introduced by Robinson (1991) and developed by Granger and Ding (1995) and other authors. The general form of an ARCH(∞) process is:

$$R_t = m(R_t) + \sigma_t \varepsilon_t, \quad \varepsilon_t \sim D(0,1), \quad \sigma_t^\delta = \omega + \psi(L)g(\varepsilon_t) \tag{3.1}$$

where $m(R_t)$ denotes the regression function, $\psi(L) = \sum_{i=1}^{\infty} \psi_i L^i$ is an infinite order lag polynomial the coefficients of which are positive and have asymptotically the following hyperbolic slow rate of decay $\psi_j = O\left(j^{-(1+d)}\right)$, $g(\varepsilon_t)$ is a function of the innovations ε_t including non-linear transformations, see Teyssière (2001), $D(0,1)$ is a distribution with mean equal to zero and variance equal to one, and $\sigma_t^\delta - g(\varepsilon_t)$ is a martingale difference sequence. The memory properties of the class of ARCH(∞) model have been studied by Giraitis, Kokoszka and Leipus (2000),

[8]These properties are shared by the high frequency data from the London FOREX market, see Goodhart *et al.* (1993).

Kazakevičius and Leipus (2002), Kazakevičius, Leipus and Viano (2000), and Giraitis and Surgailis (2002). This long-range dependence in the conditional second moments can alternatively be modeled by the class of long-memory stochastic volatility processes, interested readers are referred to Robinson (2001) for a complete theory on the memory properties of LMSV models.

However, the first moments of the series $|R_t|^\delta$ differ from the ones of a long-memory process: while a long-memory process exhibits local trends, the slope of which is increasing with the parameter d, the series $|R_t|^\delta$ is not trended.[9] For that reason, we term this type of processes as "pseudo long-memory processes".

The parameters of the model are tuned so that these properties are satisfied. As in Kirman and Teyssière (2002), we test that the returns series is $I(0)$ by using the statistics proposed by Lo (1991), Kwiatkowski *et al* (1992), and Giraitis, Kokoszka, Leipus and Teyssière (2001). These statistics are based on the partial sum process $S_k = \sum_{t=1}^{k}(y_t - \bar{y})$ and the assumption that under the null hypothesis of $I(0)$, the standardised partial sum process satisfies a functional central limit theorem. Lo (1991) considered the standardised range of S_k, i.e.,

$$\tilde{Q}_n(q) = \frac{1}{\hat{\sigma}(q)} \left[\max_{1 \le k \le T} S_k - \min_{1 \le k \le T} S_k \right] \tag{3.2}$$

Kwiatkowski *et al* (1992) considered the standardised second moment of S_k:

$$KPSS(q) = \frac{1}{T^2 \hat{\sigma}^2(q)} \sum_{k=1}^{T} S_k^2 \tag{3.3}$$

while Giraitis *et al.* (2001) considered the standardised variance of S_k:

$$V/S(q) = \frac{1}{T^2 \hat{\sigma}^2(q)} \left[\sum_{k=1}^{T} S_k^2 - \frac{1}{T} \left(\sum_{k=1}^{T} S_k \right)^2 \right] \tag{3.4}$$

where $\hat{\sigma}^2(q)$ is the Newey and West (1987) heteroskedastic and autocorrelation consistent (HAC) variance estimator:

$$\hat{\sigma}^2(q) = \hat{\gamma}_0 + 2 \sum_{i=1}^{q} \omega_i(q) \hat{\gamma}_i \quad \text{with} \quad \omega_i(q) \equiv 1 - \frac{1}{q+1} \tag{3.5}$$

where the sample auto-covariances $\hat{\gamma}_i$ at lag i account for the possible short-range dependence up to the q^{th} order. The V/S statistic is less sensitive to the choice of the truncation order q than Lo's (1991) statistic, and is more powerful than the KPSS statistic. We use the same tests for checking that the power transformation of returns display long-memory.

In Kirman and Teyssière (2002), we have shown that this model is able to generate the sort of strong dependence observed in the volatility of asset returns, and that the degree of long–memory is controlled by the swings in opinions, i.e., the evolution of the process governing k_t. It is well known that statistical tests wrongly detect long–range dependence when the true DGP is a change–point process. In particular, according to Mikosch and Stărică (1999), long-range dependence in the volatility process can be spurious and the consequence of the concatenation of short-range dependent GARCH(1,1) processes with changing coefficients. Given that in our

[9] We are grateful to Clive Granger for pointing us out these features.

model, asset prices P_t are a varying combination of the previous prices P_{t-1} and the fundamentals $\bar{P}_t$ and $\bar{P}_{t-1}$, it is interesting to check whether we are able to detect the changes in the proportion of fundamentalists k_t, i.e., the swings in opinion, which generate the long-memory in the volatility process. There is a substantial literature on change–point analysis, see e.g., Csörgő and Horváth (1997), Besseville and Nikifirov (1993) for recent references, which focuses on conditional mean processes. In our case, we are interested in the conditional variance processes, and we then use the tests for change–point in the conditional variance proposed by Kokoszka and Leipus (2000), Horváth, Kokoszka and Teyssière (2001) and Kokoszka and Teyssière (2001). A recent survey on change–point tests is given in Kokoszka and Leipus (2002). We compare the relative performance of these tests with respect to our non–standard DGP.

Kokoszka and Leipus (2000) proposed a CUSUM based estimator for change-point in ARCH(∞) processes at unknown time t. This estimator is defined by:

$$\hat{t} = \min\left\{t : |C_t| = \max_{1\leq j\leq T} |C_j|\right\} \tag{3.6}$$

where

$$C_t = \frac{t(T-t)}{T^2}\left(t^{-1}\sum_{j=1}^{t} R_j^2 - \frac{1}{T-t}\sum_{j=t+1}^{T} R_j^2\right) \tag{3.7}$$

Horváth, Kokoszka and Teyssière (2001) proposed several tests for change–point in ARCH sequences, based on the empirical process of standardised residuals. According to Kokoszka and Teyssière (2001), these tests work well for GARCH(1,1) sequences and do have a limit distribution under the null hypothesis, although a bootstrap based inference gives better results than asymptotic inference. The first statistic is a Kolmogorov-Smirnov type statistic. For $1 \leq k \leq T$, define

$$\hat{T}(k,t) = \sqrt{T}\,\frac{k}{T}\left(1-\frac{k}{T}\right)\left|\hat{F}_k(t) - \hat{F}_k^*(t)\right| \tag{3.8}$$

with

$$\hat{F}_k(t) = \frac{1}{k}\#\{i \leq k : \hat{\varepsilon}_i^2 \leq t\}, \quad \hat{F}_k^*(t) = \frac{1}{T-k}\#\{i > k : \hat{\varepsilon}_i^2 \leq t\} \tag{3.9}$$

where the $\hat{\varepsilon}_t^2$ denote the sequence of squared standardised residuals of a GARCH(1,1) process fitted on the simulated returns, i.e.,

$$R_t = m + \varepsilon_t, \quad \varepsilon_t \sim N(0,\sigma_t^2), \quad \sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\varepsilon_{t-1}^2 \tag{3.10}$$

The K–S statistic is defined as

$$\hat{M} = \sup_{0\leq t\leq\infty}\ \max_{1\leq k\leq T} |\hat{T}(k,t)| \tag{3.11}$$

According to Horváth, Kokoszka and Teyssière (2001), for ARCH(p) models with finite fourth moment the asymptotic distribution of $\hat{M}$ is the same as for Picard's (1985) generalized Kolmogorov-Smirnov statistic.

Horváth, Kokoszka and Teyssière (2001) proposed also a Cramér – von Mises statistic:

$$\hat{B} = \int_0^1 \left\{\frac{1}{T}\sum_{i=1}^{T}[\hat{T}([Ts],\hat{\varepsilon}_i^2)]^2\right\} ds \tag{3.12}$$

The distribution function of B can be derived from Blum, Kiefer and Rosenblatt (1961). Kokoszka and Teyssière (2001) proposed other tests for change–point in GARCH(1,1) processes, that are considered in Kirman and Teyssière (2001).

These tests consider the case of a single change–point. Test for multiple change–points in conditional mean processes have been proposed by Lavielle (1999) and Lavielle and Moulines (2000) among others. Given that the series of k_t switch from one extreme to the other, it is to be expected that the simulated process P_t will have several change–points for large series. Thus, we restrict our analysis to samples of 500 observations to reduce the probability of occurrence of multiple change–points.

The details of the simulation are as follows:

- $T = 500$, (sample size),
- number of simulated series = 10000,
- number of agents, $N = 1000$,
- the number of fundamentalists at the beginning of the process = $N/2$, thus $k_0 = 0.5$,
- $P_0 = 1000$, $\bar{P}_0$ =1050,
- $h_0 = 0.625$, $h_1 = 0.375$, $\nu_1 = 0.62$,
- $\delta = 0.10$,
- annual foreign interest rate $\rho = 0.07$,
- annual domestic interest rate $r = 0.04$,

The series of error terms used in this simulation have been generated by using the Box-Muller transformation of a sequence of uniform deviates which succeeds Marsaglia's (1996) DIEHARD randomness tests.

A series of standard tests were now run on the simulated data to see if it was possible to reject alternative specifications which have different statistical characteristics from those of the process used in the model. In particular it is interesting to see if an econometrician faced with this data would have been led to reject the random walk hypothesis and whether he would have been able to detect the changes in regimes. Once again, it is important to emphasize that the interest of this exercise is to see what the results are when the test is applied to data which are, in fact, generated by an "inappropriate" process, although this would not be known a priori to the tester.

Table 1 below reports the results of the tests for long-range dependence on the series of simulated returns, absolute and squared returns. The results for the series of returns are close to the ones reported in Giraitis *et al.* (2001) for *i.i.d.* white noise processes of the same sample size. These tests detect long–memory in the simulated series of absolute returns and squared returns. The tests reject the null hypothesis of $I(0)$ more often for absolute returns than for squared returns, in accordance with empirical results on financial series. As with standard long range dependent processes, the V/S statistic appears to be more powerful than the KPSS statistic and less sensitive to the choice of q than Lo's (1991) statistic.

Table 2 below reports the empirical power of the CVM and KS tests for change–point in the conditional variance of the simulated returns. These tests detect a change in regime, although

Table 1: Tests for long-range dependence on the series of simulated returns R_t, absolute simulated returns $|R_t|$, and squared simulated returns R_t^2. Test size 5%.

	R_t: Pr$(d=0)$			$\|R_t\|$: Pr$(d>0)$			R_t^2: Pr$(d>0)$		
q	Lo	KPSS	V/S	Lo	KPSS	V/S	Lo	KPSS	V/S
0	0.9499	0.9485	0.9480	0.9447	0.9054	0.9441	0.9296	0.8917	0.9331
1	0.9556	0.9474	0.9519	0.9281	0.8894	0.9315	0.9043	0.8712	0.9155
2	0.9633	0.9447	0.9537	0.9166	0.8774	0.9228	0.8874	0.8564	0.9049
3	0.9666	0.9441	0.9562	0.9054	0.8661	0.9157	0.8718	0.8417	0.8950
4	0.9699	0.9444	0.9573	0.8924	0.8544	0.9090	0.8557	0.8297	0.8843
5	0.9709	0.9438	0.9594	0.8799	0.8445	0.9002	0.8388	0.8184	0.8749
6	0.9733	0.9428	0.9607	0.8679	0.8325	0.8912	0.8245	0.8067	0.8653
7	0.9756	0.9424	0.9607	0.8575	0.8200	0.8834	0.8103	0.7942	0.8556
8	0.9759	0.9430	0.9615	0.8423	0.8099	0.8732	0.7919	0.7828	0.8450
9	0.9758	0.9417	0.9623	0.8292	0.8009	0.8623	0.7729	0.7728	0.8348
10	0.9770	0.9420	0.9629	0.8149	0.7903	0.8528	0.7551	0.7627	0.8267

these tests are very sensitive to the magnitude of the change in the unconditional variance, i.e., the power of the test is equal to its size when the unconditional variance is constant. Interested readers are referred to Kokoszka and Teyssière (2001) for further details.

Table 2: Tests for change–point in the conditional variance of simulated returns R_t. Probability of a change–point. Test size 5%.

CVM	KS
0.2062	0.2558

Interestingly, the parameters of the estimated GARCH(1,1) process fitted on the series of simulated returns R_t are close to the empirical values of the estimated parameters on real data, i.e., $\beta + \alpha$ is close to one. On average $\hat{\beta}$ is equal to 0.7988 while $\hat{\alpha}$ is equal to 0.0934, i.e., $\hat{\alpha} + \hat{\beta}$ is close to 0.9 and tends to 1 when the sample size increases, e.g., with samples of 1000 observations $\hat{\beta} + \hat{\alpha} = 0.964866$. Our model is able to replicate the standard property of occurence of an IGARCH process, i.e., $\alpha + \beta = 1$ for large samples. Interested readers are referred to Kirman and Teyssière (2002, 2001) for the large sample properties of this model.

Figures 1 and 2 below give two examples of the test by Kokoszka and Leipus (2000) for detecting a change in regime in the conditional variance process. These pictures display the series of k_t, i.e., the proportion of fundamentalists. The vertical line shows the point where the change in regime has been detected. These examples show that changes in regime are detected, which correspond to a switch in the evolution of the process k_t. Thus, the change–point test by Kokoszka and Leipus (2000) can be used for detecting bubbles in asset prices.

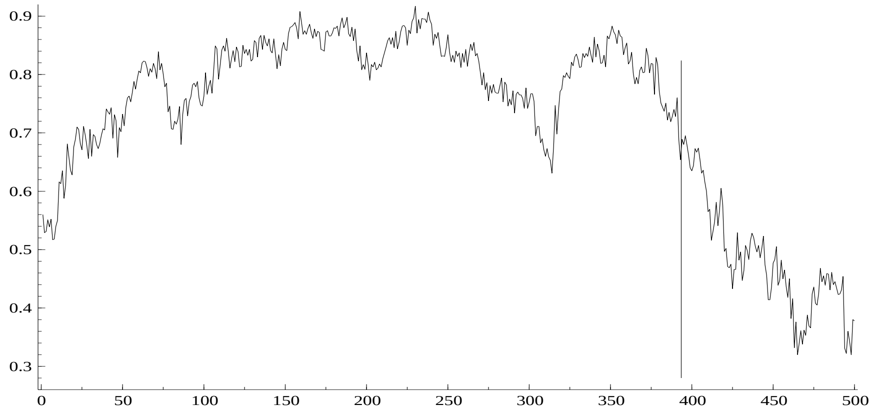

Figure 1: A change–point has been detected at time $t = 394$

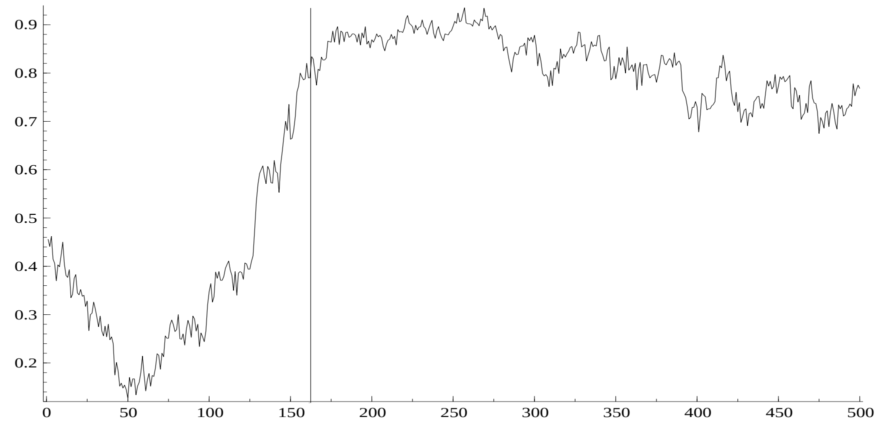

Figure 2: A change–point has been detected at time $t = 166$

4 Conclusion

In this paper we have used a specific model of the microbehaviour of interacting individuals to generate aggregate price behaviour for the market for a financial asset. The prices are generated by a rather complicated stochastic process and exhibit bubble-like phenomena. Standard tests used to detect bubbles in the past would have led one to deny the existence of these bubbles. What is really happening in the model is that there are switches of regime and therefore that one should use tests to detect these rather than trying to decide if the process over the whole period exhibits the characteristics of a random walk. Such tests which have recently been developed by one of the authors and his collaborators are used here and detect the onset and bursting of bubbles. Furthermore the process generates time series whose properties correspond very well to those of empirical data.

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