

# Loss Reduction and Implicit Deductibles in Medical Insurance <sup>1</sup>

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## Abstract

This note presents a modest extension of the very useful “theorem of the deductible” (Arrow, *American Economic Review*, 1963). The extension concerns *ex post* moral hazard in medical insurance. Under full insurance above a deductible, the marginal cost of treatment to the insured is zero, resulting in over-consumption. Co-insurance is the standard approach to mitigate that problem. Assuming that resources and consumption preferences are independent of health (to separate medical insurance from disability insurance), the optimal co-insurance contract results in the same indemnities as a contract with 100% coverage above a variable deductible, related positively to the elasticity of medical expenditures with respect to the coverage rate.

**Key words:** health insurance, deductibles, moral hazard.

# 1 Introduction

“If an insurance company is willing to offer any insurance policy against loss desired by the buyer at a premium which depends only on the policy’s actuarial value, then the policy chosen by a risk-averting buyer will take the form of 100% coverage above a deductible minimum.” (Arrow, 1963).

The elegant “theorem of the deductible” is one of the more practically useful pieces of economic theorising known to me. Unfortunately, its realm of applicability remains limited. First, the theorem does not hold when preferences are state dependent. Second, it does not apply under moral hazard. Accordingly, it does not apply to medical insurance - the very application which motivated the formulation of the theorem!

Moral hazard affects health insurance at two levels: prevention and treatment, corresponding respectively to moral hazard *ex ante* (before occurrence or diagnosis of sickness) and *ex post*; see, e.g. Eeckhoudt *et al* (1998). This note is primarily addressed to *ex post* moral hazard. Typically, the insurer observes medical expenses, but not diagnostics or patient conditions. If the policy stipulates full reimbursement of expenses beyond a deductible, the *ex post* marginal cost to the insured of additional treatment is zero, leading to over-consumption. When there exist alternative treatments (for instance, hospitalisation versus home care), their costs and benefits should be assessed, with additional costs accepted only when justified by the additional benefits.<sup>1</sup> “100% coverage above a deductible” eliminates the incentives (of both patients and providers of care) to arbitrate costs and benefits at the margin.

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<sup>1</sup>See remark (iii) in section 2.2.

The standard avenue to cope with this moral hazard problems is “co-insurance”: the policy stipulates the fraction  $\alpha$ ,  $1 \geq \alpha \geq 0$ , of expenses reimbursed by the insurer. Second-best analysis investigates the level  $1-\alpha$  of co-insurance that strikes the optimal balance between risk sharing (calling for  $\alpha$  close to 1) and incentives (calling for  $\alpha$  close to 0 in order to avoid over-consumption, i.e. to implement “loss reduction”); see e.g. Winter (1992, 2000), Zweifel and Breyer (1997) or Feldman and Manning (1997).

The purpose of this note is to bring to light the property, apparently not on record to date, that *the logic of the theorem of the deductible remains at work under second-best co-insurance*. It takes the form of the “implicit deductible property” stated and documented in section 3 below. Section 2 paves the ground by defining a medical insurance problem and noting three relevant features. Section 4 deals with implementation and extensions.

## **2 A Medical Insurance Problem**

### **2.1 Utility analysis of medical expenditures**

A simple medical insurance problem concerns an individual facing uncertainty about her future health condition. There are  $S + 1$  possible “states of health” indexed  $s = 0, 1, \dots, S$ . State 0 calls for no treatment. For  $s \geq 1$ , medical expenditures  $M_s$  are apt to improve the well-being of the individual.

These features are readily modeled within the expected utility frame-

work. One starts from conditional preferences between vectors  $(M_s, C_s) \in R_+^2$ , where  $M_s$  stands for medical expenditures and  $C_s$  for expenditures on consumption exclusive of medical expenditures. It is assumed that conditional preferences are represented by state-dependent utility functions  $U^s(M_s, C_s)$ , endowed with standard properties (continuity, concavity, here also differentiability).<sup>2</sup> The theory of decision with state-dependent preferences, surveyed for instance in Drèze and Rustichini (2001), leads to an expected-utility representation of *ex ante* preferences among  $(S + 1)$ -tuples of vectors  $(M_s, C_s)$ , namely:

$$V\left((M_s, C_s), s = 0, 1, \dots, S\right) = \sum_s p_s U^s(M_s, C_s), \quad (1)$$

where  $p = (p_0, p_1 \dots p_S)$  denotes *ex ante* probabilities of the states. In the additive representation (1),  $V$  is defined up to a single, positive affine transformation:  $aV + b$ ,  $a > 0$ , is equally acceptable.<sup>3</sup>

This general formulation places no cross-restrictions on the conditional preferences: the functions  $U^s(\cdot)$ ,  $U^t(\cdot)$ ,  $s \neq t$ , need not bear any particular relation to each other. In particular consumption preferences given state  $s$  and medical expenditures  $\overline{M}_s$  are represented by the car-

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<sup>2</sup>A trivial extension of the formulation recognises that the effect of medical expenditures on the person's "state of health" is itself uncertain.

<sup>3</sup>The state-dependent utilities  $U^s, U^t$  are thus defined up to a *common* affine transformation, so that marginal rates of substitution are defined uniquely – for instance,

$$\frac{\frac{\partial V}{\partial C_s}}{\frac{\partial V}{\partial C_t}} = \frac{p_s \frac{\partial U^s}{\partial C_s}}{p^t \frac{\partial U^t}{\partial C_t}}$$

is invariant to  $(a, b)$ . As explained, for instance, in Gollier (2001, sections 1.2-1.3), increasing non-linear transformations of the functional  $V$  are order-preserving, but incompatible with the *additive* representation (1).

dinal utility function  $U^s(C_s; \overline{M}_s)$  whose properties (risk aversion) are unrestricted and could be quite different from those of  $U^t(C_t; \overline{M}_t)$ ,  $t \neq s$ , or of  $U^s(C_s; \hat{M}_s)$ ,  $\hat{M}_s \neq \overline{M}_s$ . To illustrate, if state  $s$ , and only state  $s$ , were to stipulate that the individual is blind, it could be that  $U^s(M_s, C_s) = U^s(0, C_s)$  identically in  $M_s \geq 0$ ; yet,  $U^0(0, C_0)$  could be quite different from  $U^s(0, C_s)$ . For instance,  $U^s(0, C_s)$  could exhibit more risk aversion at low values of  $C_s$  (reflecting fixed expenses for daily assistance, starting with a trained dog) but less risk aversion at high values of  $C_s$  (reflecting limited access to more sophisticated consumption patterns, like skiing vacations). A special case of substantive interest imposes that the functions  $U^s(M_s, C_s)$  be additive. More on this below; see (10)-(11).

A more restricted formulation, that implies cross-restrictions on conditional preferences, introduces a state-dependent “health variable” related to medical expenditures,  $h^s(M_s)$ . It is then assumed that

$$U^s(M_s, C_s) \equiv U\left(h^s(M_s), C_s\right), \quad (2)$$

where the function  $U(h^s, C_s)$  is state-independent – subject to the remark that the range of the health variable  $h^s(\cdot)$  is apt to be state-dependent. If moreover the function  $U$  is additive, then consumption preferences are independent of health state.

## 2.2 First-best medical insurance

Let non-medical consumption in state  $s$  be equal to wealth  $W_s$  minus medical expenditures  $M_s$ , in the absence of medical insurance. And let insurance provide indemnities  $I = (I_0, I_1, \dots, I_S)$  against a premium  $\pi(I)$ .

Expected utility is then given by

$$V(\cdot) = \sum p_s U^s(M_s, W_s - M_s - \pi + I_s). \quad (3)$$

The problem of optimal insurance design, for a given relationship between premium and indemnities, calls for maximising (3) with respect to  $M$  and  $I$  subject to  $\pi = \pi(I)$ .

The first-order conditions are

$$\frac{\partial V}{\partial M_s} = p_s(U_M^s - U_W^s) = 0 \quad (4.a)$$

$$\frac{\partial V}{\partial I_s} = p_s U_W^s - \sum_t p_t U_W^t \frac{\partial \pi}{\partial I_s} = 0 \quad (4.b)$$

where  $U_M^s, U_W^s$  denote the partial derivatives of  $U$  with respect to its first and second arguments respectively. ( $U_W^s$  is equivalent to  $U_C^s$ , but more suggestive.)

In the special case of a linear premium,

$$\pi(I) = (1 + \lambda) \sum_s p_s I_s, \quad (5)$$

where  $\lambda \geq 0$  is a loading factor. Expressing  $I_s$  in terms of co-insurance,<sup>4</sup>

$$I_s = \alpha_s M_s, \quad C_s = W_s - \pi - M_s(1 - \alpha_s). \quad (6)$$

Using (5) and (6), the problem (3) takes the explicit form

$$\begin{aligned} \max_{M \geq 0, \alpha \geq 0} \Lambda = & \sum_s p_s U_s(M_s, W_s - \pi - M_s(1 - \alpha_s)) \\ & - \lambda^1 \left( (1 + \lambda) \sum_s p_s \alpha_s M_s - \pi \right) \end{aligned} \quad (7)$$

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<sup>4</sup>My use of the term ‘‘co-insurance’’ allows for  $\alpha_s \neq \alpha_t$ .

with first-order conditions

$$\frac{\partial \Lambda}{\partial M_s} = p_s [U_M^s - (1 - \alpha_s)U_W^s] - \lambda^1(1 + \lambda)p_s\alpha_s \leq 0, \quad M_s \frac{\partial \Lambda}{\partial M_s} = 0 \quad (8.a)$$

$$\frac{\partial \Lambda}{\partial \alpha_s} = p_s U_W^s M_s - \lambda^1(1 + \lambda)p_s M_s \leq 0, \quad \alpha_s \frac{\partial \Lambda}{\partial \alpha_s} = 0. \quad (8.b)$$

Taken together, these conditions imply,  $\forall s, t = 0, 1 \dots S$  such that  $M_s > 0, M_t > 0$ :

$$U_M^s = U_W^s = U_W^t. \quad (9)$$

These conditions characterise the first-best solution. They have two aspects. First  $U_M^s = U_W^s$  means that the marginal rate of substitution between medical expenditure and other consumption is unity, so there is *no over-consumption*. Second,  $U_W^s = U_W^t$  means state-independent marginal utility of consumption, a property further discussed under (ii) below. These conditions ignore the *ex post* moral hazard problem: once the premium  $\pi$  has been paid, the individual may disregard the link between the premium and expected expenditures, and  $\lambda^1 = 0$  in (8.a). Hence,  $U_M^s = (1 - \alpha_s)U_W^s$ . In the limit, if  $\alpha_s = 1$ , the over-consumption brings  $U_M^s$  to zero. The design of a second-best insurance policy, taking the *ex post* moral hazard into account, is discussed in section 3.

The characterisation of the first-best insurance contract invites three remarks.

(i) The characterisation is couched in terms of marginal utilities, or rather marginal rates of substitution. Utility *levels*  $U^s(M_s, C_s), U^t(M_t, C_t)$  play no role: they are irrelevant to the insurance problem. Optimal insurance does not compensate the individual for the utility loss associated with blindness; it simply redistributes resources across states so as

to equate marginal utilities of consumption. The absolute utility levels matter only to decisions on *prevention*, namely effort or expenditures aimed at reducing the *probabilities* of states with low levels of utility.

(ii) Equating marginal utilities across states has two aspects. *First*, indemnities covering medical expenditures stabilise resources available for consumption. Other things equal (including  $W_s = W_t \forall s, t$ ), coverage rates implying  $M_s(1 - \alpha_s) = M_t(1 - \alpha_t)$ ,  $C_s = C_t \forall s, t$  would be optimal on this score. *Second*, an optimal policy implements transfers from states with low marginal utilities to states with high marginal utilities *at equal consumption levels*. These transfers are *additional to* and *independent of* coverage of medical expenditures. They do not correspond to “medical insurance” in the ordinary sense of the word. Rather, they correspond to a form of “disability insurance”; a form limited, however by remark (i) above: these transfers equate marginal utilities, not utility levels. It is natural to draw a logical distinction between coverage of medical expenditures, i.e. medical insurance *sensu stricto*; and equalisation of marginal utilities across states, i.e. a form of disability insurance.<sup>5</sup> Such a distinction is immediate (unnecessary) when the state-dependent utilities are *additive with state-independent consumption preferences*, i.e. when

$$U^s(M_s, C_s) = h^s(M_s) + g(C_s). \quad (10)$$

The element of “disability insurance” disappears, since (10) implies

$$U_W^s = U_W^t \iff C_s = C_t. \quad (11)$$

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<sup>5</sup>For a related distinction in property insurance, but without *ex post* moral hazard, see Garratt and Marshall (1996).

Note that in general the “disability” transfers are unrestricted as to sign. Zweifel and Breyer (1997, p. 187) assume that “with improvement in health the capacity to enjoy consumption increases”. That is clearly a special assumption – as suggested above by the remark that a blind person may face non-medical expenses for daily assistance. Also, labour income may be linked to health condition, so that  $W_s \neq W_t \forall s, t$ . Compensation of income losses due to poor health is another compartment of disability insurance, as distinct from medical insurance.

(iii) As a trivial implication of the statement of problem (7), a medical expenditure in state  $s$  is warranted, not if the individual experiencing state  $s$  is willing to incur the expenditure, but rather if the individual contracting an insurance policy *ex ante* is willing to face in all states the incremental premium linked to the contingent expenditure. The availability of insurance thereby expands significantly the scope for treatment.

### 3 The Implicit Deductible Property

#### 3.1 Second-best co-insurance

As noted above, the first-order condition for optimal *ex post* medical expenditure in state  $s$  is simply

$$\frac{dU^s}{dM_s} = U_M^s - (1 - \alpha_s)U_W^s \leq 0, \quad M_s \frac{dU^s}{dM_s} = 0. \quad (12)$$

The second-order condition is

$$\frac{d^2U^s}{dM_s^2} = U_{MM}^s - 2(1 - \alpha_s)U_{WM}^s + (1 - \alpha_s)^2 U_{WW}^s < 0. \quad (13)$$

Differentiation of (12) in equality form yields, for  $M_s > 0$  :

$$\begin{aligned}\frac{dM_s}{d\alpha_s} &= -\frac{M_s[U_{WM}^s - (1 - \alpha_s)U_{WW}^s] + U_W^s}{\frac{d^2U^s}{dM_s^2}} \\ &= -\frac{[M_s\frac{dU_W^s}{dM_s} + U_W^s]}{\frac{d^2U^s}{dM_s^2}} > 0.\end{aligned}\quad (14)$$

Taking the *ex post* incentive compatibility condition (12) into account, the optimal co-insurance contract with  $1 \geq \alpha_s \geq 0 \forall s$  solves

$$\begin{aligned}\max_{M \geq 0, \alpha \geq 0} \Lambda &= \sum_s p_s U^s(M_s, W_s - \pi - (1 - \alpha_s)M_s) \\ &\quad - \lambda^1[(1 + \lambda) \sum_s p_s \alpha_s M_s - \pi] \\ &\quad - \sum_s \lambda_s M_s [U_M^s - (1 - \alpha_s)U_W^s] \\ &\quad + \sum_s \mu_s (1 - \alpha_s).\end{aligned}\quad (15)$$

The first-order conditions are:

$$\frac{\partial \Lambda}{\partial M_s} = -\lambda^1(1 + \lambda)p_s \alpha_s - \lambda_s M_s \frac{d^2U^s}{dM_s^2} \leq 0, \quad M_s \frac{\partial \Lambda}{\partial M_s} = 0 \quad (16.a)$$

$$\begin{aligned}\frac{\partial \Lambda}{\partial \alpha_s} &= p_s U_W^s M_s - \lambda^1(1 + \lambda)p_s M_s - \lambda_s M_s \left[ M_s \frac{dU_W^s}{dM_s} + U_W^s \right] - \mu_s \leq 0, \\ \alpha_s \frac{\partial \Lambda}{\partial \alpha_s} &= 0.\end{aligned}\quad (16.b)$$

To lift inconsequential indeterminacy, I set  $\alpha_s = 0$  whenever  $M_s = 0$ .

*Proposition 1: There exists a partition of the set of states  $\Sigma$  into three subsets  $\{\Sigma_0, \Sigma_\alpha, \Sigma_1\}$  such that:*

*on  $\Sigma_0$ ,  $\alpha_s = 0$ , and  $U_W^s < \lambda^1(1 + \lambda)(1 + \eta_s)$*

*on  $\Sigma_\alpha$ ,  $1 > \alpha_s > 0$  and  $U_W^s = \lambda^1(1 + \lambda)(1 + \eta_s)$*

*on  $\Sigma_1$ ,  $\alpha_s = 1$  and  $U_W^s > \lambda^1(1 + \lambda)(1 + \eta_s)$*

*where  $\eta_s := \frac{\alpha_s}{M_s} \frac{dM_s}{d\alpha_s}$  = the elasticity of medical expenditure with respect to the coverage rate  $\alpha_s$ .<sup>6</sup>*

*Proof:* Let  $1 > \alpha_s > 0$ . Substituting for  $\lambda_s$  from (16.a) into (16.b), using (14), and collecting terms,

$$U_W^s = \lambda^1(1 + \lambda) \left\{ 1 - \frac{\alpha_s}{M_s} \frac{[M_s \frac{dU_W^s}{dM_s} + U_W^s]}{\frac{d^2 U^s}{dM_s^2}} \right\} := \lambda^1(1 + \lambda)(1 + \eta_s).$$

On  $\Sigma_0$ ,  $U_W^s$  is  $\leq$  the r.h.s.; on  $\Sigma_1$  one adds  $\frac{\mu_s}{p_s M_s}$ . ■

When  $\eta_s \equiv 0 \forall s$ , proposition 1 characterises the first-best solution.

### 3.2 Implicit deductibles

More specific results follow from special assumptions, motivated by the desire to concentrate on medical insurance as distinct from disability insurance.

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<sup>6</sup>A similar characterisation applies to optimal co-insurance under *ex ante* moral hazard with observable preventive effort and no *ex post* moral hazard (in which case  $\eta_s \equiv 0$ .)

*Corollary 1* (implicit deductible property)

Assume (A1) resources are state-independent i.e.  $W_s = W_t \forall s, t$ ; and (A2) preferences are separable, with state-independent consumption preferences, i.e.  $U^s(M_s, C_s) = f^s(M_s) + g(C_s)$ ; then the optimal co-insurance contract results in the same indemnities as a contract with 100% coverage above a variable deductible related positively to  $\eta_s$ , the elasticity of medical expenditures with respect to the coverage rate  $\alpha_s$ .

*Proof* The property follows from proposition 1, since the assumptions (A1) and (A2) imply that  $U_W^s$  is a function of  $M_s(1 - \alpha_s)$  alone. The positive association follows from  $U_{WW}^s < 0$  (risk aversion).  $\sum_1 = \emptyset$  follows from the fact that, for  $s \in \sum_\alpha$ ,  $t \in \sum_1$ ,  $U_W^t > U_W^s$  and  $\alpha_t = 1$  imply  $W_0 - M_s(1 - \alpha_s) > W_0$ , a contradiction. ■

In other words, to a set of states  $\bar{\Sigma}$  such that  $\eta_s = \bar{\eta}$  for all  $s$  in  $\bar{\Sigma}$ , there corresponds a deductible  $\bar{k}$ , and  $M_s(1 - \alpha_s) = \min(M_s, \bar{k}) \forall s \in \bar{\Sigma}$ . The property is implemented by adjusting the coverage rates  $\alpha_s$  in such a way that  $M_s(1 - \alpha_s) \leq \bar{k}$ ,  $\alpha_s = \max(0, 1 - \frac{\bar{k}}{M_s})$ . As a function of medical expenses  $M_s \geq \bar{k}$ ,  $\alpha_s$  is a *concave increasing function*, with  $\alpha_s$  tending to 1 as  $M_s$  tends to infinity. But different values of  $\bar{\eta}$  lead to different coverage-rate functions, since  $\bar{k} = k(\bar{\eta})$ .

The need to assume state-independent resources (A.1) and consumption preferences (A.2) is obvious from the statement of Proposition 1. Otherwise,  $U_W^s$  will depend upon  $s$  at given levels of  $C_s$ , also across states with identical  $\eta_s = \bar{\eta}$ , and the deductible property is lost.

To illustrate the corollary, define  $W_0 = W - \pi$  and consider the first-

order approximation

$$\begin{aligned} U_W^s &= U_W^s(W_0) - M_s(1 - \alpha_s)U_{WW}^s(W_0) \\ &= U_W^s(W_0)[1 + \rho_s R_R^0] \end{aligned} \quad (17)$$

where  $R_R^0$  is the Arrow-Pratt measure of relative risk aversion,  $-\frac{U_{WW}^s \cdot W_0}{U_W^s}$  evaluated at  $W_0$ , and  $\rho_s = \frac{M_s(1-\alpha_s)}{W_0}$  is the share of medical expenditures in disposable income under state  $s$ . Then, for all  $s \in \Sigma_\alpha$ ,

$$U_W^s(W_0)(1 + \rho_s R_R^0) = \lambda^1(1 + \lambda)(1 + \eta_s), \quad \rho_s = \frac{(K - 1) + K\eta_s}{R_R^0} \quad (18)$$

where  $K = \frac{\lambda^1(1+\lambda)}{U_W^s(W_0)}$ , a pure number (see below). Thus, to a first approximation, *the relative implicit deductible is inversely proportional to relative risk aversion and linearly increasing in  $\eta$ .*

The foregoing invites further interpretation of the endogenous parameters  $\lambda^1$  and  $\eta_s$ . Regarding  $\lambda^1$ , it suffices to note from the formulation of problem (15) that

$$\frac{d\Lambda}{d\pi} = -\sum_s p_s U_W^s + \lambda^1, \quad (19)$$

so that  $\lambda^1$  has the dimension of the expected marginal utility of income, and  $\frac{U_W^s}{\lambda^1}$  is a pure number.

Turning to  $\eta_s$ , it is shown in Appendix that, when  $U_{WM}^s = 0$ , then

$$\eta_s = \frac{\alpha_s}{1 - \alpha_s} \frac{1 + \rho_s R_R^s}{\rho_s R_r^s - \frac{U_{MM}^s \cdot M_s}{U_M^s}} \quad (20)$$

The (positive) term  $-\frac{U_{MM}^s \cdot M_s}{U_M^s} := \Gamma_M$  is a measure of the (relative) concavity of  $U^s(M_s, C_s)$  as a function of  $M_s$  - in exactly the same way that  $R_R^s$  is a measure of (relative) concavity with respect to  $C_s$  (or  $W_s$ ).

Empirical estimates are typically reported for elasticities with respect to rates of co-insurance,  $1 - \alpha_s$ , rather than with respect to  $\alpha_s$ . And  $\eta_{M \cdot (1-\alpha)} = \frac{1-\alpha}{\alpha} \eta_{M \cdot \alpha}$ . Given empirical estimates of  $\eta_s \frac{1-\alpha_s}{\alpha_s}$  in the range (.1, .2),<sup>7</sup> one would surmise that  $\Gamma_M$  is definitely greater than 5, possibly by a wide margin. I am not aware of direct empirical estimates of that intriguing parameter.

## 4 Implementation and Extensions

The formulae in section 3 do not lend themselves easily to implementation. Their usefulness is qualitative. First, they explain how the logic of the theorem of the deductible remains at work under *ex post* moral hazard. Second, they validate the practice of higher *coverage rates* (not only indemnities) for major medical expenses. Third, they confirm the intuition that coverage rates should be inversely related to demand elasticity (the elasticity of expenditures with respect to coverage), and positively related to risk aversion.

A contract capturing the gist of proposition 1 would rely on a two-way classification of medical expenditures: by *amounts*, and by *types of treatment*. The treatments should be partitioned on a basis apt to reflect demand elasticity. Easier said than done – but a clear invitation to collect data on demand elasticities by types of treatment.

When coverage rates are held constant over a set of states, the properties listed in proposition 1 hold in terms of averages. To illustrate,

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<sup>7</sup>See, e.g., Manning *et al* (1987) or Feldman and Manning (1997).

let there be three coverage rates, namely 0,  $\alpha$ , and 1, with  $1 > \alpha > 0$ , applied to states  $s$  in  $\Sigma_0, \Sigma_\alpha$  and  $\Sigma_1$ , respectively. The first-order condition on  $\alpha$  is then obtained from summing over  $s$  in  $\Sigma_\alpha$  the right-hand side of (16.b), which yields

$$\sum_{s \in \Sigma_\alpha} p_s M_s [U_W^s - \lambda^1 (1 + \lambda) (1 + \eta_s)] = 0. \quad (21)$$

This, together with  $U_W^r > U_W^s > U_W^t$  for  $r \in \Sigma_1, s \in \Sigma_\alpha, t \in \Sigma_0$ , characterises the third-best contract.

Two analytical extensions are needed. The first concerns aggregation over time; the second concerns aggregation over individuals.

The simple logic of deductibles says that, ideally, they should be defined *across* the various risks borne by an individual. For instance, a *single* deductible should be applied *simultaneously* to home insurance, liability insurance, health insurance, a.s.o.. Short of that ideal, a deductible for health insurance should be applied to, say, annual expenditures rather than case-by-case expenditures. To implement a contract with that feature one would normally proceed in two stages, defining minimal coverage rates applicable to case-by-case expenditures, to be complemented by an end-of-year (say) supplementary progressive coverage based on overall annual expenditures. Modeling such a contract raises new complications, if only because the insured will have to decide on case-by-case medical expenditures under uncertainty about the final coverage rate applicable to them. That extension lies beyond the scope of this paper.<sup>8</sup>

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<sup>8</sup>In Belgium, a “social deductible” was introduced in 1994, whereby co-insurance payments on a class of medical expenses faced by socially disadvantaged households

Aggregation over individuals is more straightforward. Let there be  $H$  individuals indexed  $h = 1 \cdots H$ , each facing the same comprehensive set of health states  $s$ ,  $s = 0, 1 \cdots S$ . Let the coverage rates  $\alpha_s$  and the premium  $\pi$  be identical for all individuals. Indexing utilities, probabilities and medical expenditures by  $h$ , and using unspecified multipliers  $\tau^h$  to aggregate preferences (to characterise second-best Pareto efficiency), the optimal proportional insurance contract with  $1 \geq \alpha_s \geq 0$  solves

$$\begin{aligned}
\max_{M^h \geq 0, \alpha \geq 0} \quad \Lambda_\tau = & \sum_h \tau^h \sum_s p_s^h U^{hs} (M_s^h, W_s^h - \pi - (1 - \alpha_s) M_s^h) \\
& - \lambda^1 \left[ (1 + \lambda) \sum_h \sum_s p_s^h \alpha_s M_s^h - H\pi \right] \\
& - \sum_h \sum_s \lambda_s^h M_s^h [U_M^{hs} - (1 - \alpha_s) U_W^{hs}] \\
& + \sum_s \mu_s (1 - \alpha_s)
\end{aligned} \tag{22}$$

The first-order conditions on  $\alpha_s$  become

$$\begin{aligned}
\frac{\partial \Lambda_\tau}{\partial \alpha_s} = & \sum_h \tau^h p_s^h U_W^{hs} M_s^h - \lambda^1 (1 + \lambda) \sum_h p_s^h M_s^h \\
& - \sum_h \lambda_s^h M_s^h \left[ M_s^h \frac{dU_W^{hs}}{dM_s^h} + U_W^{hs} \right] - \mu_s \leq 0, \quad \alpha_s \frac{\partial \Lambda}{\partial \alpha_s} = 0 \\
& \sum_h p_s^h M_s^h [\tau^h U_W^{hs} - \lambda^1 (1 + \lambda) (1 + \eta_s^h)] - \mu_s \leq 0.
\end{aligned} \tag{23}$$

This leads to a natural generalisation of proposition 1 in terms of weighted averages.

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are limited by a flat annual ceiling; expenses above the ceiling are treated as prepaid income taxes. Although the implicit 100% coverage above the ceiling exposes the system to overconsumption, the total cost has proved small, due to the ancillary conditions.

## Appendix

When  $U_{WM}^s = 0$ , (14) simplifies to

$$\frac{dM_s}{d\alpha_s} = -\frac{U_W^s - M_s(1 - \alpha_s)U_{WW}^s}{U_{MM}^s + (1 - \alpha_s)^2 U_{WW}^s}.$$

Using (12)

$$\begin{aligned}\eta_s &= \frac{\alpha_s}{M_s} \frac{dM_s}{d\alpha_s} = \frac{\alpha_s(1 + \rho_s R_R^s)}{-\frac{M_s U_{MM}^s (1 - \alpha_s)}{U_M^s} + \rho_s R_s (1 - \alpha_s)} \\ &= \frac{\alpha_s}{1 - \alpha_s} \frac{1 + \rho_s R_R^s}{\rho_s R_R^s - \frac{U_{MM}^s M_s}{U_M^s}},\end{aligned}$$

namely (20).

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