Fiscal policy with agents differing in altruism and in ability[1]

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Abstract

This paper presents an overlapping generations model of growth with individuals differing in productivity and altruism. Within such a model wealth is entirely held in the steady-state by the families with the highest degree of altruism. We then look at the macroeconomic and distributive effects of three fiscal policies: public debt, pay-as-you-go social security and estate taxation. Under plausible assumption we show that both public debt and social security are neutral *à la Ricardo* but increase inequality. We also show that estate taxation can be Pareto worsening even though it can foster income equality.

1 Introduction

The literature on the effects of fiscal policy is founded on two canonical models: the Barro-Ramsey model of infinitely lived families and the Diamond-Samuelson model of overlapping generations.[1] In the first model, any attempt by the government to reallocate resources among generations through public borrowing, pays-as-you-go social security or estate taxation is totally or partially neutralized by families who want to smooth their consumption over time through their bequests. In the second model, individuals smooth consumption only over their lifetimes. They can make bequests but not of the type which can smooth consumption among generations. As a consequence fiscal policy cannot be neutralized and has real effects.

For example, government debt is completely neutral within the Barro-Ramsey model; in the Diamond-Samuelson model, it crowds out capital and reduces steady-state utility levels.[2]

As argued by a number of people, these models are not appropriate for a realistic analysis of fiscal policy. One of the reasons is that real societies don't consist of identical individuals, perfectly altruistic or perfectly life-cyclers. There is heterogeneity and by taking that simple fact into account we might get a better understanding of fiscal policy.

In this paper we consider a society consisting of life-cyclers, what Mankiw (2000) calls "spenders" and of dynastic households, what he calls "savers". We thus develop two arguments: as society gets indebted, wealth disparity increases and under some conditions an estate tax may be Pareto worsening.

We generalize an earlier paper (see Michel-Pestieau (1998)) in which we distinguish two types of individuals, altruists and non altruists and we use Cobb-Douglas production functions and loglinear utility functions. We here consider a number of types of individuals differing in their degree of altruism and their level of productivity. We also use more general utility and production functions.[3]

The setting is that of two-overlapping generations with individuals working in a first period and retiring in a second. Individuals save for retirement but some also save to make bequests. The rest of the paper is organized as follows. Section 2 presents the basic model. Section 3 gives the steady-state market solution and optimality conditions. Section 4 introduces debt policy and also pay-as-you-go social security and studies their impact on the long run distribution of wealth. Finally section 5 considers the case of a progressive

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[1] See Barro (1974), Ramsey (1928), Diamond (1965), Samuelson (1958).

[2] Granted that there is underaccumulation.

[3] This bears some resemblance with Cukierman and Meltzer (1989) who however posit at the outset that public debt crowds out some capital which is not the case here.

estate taxation which can be rather regressive.

2 The model

2.1 Households behavior

Type i's individuals are characterized by a degree of altruism γ_i and an index of productivity h_i. We can write both the utility function and the budget constraints of type i's individuals as:

$$v_{it} = u\left(c_{it}, d_{it+1}\right) + \gamma_i \; v_{it+1} \tag{1}$$

$$c_{it} = x_{it} + h_i \, w_t - s_{it} \tag{2}$$

$$d_{it+1} = R_{t+1} \, s_{it} - (1+n) \; x_{it+1} \tag{3}$$

where v_{it} is the utility of type i's individuals belonging to generation t; u is strictly quasi-concave, continuously differentiable, satisfying Inada conditions and excluding inferior goods; c and d are respectively first and second period consumption; x is the amount of bequest received; w and R, the wage rate and one plus the rate of interest; s is saving and $1+n$ the number of children per parent or alternatively n is the rate of population growth. It is assumed that bequests cannot be negative:

$$x_{it+1} \geqslant 0. \tag{4}$$

Below we distinguish lifetime resources given by $x_{it} + h_i \, w_t$ and lifetime consumption spending denoted by Ω_{it} with

$$\Omega_{it} = c_{it} + d_{it+1}/R_{t+1}.$$

We assume that type i's agents have consistently children of the same type; in other words, types are hereditary. The problem of a type i's agent at time 0 can thus be expressed as:

$$Max \; \sum_{t=0}^{\infty} \gamma_i^t \, u\left(c_{it}, d_{it+1}\right)$$

subject to (2), (3), (4) for $t \geqslant 0$ and given x_{i0}.

2.2 Equilibrium solution

Solving the above problem one obtains the following optimality condition for s_{it} and x_{it+1} respectively:

$$-u_c'\left(c_{it}, d_{it+1}\right) + R_{t+1}\, u_d'\left(c_{it},\, d_{it+1}\right) = 0$$

and

$$-\left(1+n\right)\, u_d'\left(c_{it},\, d_{it+1}\right) + \gamma_i\, u_c'\left(c_{it+1}, d_{it+2}\right) \leqslant 0 \qquad \left(= 0 \text{ if } x_{it+1} > 0\right).$$

In the steady-state with $w_t = w$ and $R_t = R$ the variables c, d, s and x are characterized by the following 4 conditions:

$$\left.\begin{array}{c} x_i + h_i w = c_i + s_i, \\ R s_i = d_i + \left(1+n\right) x_i, \\ u_c'\left(c_i, d_i\right) = R\, u_d'\left(c_i, d_i\right), \\ \gamma_i\, R \leqslant \left(1+n\right) \qquad \left(= 1+n \text{ if } x_i > 0\right) \end{array}\right\} \tag{SC1}$$

2.3 Firms and production

We assume a CRS production function $F\left(K_t, H_t\right)$ with labor H_t in efficiency units and capital K_t as inputs. We also assume total depreciation. At equilibrium factor prices equal to marginal productivity:

$$\left.\begin{array}{c} w_t = F_H'\left(K_t, H_t\right) = \omega\left(k_t\right) \\ R_t = F_K'\left(K_t, H_t\right) = \varrho\left(k_t\right) \end{array}\right\} \tag{F$_t$}$$

with $k_t = K_t/H_t$ and for further use $f\left(k_t\right) = F\left(k_t, 1\right)$.

2.4 Intertemporal equilibrium

We define p_i as the proportion of type i's agents in society with $\sum p_i = 1$. We denote N_t the size of generation t. Thus we write:

$$H_t = \sum p_i\, h_i\, N_t = \bar{h}\, N_t$$

where $\bar{h}$ is average productivity. The resource constraint in time t is:

$$F\left(K_t,H_t\right)=\sum p_i\,N_t\,c_{it}+\sum p_i\,N_{t-1}\,d_{it}+K_{t+1}.$$

This constraint is shown to be equivalent to the capital accumulation equation:

$$K_{t+1}=\sum p_i\,N_t\,s_{it},$$

after use of the Euler condition $F\left(K_t,H_t\right)=R_tK_t+w_t\,H_t$ and consumer's constraints $x_{it}+h_iw_{it}=c_{it}+s_{it}$ and $R_ts_{it-1}=d_{it}+\left(1+n\right)x_{it}$.

In the steady-state one obtains:

$$\left(1+n\right)k\,\bar{h}=\sum p_i\,s_i \tag{5}$$

with

$$w=\omega\left(k\right)\quad\text{and}\quad R=\varrho\left(k\right) \tag{6}$$

along with condition $(SC1)$ for the various types of agents.

3 Constrained and unconstrained dynasties

3.1 Market equilibrium

We assume that all the γ_i are different and further that $\gamma_1>\gamma_2>...>\gamma_m\geqslant 0$. In other words individuals are ranked with decreasing altruism with the possibility that $\gamma_m=0$, implying that type m's individuals are pure life-cyclers. We now present our first proposition.

Proposition 1 *: For all $i\geqslant 2$, dynasties are constrained in the long run with $x_i=0$ and their saving is given by:*

$$s_i=\sigma\left(wh_i,R\right)=\arg\max\nolimits_{\sigma}\,u\left(wh_i-\sigma,R\sigma\right). \tag{7}$$

Proof. : In the long run, one has $\gamma_1R\leqslant 1+n$ and hence $\gamma_iR<1+n$ for $i\geqslant 2$, which implies $x_i=0$. Then, $h_iw=c_i+s_i,d_i=Rs_i$ and $u_c'\left(c_i,d_i\right)=R\,u'\left(c_i,d_i\right)$ with s_i given by (7). ■

Condition (7) gives the optimal level of saving of "life-cyclers" as in Diamond (1975) model with $\sigma\left(w\,h_i,R\right)$.

Proposition 2 *: There exists a steady-state equilibrium with $x_1 > 0$ iff the stock of capital $\hat{k}$ consistent with the modified golden rule $\gamma_1 \varrho(\hat{k}) = 1 + n$ is such that:*

$$(1+n)\,\hat{k}\,\bar{h} > \sum_{i=1}^{m} p_i\,\sigma\left(\hat{w}\,h_i, \hat{R}\right) \tag{8}$$

with $\hat{w} = \omega(\hat{k})$ and $\hat{R} = \varrho(\hat{k}) = (1+n)\,/\gamma_1$.

In words, there are bequests in this economy ($x_1 > 0$) if the stock of capital $\hat{k}$ yielding the modified golden rule is larger to the amount that could be accumulated in an economy *à la Diamond.* This is pretty intuitive. If this is the case, altruistic parents contribute to capital accumulation through positive bequests.

Proof. : Assume $x_1 > 0$ which implies that $R = \hat{R} = \dfrac{\gamma_1}{1+n}$. The life-cycle budget constraint of type 1's agents is then:

$$c_1 + \frac{d_1}{\hat{R}} = \hat{w}\,h_1 + (1-\gamma_1)\,x_1 = \Omega_1.$$

These consumptions are independent on the way income is distributed across the two periods; they only depend on Ω_1. And one may write:

$$c_1 = \Omega_1 - \sigma\left(\Omega_1, \hat{R}\right) \text{ and } d_1 = \hat{R}\,\sigma\left(\Omega_1, \hat{R}\right).$$

Henceforth,

$$x_1 = c_1 + s_1 - h_1\hat{w} = s_1 + (1-\gamma)\,x_1 - \sigma\left(\Omega_1, \hat{R}\right),$$

or

$$s_1 = \gamma_1 x_1 + \sigma\left(\hat{w}h_1 + (1-\gamma_1)\,x_1, \hat{R}\right) \equiv \hat{\varphi}\,(x_1).$$

One observes that $\hat{\varphi}\,(x_1)$ is an increasing function of x_1 going from $\hat{\varphi}\,(0) = \sigma\left(\hat{w}\,h_1, \hat{R}\right)$ to $\hat{\varphi}\,(\infty) = \infty$.

On the other hand, equilibrium condition (5) along with $k = \hat{k}$ and $s_i = \sigma\left(\hat{w}\,h_i, \hat{R}\right)$ for $i \geqslant 2$ is equivalent to

$$p_1 s_1 = (1+n)\,\hat{k}\,\bar{h} - \sum_{i=2}^{m} p_i \sigma\left(\hat{w}\,h_i, \hat{R}\right),$$

which determines s_1 as a function of $\hat{k}$.

By assumption, there is a value of $x_1 > 0$ such that $\hat{\varphi}(x_1) = s_1$. Namely,

$$p_1 \hat{\varphi}(x_1) = (1+n)\,\hat{k}\,\bar{h} - \sum_{i=2}^{m} p_i \sigma\left(\hat{w}\,h_i, \hat{R}\right). \tag{9}$$

This implies:

$$p_1 \hat{\varphi}(0) < (1+n)\,\hat{k}\,\bar{h} - \sum_{i=2}^{m} p_i \sigma\left(\hat{w}\,h_i, \hat{R}\right)$$

and with $\hat{\varphi}(0) = \sigma\left(\hat{w}\,h_1, \hat{R}\right)$ one verifies the necessary condition (8).

Conversely, when condition (8) is verified, equation (9) implies a unique solution $x_1 > 0$. With $\hat{k}, \hat{R}\gamma_1 = 1+n$, $\hat{w}$ and the corresponding values for $\Omega_1, c_1, d_1, s_1, \hat{\varphi}(x_1)$ as well as the choices of constrained dynastic constitute a steady-state equilibrium for this economy. ■

3.2 Welfare analysis

In the following, we assume that (8) is satisfied. Then, the steady-state values of $\hat{k}$ and factor prices $\hat{w}$ and $\hat{R} = \dfrac{1+n}{\gamma_1}$ only depends on γ_1. These values are determined by the most altruistic agents regardless of productivity h_i and frequencies p_i.

Dynasties of type i with $i \geqslant 2$ are constrained and their life-cycle utility $u(c_i, d_i)$ is an increasing function of h_i. Yet, their long term welfare

$$\sum_{t=0}^{\infty} \gamma_i^t u(c_i, d_i) = \frac{1}{1-\gamma_i} u(c_i, d_i)$$

depends on both h_i and γ_i.

Type 1's individuals have a life-cycle utility which can be compared with that of other types:

$$u(c_1, d_1) \lesseqgtr u(c_i, d_i) \Longleftrightarrow \Omega_1 = \hat{w}\,h_1 + (1-\gamma_1)\,x_1 \lesseqgtr \hat{w}\,h_i.$$

It is thus not impossible that constrained but highly productive individuals are better off than unconstrained individuals if the latter have a low productivity. However, one can show that if the proportion of type 1's individuals is small, their life-cycle income is the highest. It can indeed be shown that the level of x_1 decreases with p_1.

Let us change p_1 without modifying the relative proportions of $i \geqslant 2$. We write:

$$p_i = (1 - p_1)\, q_i \qquad i = 2, ..., m$$

with $\sum_2^m q_i = 1$. Then from (9) we have:

$$p_1 \left[\hat{\varphi}\,(x_1) - \sum_{i=2}^{m} q_i\, \sigma\left(\hat{w}\, h_i, \hat{R}\right)\right] = (1+n)\, \hat{k}\, \bar{h} - \sum_{i=2}^{m} q_i\, \sigma\left(\hat{w}\, h_i, \hat{R}\right). \tag{10}$$

One sees right away that x_1 decreases as p_1 increases and that when $p_1 \to 0$, $x_1 \to \infty$. Henceforth one can assert that for p_1 sufficiently small,

$$\Omega_1 = \hat{w}\, h_1 + (1 - \gamma_1)\, x_1 > \hat{w}\, h_i \qquad i = 2, ...m.$$

In words, even if one of the constrained types is highly productive this will be more than offset by the wealth of the unconstrained individuals. In this case, their life-cycle utility and also their long run welfare given by $u\,(c_1, d_1) \,/\, (1 - \gamma_1)$ dominate those of the other types. Naturally if $h_1 \geqslant h_i\, (i \geqslant 2)$ one does not need to assume a small value for p_1. In particular if abilities are identical, $h_i = h_1\, (\forall_i \geqslant 2)$, the life-cycle income and utility of the most altruistic agents are larger than the others, whatever the proportions p_i.

4 Intergenerational fiscal policy

4.1 Public debt

Let us now introduce some public debt B_t the service of which is financed by lump-sum taxes levied on the workers τ_{it}^{y} and on the retirees τ_{it}^{0} (τ^y and τ^0 are both non negative). If there is no other public spending the government revenue constraint is equal to:

$$B_t = R_t\, B_{t-1} - \sum_i p_i \left(\tau_{it}^{y}\,(1+n) + \tau_{it}^{0}\right) N_{t-1}$$

or in the steady-state

$$b\left(R-(1+n)\right)=\sum_i p_i\left(\tau_i^y\left(1+n\right)+\tau_i^0\right) \tag{11}$$

where $b_t = B_t/N_t$ is constant: $b_t = b$.

As taxes are non distortionary the optimal conditions for individuals' choices of c, d and x are not changed. If, as assumed, $x_1 > 0$ one has as before:

$$R=\hat{R}=\left(1+n\right)/\gamma_1\ ;\ k=\hat{k}$$

and

$$x_i = 0 \qquad \text{for } i \geqslant 2.$$

In other words, debt policy has no impact on prices and aggregate variables but it has some on individuals' resources. For $i \geqslant 2$, life-cycle income with debt b can be written as:

$$\Omega_i^b = h_i\,\hat{w} - \tau_i^y - \tau_i^0/\hat{R}.$$

As a consequence, if the life-cyclers have to pay for the interest payments on the debt, their lifetime income, their two consumptions and their utility decrease. Note that when the debt is issued, its short run effect allows the life-cyclers to consume relatively more than in the absence of debt.

What about type 1's individuals? their initial response is to look ahead to their future tax liabilities and to adjust their bequests accordingly. Let us consider the resource equilibrium constraint:

$$\left[f\left(\hat{k}\right)-\left(1+n\right)\hat{k}\right]\bar{h}=\sum_{i=1}^{m}p_i\left(c_i+\frac{d_i}{1+n}\right) \tag{12}$$

and observe that disposable income has not changed. Given that the consumption of type i for $i \geqslant 2$ decreases, one knows that the consumption of type 1 increases. It also implies that x_1 increases along with the life-cycle income of type 1's individuals. This leads us to our third proposition.

Proposition 3 : *When debt is financed by non distortionary taxes on all, the constrained dynasties lose and the unconstrained dynasty gains. At the aggregate level, nothing changes in the long run.*

4.2 Pay-as-you-go social security with redistribution

We now turn to a pay-as-you-go pension system which provides the same benefits θ_t to all and finance them by a proportional payroll tax of rate τ. We thus have:

$$N_{t-1}\,\theta_t = N_t\,\tau\,w_t\,\bar{h},$$

or

$$\theta_t = (1+n)\,\tau w_t\,\bar{h}.$$

In the steady-state with $x_1 > 0, w = \hat{w}$ and $R = \hat{R} = (1+n)/\gamma_1$, one can write the life-cycle income of constrained dynasties ($i \geqslant 2$):

$$\begin{aligned} \Omega_i^\theta &= (1-\tau)\,\hat{w}\,h_i + \theta/\hat{R}, \\ &= \hat{w}\,h_i + \tau\hat{w}\left(\gamma_1\,\bar{h} - h_i\right). \end{aligned} \tag{13}$$

One sees that $\Omega_i^\theta \lesseqgtr \Omega_i$ as $h_i \lesseqgtr \gamma_1\,\bar{h}$. In words, the introduction of social security has a positive (negative) effect on type i's individuals ($i \geqslant 2$) if their productivity index is inferior (superior) to the average productivity times the altruism factor of type 1. There are two redistributive effects involved here.

There is first some redistribution from all constrained individuals to the unconstrained ones. It is made clear when $h_i = \bar{h}$ for all i. Then social security likes public debt penalizes in the steady-state the constrained individuals who have to finance the free lunch offered when the system was introduced. There is also some redistribution from high productivity individuals to low productivity workers. In the limit case when γ_1 tends to 1, any individual with productivity below average benefits from this redistributive pension system.

Let us consider as a benchmark the case where $h_i = \gamma_1\,\bar{h}$ for $i \geqslant 2$. From (13), $\Omega_i^\theta = \Omega_i^0$; in other words, social security has no effect on agents i's lifetime income and thus one their consumption. Redistributive pension benefits compensate for the burden a pay-as-you-go system imposes upon constrained households. Given the overall resource constraint the welfare of the unconstrained dynastic is also unaffected. We turn now to the other cases where some agents loose and others gain.

In the extreme case where all constrained dynasties have a relatively low productivity such that $h_i < \gamma_1\,\bar{h}$ for all $i \geqslant 2$, then they all gain and the welfare of the unconstrained dynasty necessarily decreases.

Consider now the case when $h_i > \gamma_1 \bar{h}$ for $i \geqslant 2$. From (13) it is clear that the welfare of constrained individuals decrease and thus that of the unconstrained one increase. This case includes the particular situation with $h_i = \bar{h}$ for $i \geqslant 2$ which has been studied by Michel and Pestieau (1998). Thus,

$$\Omega_i^\theta < \Omega_i^0 \qquad \text{for } i \geqslant 2$$

and both levels of consumption as well as utility decreases. By the same token the welfare of the unconstrained households increases. Naturally we can have a wide dispersion among the h_i's $(i \geqslant 2)$ so that those with low $h_i \left(< \gamma_1 \bar{h}\right)$ would benefit from social security and those with higher $h_i \left(> \gamma_1 \bar{h}\right)$ would be hurt by social security. If we have homothetic preferences[4], $c_i + \dfrac{d_i}{1+n} = \lambda\left(\hat{R}\right)\Omega_i$ for all agents. Then, the sum $\sum_{i=1}^{m} p_i \left(\Omega_i^\theta - \Omega_i^0\right)$ is equal to 0 (since the total consumption is unchanged). Consequently, $\Omega_1^\theta - \Omega_1^0$ is positive iff $\sum_{i=2}^{m} p_i \left(\Omega_i^\theta - \Omega_i^0\right) = \tau \hat{w}\left[\gamma_1 \bar{h}\left(1-p_1\right) - \bar{h} + p_1 h_1\right]$, is strictly negative. And this will be the case iff $p_1 h_1 < \bar{h}\left(1 - \gamma_1\left(1-p_1\right)\right)$. Proposition 4 summarizes the above findings.

Proposition 4 *: A redistributive pay-as-you-go pension system is neutral when* $h_i = \gamma_1 \bar{h}$ *for* $i \geqslant 2$. *It improves the welfare of constrained dynasties for which* $h_i < \gamma_1 \bar{h}$ *and worsens that of those with* $h_i > \gamma_1 \bar{h}$. *Assuming homothetic preferences it improves the welfare of the unconstrained dynasty if and only if* $h_1 < \left(\gamma_1 + \dfrac{1-\gamma_1}{p_1}\right)\bar{h}$.

Redistributive social security is thus shown to have a positive influence on income equality. It does not have such an influence on the distribution of inherited wealth. In fact, Propositions 4 and 5 imply that societies with growing public endebtment and social security schemes should experience all things being equal increasing wealth inequality. This point is made by Laitner (2000).

5 Estate taxation

One of the interests of studying a society with heterogenous individuals with unequal wealth is to study the redistributive incidence of estate taxation.

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[4] See the appendix.

We here assume that a flat tax is levied on estate and that its proceeds are redistributed uniformly to all households. Denoting by τ the estate tax rate and by T the uniform transfer, we rewrite the individuals' budget constraints:

$$(1-\tau)\,x_{it} + h_i\,w_t + T_t = c_{it} + s_{it}$$

$$R_{t+1}\,s_{it} = d_{it+1} + (1+n)\,x_{it+1}$$

The revenue constraint is:

$$T_t = \tau \sum_i p_i x_{it}.$$

Quite clearly the optimal condition for saving is unchanged but that for bequest is now distorted:

$$\begin{aligned} -(1-n)\,u'_d\,(c_{it}, d_{it+1}) + \gamma_i\,(1-\tau)\,u'_c\,(c_{it}, d_{it+1}) &\leqslant 0 \\ &= 0 \text{ if } x_{it+1} > 0. \end{aligned}$$

In the steady-state, the sufficient conditions for an optimum are:

$$\left.\begin{array}{c} (1-\tau)\,x_i + h_i w + T = c_i + s_i \\ R s_i = d_i + (1+n)\,x_i \\ u'_c\,(c_i, d_i) = R\,u'_d\,(c_i,\, d_i) \\ (1-\tau)\,\gamma_i\,R \leqslant 1+n \qquad (= 1+n \ \text{ if } x_i > 0)\,. \end{array}\right\} \tag{SC2}$$

As above, for $i \geqslant 2$, $x_i = 0$ and $s_i = \sigma\,(wh_i + T, R)$. Assuming $x_1 > 0$, we have in the steady-state $\hat{k}_\tau$ a level of capital stock defined by $(1-\tau)\,\gamma_1\,\varrho(\hat{k}_\tau) = 1+n$. We note that $\hat{k}_\tau < \hat{k}$, the capital stock consistent with the modified golden rule when there is no estate tax. Disposable income thus decreases: $\bar{h}(f(\hat{k}_\tau) - (1+n)\,\hat{k}_\tau)$. The revenue constraint implies that

$$T = \tau\,p_1\,x_1.$$

Then for $i \geqslant 2$, we have in the steady-state:

$$c_i + d_i/R = \Omega_i\,(\tau) = wh_i + \tau p_1 x_1$$

where $w = w\left(\hat{k}_\tau\right)$ et $R = \varrho\left(\hat{k}_\tau\right) = \dfrac{1+n}{(1-\tau)\,\gamma_1}$.

And for the most altruistic agents ($i = 1$), we have:

$$c_1 + d_1/R = \Omega_1\,(\tau) = wh_1 + \tau p_1 x_1 + \left(1 - \tau - \frac{1+n}{R}\right) x_1. \tag{14}$$

5.1 Bequest with homothetic preferences

For now on we assume homothetic preferences which imply that consumption levels are proportional to life-cycle consumption expenditures Ω_i. We have for all agents i:

$$c_i + \frac{d_i}{1+n} = \lambda\left(R\right)\Omega_i\left(\tau\right)$$

where $\lambda\left(R\right)$ is increasing for $R > 1+n$. More precisely the equilibrium condition $u'_c = Ru'_d$ is equivalent to $c/d = \phi\left(R\right)$, where $\phi\left(R\right)$ is decreasing. Also $\lambda\left(R\right)$ is defined by:

$$\lambda\left(R\right) = \frac{\phi\left(R\right) + 1/\left(1+n\right)}{\phi\left(R\right) + 1/R}$$

as shown in the appendix.

We now calculate the value of x_1. We start with the resource constraint:

$$\bar{h}\left(f\left(k\right) - \left(1+n\right)k\right) = \sum_1^m p_i\left(c_i + d_i/\left(1+n\right)\right) = \lambda\left(R\right)\sum_1^m p_i\Omega_i\left(\tau\right).$$

Furthermore, we have:

$$\sum_{i=1}^m p_i\Omega_i\left(\tau\right) = w\ \bar{h} + \tau p_1 x_1 + \left(1 - \tau - \frac{1+n}{R}\right)p_1 x_1 = w\ \bar{h} + \left(1 - \frac{1+n}{R}\right)p_1 x_1.$$

Thus

$$\begin{aligned}\left(1 - \frac{1+n}{R}\right)\frac{p_1 x_1}{\bar{h}} &= \frac{1}{\lambda\left(R\right)}\left[f\left(k\right) - \left(1+n\right)k\right] - w\\ &= \frac{1}{\lambda\left(R\right)}\left(Rk + w - \left(1+n\right)k\right) - w.\end{aligned}$$

Using the expression for $\lambda\left(R\right)$, we obtain:

$$\frac{p_1\ x_1}{\bar{h}} = \frac{1}{\phi\left(R\right) + 1/\left(1+n\right)}\left(\left(R\phi\left(R\right) + 1\right)k - w/\left(1+n\right)\right). \qquad (15)$$

One notes that the value of x_1 does not depend on the distribution of $\gamma_i\left(i \geqslant 2\right)$ nor of h_i; $p_1 x_1/\bar{h}$ only depends on $\left(1-\tau\right)\gamma_1$. In other words x_1 can be studied as if we were in a single agent economy.

Remark 5 *In the following we use the assumption that* $\frac{\partial x_1}{\partial \tau} < 0$. *Namely estate taxation does not only depress per capita accumulation but also bequests. At first sight this is what we expect. Yet one can construct examples that yield the opposite outcome. We illustrate this by considering two extreme cases. In the first,* $c = 0$. *Then,* $w + x = s = (1+n)\,k$ *or* $(1+n)k - f(k) + f'(k)k = x$. *Thus*

$$\frac{dx}{dk} = 1 + n + f''(k)k.$$

In the second case, $d = 0$. *Then* $f'(k)k = x$ *and thus* $\frac{dx}{dk} = f'(k)\left(1 + \frac{f''(k)k}{f'(k)}\right)$. *There exist well behaved production functions that give the two possibilities* $\frac{\partial x_1}{\partial \tau} \gtrless 0$.

We now turn to incidence of τ on Ω_i and u_i for $i \geqslant 2$ and then on Ω_1 and u_1.

5.2 Incidence of τ on Ω_i and u_i for $i \geqslant 2$.

Life-cycle income is $\Omega_i = w\ h_i + \tau p_1 x_1$ and thus:

$$\frac{\partial \Omega_i}{\partial \tau} = h_i \frac{\partial w}{\partial \tau} + p_1 x_1 + \tau p_1 \frac{\partial x_1}{\partial \tau}.$$

We note that

$$\frac{\partial R}{\partial \tau} = f'' \frac{\partial k}{\partial \tau} = \frac{1+n}{\gamma_1 (1-\tau)^2} = \frac{R}{1-\tau}$$

and

$$\frac{\partial w}{\partial \tau} = -k f'' \frac{\partial k}{\partial \tau} = -\frac{Rk}{1-\tau}.$$

Hence,

$$\left.\frac{\partial \Omega_i}{\partial \tau}\right|_{\tau=0} = -Rkh_i + p_1 x_1 = -Rk\left(h_i - \bar{h}\right) + p_1 x_1 - Rk\bar{h} \qquad (16)$$

and using (15) we know that for $R > 1 + n$.

$$p_1 x_1 - Rk\bar{h} = \frac{\bar{h}}{\phi(R) + 1/(1+n)} \left[-k\left(\frac{R}{1+n} - 1\right) - \frac{w}{1+n}\right] < 0. \qquad (17)$$

From (16) and (17) we have that for $h_i = \bar{h}$ the estate tax has a negative effect on the life-cycle income of individuals i. The marginal gain from the transfer is more than offset by the loss in earnings.[5] In fact this conclusion applies to any i with $h_i > \tilde{h}$ where $\tilde{h}$ is defined by:

$$Rk\left(\tilde{h} - \bar{h}\right) = p_1 x_1 - Rk\bar{h} < 0.$$

We now turn to the tax incidence on u_i. In the appendix, we show that:

$$\frac{\partial u_i}{\partial \tau} = u_c' \left[\frac{\partial \Omega_i}{\partial \tau} + \frac{\Omega_i}{R^2\phi + R}\frac{\partial R}{\partial \tau}\right],$$

and thus:

$$\left.\frac{\partial u_i}{\partial \tau}\right|_{\tau=0} = u_c' \left[R\ k\left(\bar{h} - h_i\right) - \frac{\bar{h}}{\phi + 1/\left(1+n\right)}\left(k\frac{R}{1+n} - 1\right) + \frac{w}{1+n}\right] \tag{18}$$

given that for $\tau = 0$:

$$\frac{\Omega_i}{R^2\phi + R}\frac{\partial R}{\partial \tau} = \frac{w\ h_i + \tau\ p_1 x_1}{\left(R\ \phi + 1\right)\left(1 - \tau\right)} = \frac{w\left(h_i - \bar{h}\right)}{R\phi + 1} + \frac{w\ \bar{h}}{R\phi + 1}.$$

We can rewrite (18):

$$\left.\frac{1}{u_c'}\frac{du_i}{d\tau}\right|_{\tau=0} = A\left(\bar{h} - h_i\right) - B\ \bar{h},$$

with

$$A = Rk - \frac{w}{R\ \phi + 1} > 0 \qquad \left(x_1 > 0 \text{ implies } \left(R\ \phi + 1\right)k > \frac{w}{1+n} > \frac{w}{R}\right)$$

and

$$B = \frac{k\left(\frac{R}{1+n} - 1\right)}{\phi + \frac{1}{1+n}} + \frac{w\ \phi\left(R - \left(1+n\right)\right)}{\left(\left(1+n\right)\phi + 1\right)\left(R\ \phi + 1\right)} > 0.$$

This leads us to our next proposition.

Proposition 6 *: Assuming homothetic preferences, if all workers have the same productivity, introducing estate taxation has a negative incidence on all constrained workers. If productivity is not uniform, constrained households with sufficiently low productivity can benefit from the tax. They gain if and only if* $h_i < \bar{h}\left(1 - B/A\right)$.

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[5] This result holds for $\tau > 0$ if $\frac{\partial x_1}{\partial \tau} \leqslant 0$.

5.3 Incidence of τ on Ω_1 and u_1.

$$\frac{1}{u_c'}\frac{du_1}{d\tau} = \frac{\partial \Omega_1}{\partial \tau} + \frac{\Omega_1}{R^2+\phi R}\frac{\partial R}{\partial \tau}$$

with

$$\Omega_1 = \omega\left(\hat{k}_\tau\right) h_1 + \tau p_1 x_1 + (1-\tau)(1-\gamma_1) x_1 .$$

Hence,

$$\frac{\partial \Omega_1}{\partial \tau} = h_1 \frac{\partial w}{\partial \tau} - [1-\gamma_1 - p_1] x_1 + [(1-\tau)(1-\gamma_1) + \tau p_1] \frac{\partial x_1}{\partial \tau}.$$

We know that $\dfrac{\partial R}{\partial \tau} = \dfrac{R}{1-\tau}$ and then $\left(\dfrac{\Omega_1}{R^2\phi + R}\right)\dfrac{\partial R}{\partial \tau} = \dfrac{\Omega_1}{(R\phi+1)(1-\tau)}$. Also we have $\dfrac{\partial w}{\partial \tau} = -\dfrac{Rk}{1-\tau}$.

This gives:

$$(1-\tau)\frac{\partial \Omega_1}{\partial \tau} = -h_1 Rk - (1-\gamma_1)(1-\tau) x_1 + (1-\tau) p_1 x_1 + (1-\tau) D \frac{\partial x_1}{\partial \tau}$$

where

$$D = (1-\tau)(1-\gamma_1) + \tau p_1 > 0.$$

We now write:

$$\begin{aligned}\frac{(1-\tau)}{u_c'}\frac{\partial u_1}{\partial \tau} &= -h_1 Rk - (1-\gamma_1)(1-\tau) x_1 - \tau p_1 x_1 + p_1 x_1 + (1-\tau) D \frac{\partial x_1}{\partial \tau} \\ &\quad + \frac{1}{R\phi+1}\left[w h_1 + \tau p_1 \ x_1 + (1-\tau)(1-\gamma_1) x_1\right],\end{aligned}$$

or

$$\frac{(1-\tau)}{u_c'}\frac{\partial u_1}{\partial \tau} = -h_1\left(Rk - \frac{w}{R\phi+1}\right) + p_1 x_1 - D\left(\frac{R\phi x_1}{R\phi+1} - (1-\tau)\frac{\partial x_1}{\partial \tau}\right).$$

From (15) we have:

$$\begin{aligned}p_1 \ x_1 - \bar{h} \ Rk + \tfrac{w \ \bar{h}}{R\phi+1} &= \bar{h} \ k\left(\tfrac{R\phi+1}{\phi+\frac{1}{1+n}} - R\right) + \bar{h} w \left(\tfrac{1}{R\phi+1} - \tfrac{1}{(1+n)\phi+1}\right) \\ &= \tfrac{\bar{h}k}{\phi+\frac{1}{1+n}}\left(1 - \tfrac{R}{1+n}\right) + \tfrac{\bar{h}w}{(R\phi+1)((1+n)\phi+1)}(1+n-R)\phi < 0,\end{aligned}$$

given that $R > 1 + n$.

This implies that

$$\frac{(1-\tau)}{u_c'}\frac{\partial u_1}{\partial \tau} < \left(\bar{h} - h_1\right)\left(Rk - \frac{w}{R\phi + 1}\right) - D\left(\frac{R\phi x_1}{R\phi + 1} - (1-\tau)\frac{\partial x_1}{\partial \tau}\right).$$

We can thus conclude that $\frac{\partial u_1}{\partial \tau} < 0$ if $h_1 \geqslant \bar{h}$ and $\frac{\partial x_1}{\partial \tau} \leqslant 0$. This leads us to our last proposition.

Proposition 7 *: Under the assumption that preferences are homothetic and that estate tax discourages bequest it has a depressive effect on the utility of the unconstrained households if these have a productivity not smaller than the average one.*

Combining Propositions 5 and 6 we have that estate taxation is Pareto worsening if everyone has the same productivity $\left(h_i = \bar{h}\right)$ and $\frac{\partial x}{\partial \tau} \leqslant 0$. Both are sufficient conditions and by no means necessary ones.

6 Conclusion

The purpose of this paper was to generalize some results that are developed in Michel and Pestieau (1998) and in Mankiw (2000). The setting is one of an overlapping generations economy consisting of households who differ in two characteristics: earnings ability and intergenerational altruism. These two characteristics are assumed to be hereditary. We then show that both public debt and pay-as-you-go social security are neutral *à la Ricardo* but increase wealth inequality between households who are the most altruistic and the others. We also demonstrate that in the steady-state redistributive estate taxation is likely to be Pareto worsening even though it reduces wealth inequality.

This paradoxical result has been obtained in a different setting by Stiglitz (1978). When capital is taxed the quantity falls which in turn depresses the real wages. This effect may be large enough to make any tax on wealth transfer undesirable even from the standpoint of people who own no wealth, pay no tax and benefit from the transfer. Naturally with different productivities we expect that life cyclers with low productivity will benefit from the tax transfer.

Appendix

Homothetic preferences

Continuous homothetic preferences can be represented by an homogenous utility function of degree 1. We take a utility function $u(c,d)$ homogenous of degree 1, continuously differentiable, increasing and strictly concave with respect to c and to d, and satisfying Inada's conditions:

$$\lim_{c\to 0} u'_c(c,1) = +\infty \quad \text{and} \quad \lim_{d\to 0} u'_d(1,d) = +\infty. \tag{A.1}$$

This implies that the equation

$$u'_c(\phi,1) = R\ u'_d(\phi,1) \tag{A.2}$$

admits a unique solution $\phi(R)$ that is decreasing and one-to-one from R_+^* into R_+^* (the set of positive numbers).

Indeed $R(\phi) = u'_c(\phi,1)/u'_d(1,1/\phi)$ is decreasing and one-to-one. Thus the same holds for the inverse function $\phi(R)$.

The problem of the consumer is to maximize $u(c,d)$ subject to

$$c + d/R = \Omega. \tag{A.3}$$

Optimality conditions are given by (A.3) and

$$c/d = \phi(R). \tag{A.4}$$

Henceforth, we have:

$$\begin{aligned}
d(\Omega,R) &= \frac{\Omega}{\phi(R)+1/R} \text{ with } d'_R(\Omega,R) > 0 \ , \\
c(\Omega,R) &= \frac{\Omega\phi(R)}{\phi(R)+1/R} \text{ with } c'_R(\Omega,R) \gtrless 0 \ , \\
\lambda(R) &= \frac{c(\Omega,R)+d(\Omega,R)/(1+n)}{\Omega} = 1 + \frac{1/(1+n)-1/R}{\phi(R)+1/R}.
\end{aligned}$$

We note that $\lambda(1+n) = 1$ and $\lambda'(R) > 0$ for $R > 1+n$.

We now derive the indirect utility function:

$$\begin{aligned}
\tilde{u}(\Omega,R) &= u(c(\Omega,R), d(\Omega,R)) = c(\Omega,R)\,u'_c + d(\Omega,R)\,u'_d \\
&= u'_c(c+d/R) = \Omega\ u'_c(\phi(R),1)
\end{aligned}$$

when we use the homogeneity of degree 1.

Function $\tilde{u}$ has the following properties:

$$\frac{\partial \tilde{u}}{\partial \Omega} = u_c' \; ; \; \frac{\partial \tilde{u}}{\partial R} = u_c' \; c_R' + u_d' \; d_R' = u_c' \left(c_R' + \frac{d_R'}{R} \right).$$

Given that $c'(R) + \frac{d'(R)}{R} = \frac{d}{R^2}$, we can write:

$$\frac{\partial \tilde{u}}{\partial R} = u_c' \frac{d}{R^2} = u_c' \frac{\Omega}{R^2 \phi(R) + R}.$$

We can now give the derivative of $\tilde{u}$ with respect to τ:

$$\begin{aligned}
\frac{\partial \tilde{u}}{\partial \tau} &= u_c' \frac{\partial c}{\partial \tau} + u_d' \frac{\partial d}{\partial \tau} = u_c' \left(\frac{\partial c}{\partial \tau} + \frac{1}{R} \frac{\partial d}{\partial \tau} \right) \\
&= u_c' \left(\phi(R(\tau)), 1 \right) \left(\frac{\partial \Omega}{\partial \tau} + \frac{d}{R^2} \frac{\partial R}{\partial \tau} \right) \\
&= u_c' \left(\phi(R(\tau)), 1 \right) \left(\frac{\partial \Omega}{\partial \tau} + \frac{\Omega}{R^2 \phi(R) + R} \frac{\partial R}{\partial \tau} \right) \qquad (A.5)
\end{aligned}$$

following

$$\frac{\partial \Omega}{\partial \tau} = \frac{\partial c}{\partial \tau} + \frac{1}{R} \frac{\partial d}{\partial \tau} - \frac{d}{R^2} \frac{\partial R}{\partial \tau}.$$

Examples:

$$\begin{aligned}
u(c,d) &= c^{\alpha} d^{1-\alpha}, \; \phi(R) = \frac{c}{d} = \frac{\alpha}{(1-\alpha) R} \\
u(c,d) &= \left(c^{1-1/\sigma} + \beta \; d^{1-1/\sigma} \right)^{\frac{\sigma}{\sigma-1}}, \; \phi(R) = (\beta \; R)^{-\sigma}.
\end{aligned}$$

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