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COMMODITY TAXATION WITH NON LINEAR PRICING OLIGOPOLY

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Abstract

This paper studies commodity taxation when firms use two-part tariffs in model of competition *à la Hotelling*. Three kinds of taxes are considered: a specific tax, an *ad valorem* one on the subscription fee and an *ad valorem* one on the per usage fee. We first derive the equilibrium tariffs, market shares and profits. We show that the tax on the subscription fee is profit neutral (unlike the other two) but socially costly (like the other two) as it modifies the consumption choice of the consumers. In a context of costly public funds the *ad valorem* taxation on the variable fee dominates specific taxation. Moreover, the ranking between *ad valorem* taxation on the fixed fee and an *ad valorem* taxation on the variable fee depends on the relative magnitude of economic parameters, in particular the degree of differentation. Finally, we show that the government might prefer the use of two-part tariffs rather than the use of more general tariffs.
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1 Introduction

The study of commodity taxation when firms use linear prices is an old topic in public economics and has been the subject of numerous works[1]. The main result that emerges from this literature is that the type of tax used matters as soon as there exists imperfect competition. Then, before looking at the level a commodity should be taxed, governments have to deal with the question of the balance between different types of commodity taxes[2], in particular between the most common ones: an *ad valorem* tax and a specific taxe[3]. But, the question of commodity taxation when firms use two-part tariffs (or more general tariffs) has not received much attention by public economists even if this kind of tariffs are more and more used in practice in monopolistic and oligopolistic environments: telecommunication operators, golf clubs (member card plus green fee), theaters, night clubs, entertainment parks...

The problem with the taxation of good produced by a non linear pricing monopolist is to justify this public intervention. Indeed, by using discriminating prices a monopolist can extract all the consumers' surplus and then produces efficiently, the first best equilibrium is reached. However, Laffont (1987) shows that if there is asymmetric information between the firm and the consumers or if there exists a cost of public funds there is room for commodity taxation. Laffont then studies the sign of the optimal tax and shows that this sign depends on the cost of public funds and on the cost of the informational rent the government has to leave to the firm. But, the question of the type of tax is not addressed in this paper, a specific tax is only considered. More recently, Cheung (1998) deals with this problem and compares a specific tax and an *ad valorem* one when a monopoly uses a non linear tariff to discriminate among consumers. The author shows that the result of Skeath and Trandel (1994) could be generalized to the case of non linear pricing: the *ad valorem* tax Pareto dominates the specific one. Then, even with a non linear pricing firm, the question of the type of commodity tax to use is of relevance. These two studies, which are to my knowledge the only ones dealing with non linear pricing and commodity taxation, are driven in a monopolistic environment. The main goal of this paper is then to study, in an oligopolistic environment, the choice of tax instruments to use (the type of commodity tax).

In this paper, we consider competition between two firms in a model of the linear city due to Hotelling. Moreover, we will first focus on particular tariffs namely two-part ones. These tariffs are made of a variable fee (per unit price) and of a subscription fee. After a brief description of the model (in section 2) and of the equilibrium (in section 3), we compare, in section 4, the use of a specific taxation and an *ad valorem* one on the usage fee. In section 5, we turn to the comparison between the taxation of the subscription fee and the *ad valorem* one on the per unit price. Then, in section 6, we deal with more general tariffs and we compare their use with the use of two-part tariffs. Finally, section 7 sums up the results and gives some concluding remarks. All the proofs are detailed in the appendix.

2 The model

We consider a particular form of horizontal differentiation[4], resulting from competition between two firms in a linear city model. This framework have been used by Laffont, Rey and Tirole (1998) to study

[1] See the thorough survey of Keen (1998).

[2] See for instance Delipalla-Keen (1992), Skeath-Trandel (1994)or Suits-Musgrave (1953). Moreover, a specific is also called a unit tax.

[3] A specific tax is sometimes called a unit tax.

[4] See Eaton and Lipsey (1989) for a general overview of product differentiation.

competition between networks like mobile phone operators. The main elements of the model are as follows.

The two firms have the same cost structure. Serving a customer involves a fixed cost f. Moreover the per unit cost c is constant and is the same for all consumers[5]. Total cost is then the per customer cost multiplied by the number of customers served. The firms use a particular type of tariff, namely a two-part one denoted for firm i by

$$T_i(q) = A_i + p_i q,$$

where A_i, the fixed fee, can be viewed as a subscription fee for telephone services and p_i, the variable fee, as the price per minute of communication[6]. This variable fee will be called the usage fee. The profit per consumer served is denoted $\pi_i(p_i, A_i)$, the total profit of firm i is then this per consumer profit times the number of subscribers (the number of consumers connected to network i) and is denoted by Π_i.

Consumers are uniformly located on the segment $[0, 1]$. Moreover, firms are located[7] at the extremities of the segment, namely firm 1 is at $x_1 = 0$ and firm 2 at $x_2 = 1$. Given an income I, a consumer located at x, who subscribes and consumes q units of the good produced by firm i, has a utility

$$I + u(q) + \overline{u} - \delta \left| x - x_i \right|,$$

where $\overline{u} \geqslant 0$ is the fixed surplus[8]. This surplus is, for the telecommunication example, the utility obtained by being connected to one of the two networks[9]. The income and fixed surplus are the same for all consumers. The term $\delta \left| x - x_i \right|$ represents the cost, borne by the consumer, resulting from being not connected to her most favorite network whose firm would be located at x. Moreover, $u(q)$ is the variable gross surplus that depends on the quantity consumed and is given by

$$u(q) = \frac{q^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}}.$$

So, when a customer faces a usage fee p the quantity consumed q is such that

$$u'(q) = p \Leftrightarrow q = p^{-\eta}. \tag{1}$$

This variable gross surplus yields a constant elasticity demand function. Moreover, the elasticity of demand, namely η, is assumed to be greater than 1[10]. All the customers of firm i buy the same quantity and their net surplus is

$$w_i(p_i, A_i) = v(p_i) - A_i$$

where $v(p_i)$ is given by

$$v(p_i) = \max_q \{u(q) - p_i q\} = \frac{p_i^{-(\eta-1)}}{\eta - 1} \text{ with } v' = -q.$$

[5]Contrary to Laffont, Rey and Tirole (1998), the access charge problem is not modelised here.

[6]A could be the right to enter in an entertainment park, and then p is the price of an attraction in the park. An other possible interpretation could be to see A as the price to enter in a night-club and p the price of a beer in this night-club.

[7]Then, in this paper we do not consider the localisation game between the two firms.

[8]The utility is such that there is no income effect.

[9]This surplus might be seen as the utility to be in the park or in the night-club.

[10]These assumptions of constant elasticity of demand and that the elasticity is greater than one are made for technical convenience.

We only focus on equilibrium[11] where there is full coverage. This means that the consumer who is indifferent between buying from the two firms strictly prefers buying to not buying. Then, $\overline{u}$ has to be sufficiently high. With that kind of equilibrium, the position of the consumer indifferent between the two firms gives the market share of each firm. If the indifferent consumer is located in α, for firm i, the market share is given by

$$\alpha_i(w_i, w_j) = \frac{1}{2} + \frac{1}{2\delta}(w_i - w_j), \tag{2}$$

and $1 - \alpha_i$ is the market share of firm j.

The government sets taxes in order to maximize social welfare taking into account the value of fiscal revenue linked to the marginal cost of public funds. This cost, strictly positive and denoted λ, is due to the imperfect fiscal system. In particular, the government cannot use lump sum taxes. Then, for the government if X Euros are collected on this market, they have a positive value of $(1 + \lambda)X$ Euros because this amount has not to be levied anymore through the imperfect taxation system[12]. The government objective function[13] is then

$$\begin{aligned} SW &= \int_0^{\alpha_1} [I + u(q) + \overline{u} - \delta x - T_1(q) + \pi_1 + (1+\lambda)r]\, dx \\ &\quad + \int_{\alpha_2}^{1} [I + u(q) + \overline{u} - \delta(1-x) - T_2(q) + \pi_2 + (1+\lambda)r]\, dx \end{aligned} \tag{3}$$

where r is the fiscal revenue per consumer. Moreover, we denote by sw the social welfare per consumer.

The government can use a specific tax, t_s, an *ad valorem* tax, $t_v \in]-1, 1[$, on the usage fee and an *ad valorem* tax on the fixed fee, noted $\tau \in]-1, 1[$. Other types of taxes could have been studied, such that progressive taxes or more generally non linear ones. But, non linear taxes may be infeasible[14]in particular if the government only observe total production (and not individual consumption levels), as it is usually supposed in the literature. Moreover we suppose that a tax set by a government could not depend, here, on the location of the consumer and on individual consumption. Moreover, we suppose that there exists only one level of taxation for each type of tax: firms face the same tax system.

3 Equilibrium in two-part tariffs

The goal of this section is to derive the market equilibrium for a given tax system (τ, t_v, t_s) in order, for instance, to describe the marginal effects of taxes. As $\overline{u}$ is large enough, the market share of firm i is given by (2). Per consumer profit is

$$\pi_i(p_i, A_i) = [(1 - t_v)p_i - (c + t_s)]\, q(p_i) + (1 - \tau)A_i - f.$$

[11] See Mas Colell, Whinston and Green (1995) chapter 12 for more details on the different type of equilibrium.

[12] See, for instance, Atkinson and Stiglitz (1980) or Laffont and Tirole (1993).

[13] In their paper, Delipalla et Keen (1992) describes the government objective function in a slightly different way. The authors suppose that the government maximizes the social welfare (the difference between consumers utility and the cost of production) but faces a revenue requirement constraint. Formally, this problem is the same as the one we have in this model (see Laffont and Tirole (1993) for more details).

[14] A unit tax per consummer or per subscription fee could also be studied. But, as all consumers choose to be connected, this kind of tax becomes equivalent to a lump sum one. But we have supposed that the government cannot use this type of instrument.

Competition in fixed fee and usage fee can equivalently be viewed as competition in usage fee and net surpluses. Then, firm i maximizes

$$\max_{\{p_i,w_i\}} \Pi = \alpha(w_i,w_j)\left[(1-t_v)p_iq(p_i)-(c+t_s)q(p_i)+(1-\tau)(v(p_i)-w_i)-f\right].$$

Using (1) and the fact that $v'(q_i)=-p^{-\eta}$, it follows from the first order conditions that

$$(1-t_v)(p_i^{-\eta}-\eta p_i^{-\eta})+(c+t_s)\eta p_i^{-\eta-1}-(1-\tau)p_i^{-\eta}=0 \tag{4}$$

and

$$\frac{1}{2\delta}\left[(1-t_v)p_i^{1-\eta}-(c+t_s)p_i^{-\eta}+(1-\tau)(v(p_i)-w_i)-f\right]-(1-\tau)\alpha_i=0 \tag{5}$$

The next proposition characterizes the outcome (tariffs, profits) of the competition on the two-part tariffs.

Proposition 1 *For any $\tau \in]-1,1[$, $t_v \in]-1,1[$ and t_s there exists a unique equilibrium which is symmetric and where the symmetric usage fee is given by*

$$p_1=p_2=p^*=\frac{\eta(c+t_s)}{\eta(1-t_v)+t_v-\tau} \tag{6}$$

and the subscription fee is

$$A_1=A_2=A^*=\delta-\frac{1}{1-\tau}\left((1-t_v)p^*q^*-(c+t_s)q^*-f\right). \tag{7}$$

Moreover, the equilibrium symmetric profit is only affected by the tax on the subscription fee and is equal to

$$\Pi^*=\frac{(1-\tau)\delta}{2}.$$

One can remark that, if all the taxes are equal to zero, two-part tariffs yield pricing at marginal cost. As long as they are not all equal to zero, the usage fee is (generally) different from the marginal cost or the perceived marginal cost which is equal here to $\frac{c+t_s}{1-t_v}$. Moreover, the tax on the subscription fee is for the firm equivalent to a tax on its profit Indeed, without public intervention, the profit is equal to $\frac{\delta}{2}$, $p=c$ and $A=\delta+f$. But, from a social welfare point of view this is not the case as the usage fee depends on the level of this tax. Finally, the specific tax and the *ad valorem* one on the usage fee only affect the consumers therefore, they bear all the burden, the losses due to this kind of tax.

In this model all the taxes affect the usage fee. Comparative statics for each instrument give:

$$\frac{\partial p^*}{\partial t_s}=\frac{\eta}{\eta(1-t_v)+t_v-\tau}>0 \tag{8}$$

$$\frac{\partial p^*}{\partial t_v}=\frac{\eta(c+t_s)(\eta-1)}{\left(\eta(1-t_v)+t_v-\tau\right)^2}>0 \tag{9}$$

$$\frac{\partial p^*}{\partial \tau}=\frac{\eta(c+t_s)}{\left(\eta(1-t_v)+t_v-\tau\right)^2}=\frac{1}{\eta-1}\frac{\partial p^*}{\partial t_v}>0. \tag{10}$$

An increase of any tax leads to an increase of the usage fee. Then, all taxes have a negative effect on social welfare because they create a gap between the usage fee and the marginal cost. If this result is standard for the tax on the usage fee and the specific one, it is a little bit surprising regarding the tax on the subscription fee. Indeed, A, the fixed fee, is a lump sum transfer between the consumer and the firm, then, as long as there is full coverage, a variation of this transfer has no cost in term of social welfare. Then, even if the subscription fee is not costly itself on efficiency ground, its taxation is.

From (8), one can show that $\partial p^*/\partial t_s^* \geqslant 1$. Then specific taxation is always over-shifted: the usage fee increases more than the tax increase. For *ad valorem* taxation, over-shifting occurs if[15] $\partial p^*/\partial t_v^*$ is greater than p^*. But, from (9) $\partial p^*/\partial t_v^*$ could be greater or smaller than p^*. The sign of $\partial p^*/\partial t_v^* - p^*$ is the one of $(\eta - 1)t_v + \tau - 1$ which turns to be positive for η sufficiently large. These results are in the same spirit as the ones of Delipalla and Keen (1992), who show that, in a Cournot model, specific taxes are more often over-shifted.

In this paper we do not want to focus on the level of taxation, but we can show that starting from the 'no intervention' situation (i.e. all taxes are equal to zero), a marginal increase of any tax is welfare improving. Indeed, the social welfare per consumer is

$$sw(t_s) = I + u(q^*(t_s)) + \overline{u} - \delta x - cq^*(t_s) + \lambda t_s q^*(t_s)$$

then from (1) we have

$$\frac{\partial sw}{\partial t_s} = -\eta p^{-\eta} \frac{\partial p^*}{\partial t_s} + \eta p^{-\eta-1} c \frac{\partial p^*}{\partial t_s} + \lambda q^* - \lambda t_s \eta p^{-\eta-1} c \frac{\partial p^*}{\partial t_s}.$$

From proposition 1 we now that $p^* = c + t_s$ when $\tau = t_v = 0$, then when $t_s = 0$ we have $\frac{\partial sw}{\partial t_s} > 0$. As all the consumers have the same behavior[16], the introduction of a small specific tax raises social welfare[17]. This result is very intuitive. If all tax levels are equal to zero then the usage fee is equal to the marginal cost. A positive level of taxation leads to an increase of the usage fee but raises public funds. But, as the public funds has a positive value for the government, a marginal increase has a positive impact on the social welfare (the cost is more than compensated). In an imperfect competition world it is not always the case that a positive level of taxation is welfare improving. For instance, in a Cournot setting when firms use linear prices, the equilibrium price without public intervention is already greater than the marginal cost, the imperfect competition has then a social cost. Then, it might be the case that a positive tax decreases social welfare, as the cost of imperfect competition is too high relative to the benefits associated to a positive fiscal revenue.

The introduction of positive tax (starting from the no tax situation and whatever the type may be) is welfare improving[18]. However, the government may prefer to use one of these taxes[19] rather than the other ones. This question is addressed is the two next sections.

[15] See Delipalla-Keen (1992) for more details.

[16] They are all connected and consume the same quantity.

[17] The same result appears with the two other taxes

[18] This result does not mean that there is no room for subsidies in this model. But in order to be welfare improving, a subsidy will have to be combined with a tax.

[19] Or a combination of two type of taxes versus the use of the last one.

4 The balance between specific and *ad valorem* taxation on the usage fee

The goal of this section is to know[20] if, as when firms with market power use linear prices, taxes have different effects on the equilibrium which could create preferences for one type of tax. In other words, we want to know if for the government, the consumer or the firms, these two taxes are perfect substitutes. In general, in a model *à la Hotelling*, when firms use two-part tariffs, the usage fee is equal to the marginal cost or to the perceived marginal cost[21]. With such a price behavior, an *ad valorem* tax on the usage fee or a specific tax have the same effect[22]. From proposition 1, we know that the usage fee is different from the perceived marginal cost[23]. Then, without doing any computation, we can think that these two types of tax are not perfect substitutes.

In order to make the comparison, we will follow a slightly different approach from Delipalla and Keen (1992). There are mainly two methods in order to assess the superiority of one or another pair of taxes. The first one is to take two different pairs and to compute the difference in the associated social welfare levels. The second is to select a specific combination of tax variations and to analyze its influence in terms of social welfare. This is the method used here with changes in taxes that lead to the same usage price and individual quantities[24]. Specifically, we consider a tax change of the form

$$dt_v = -\frac{\eta}{p(\eta - 1)} dts > 0. \tag{11}$$

This reform, which will be referred to as a "p–constant" shift from specific to *ad valorem* taxation on the usage fee, does not change the usage price. Indeed, we have that

$$dp = \frac{\partial p^*}{\partial t_v} dt_v + \frac{\partial p^*}{\partial t_s} dt_s$$

which from (8) and (9) gives taking into account (11) that $dp = 0$.

Before turning to the comparison of specific and *ad valorem* taxation on the usage fee, we can study how the balance between the two affects some important variables of the model.

Proposition 2 *A "p-constant" shift from specific to ad valorem taxation on the usage fee leads to:*
i) an increase of the fiscal revenue
ii) an increase of the subscription fee
iii) no change of profits

Firms are indifferent between the two taxes as their profits are only affected by the tax on the subscription fee. But, consumers prefer a specific tax as the subscription fee is larger with an *ad valorem*

[20] The study will be driven without making the computation of the optimal level of taxes.

[21] For instance, in Laffont, Rey and Tirole (1998), the usage fee is equal to the marginal cost increased by the the access charge, which constitutes the perceived marginal cost. This kind of result is true in other framework as in Calem et Spulber (1984) for instance.

[22] See Keen (1998) or Myles (1995).

[23] For very particular cases, like when all taxes are equal to zero, the usage fee is equal to the perceived marginal cost.

[24] Delipalla and Keen consider a change in the taxation system that leave the total tax payment unchanged aat the initial price.

taxation on the usage fee[25]. These two results contrast with the ones developed by Delipalla and Keen (1992) or Keen (1998) where firms prefer specific taxation and consumers *ad valorem* taxation. For the government, the choice between the two type of taxes appears to be as a trade-off between high fiscal revenue and low subscription fee. But, an increase of the subscription fee is just an increase of a lump sum transfer between consumers and firms, then this increase has no cost in terms of social welfare. Thus, the preference of the government appears clearly.

Corollary 3 *Suppose the specific tax is restricted to be non-negative. For any* $\tau \in]-1, 1[$*, when public funds are costly the optimal rate of specific taxation is zero.*

The results concerning the superiority of an *ad valorem* taxation when firms use linear prices are still true when firms use two-part tariffs even if the preferences of economic agents (firms and consumers), in terms of taxes, are not the same. Moreover, for high level of taxation on the subscription fee, the superiority of the *ad valorem* tax is even greater. Indeed, the subscription fee is larger with the *ad valorem* tax on the usage fee, then a tax on this subscription fee generates higher fiscal revenue.

When the restriction on the level of the specific tax is relaxed, matters become more difficult as pointed out by Delipalla and Keen (1992). The study of non-constrained tax system (combination of subsidies and taxes) is a natural extension of this work. In particular, it might be relevant to verify that if, as Myles (1996) has shown for linear prices, it is possible to eliminate losses due to imperfect competition. Moreover, the question is still relevant even if there exist constraints on taxes. Indeed, it might be the case that for legal reasons (European Union laws for instance) subsidies are not permitted.

The superiority of the *ad valorem* taxation is closely linked to the fact that, in our setting, an increase of the subscription fee does not affect the number of connected consumers. An *ad valorem* on the usage fee leads to a higher subscription fee, then, if $\overline{u}$ is not sufficiently large, the market coverage could be larger with a specific tax. So, this could affect the superiority of the *ad valorem* tax on the usage fee.

If market coverage is not affected by the taxation system, then specific taxation is useless. So, we only have to compare the use of taxes on the subscription fee and on the usage fee, which is the topic of the next section.

5 Taxation of the subscription fee versus taxation on the usage fee

As a consequence of the preceding section, we compare the tax on the subscription fee and the one on the usage fee for a level of specific taxation equal to zero. To study the balance between these two instruments, it is useful to look at the effects of this two taxes and how they generate fiscal revenues. The social welfare associated with one consumer connected to firm 1 is

$$sw = I + u(q) + \overline{u} - \delta x - A_1 - pq + \pi_1 + (1+\lambda)r$$

The subscription fee has no cost and the profits are costly, in terms of social welfare, as they have a higher value for the government if they become fiscal revenue. Then, a tax on the subscription fee seems to be very interesting because it transforms profits (obtained without cost) into fiscal revenue. But, the tax on the subscription fee creates a gap between the usage fee and the marginal cost which does not generate fiscal revenue. But, from section 3, we know that the tax on the usage fee creates

[25] If consumers internalize the cost of public funds, they prefer the *ad valorem* taxation.

such a gap too. However, with the tax on the usage fee, the gap generates fiscal revenues. Thus, the choice of the government between this two instruments is not obvious.

To compare the two taxes we use the same technic as in section 4. We consider a shift in the taxation system that leads to the same usage fee (individual quantity). We then consider the marginal effect on welfare of such tax reform. The sign of this effect determine which instrument the government has to use. In order to do this comparison we have to suppose that the two taxes have the same sign, here we will consider positive taxes. The following proposition sums up the choice of the government.

Proposition 4 *Suppose that taxes are restricted to be positive. For a given usage fee $\widetilde{p}$ and a given individual consumption level $\widetilde{q}$ let $(0, \widetilde{\tau})$ be the tax system without taxation of the usage fee that leads to those equilibrium outcomes.*

If $\frac{c}{\eta-1}\widetilde{q} \leqslant f + (1-\widetilde{\tau})^2\delta$, then for this consumption level $\widetilde{q}$, the government only uses the tax on the subscription fee.

If $\frac{c}{\eta-1}\widetilde{q} \geqslant f + \delta$, then for this consumption level $\widetilde{q}$, the government only uses the tax on the usage fee.

If $\frac{c}{\eta-1}\widetilde{q} > f + (1-\widetilde{\tau})^2\delta$ et $\frac{c}{\eta-1}\widetilde{q} < f + \delta$, then for this consumption level $\widetilde{q}$, the government uses a combination of the two taxes, with strictly positive levels.

This proposition indicates if the best tax system is only composed by a tax on the subscription fee or if there exist a better tax structure.

Now, we want to know, or at least have some ideas, on the best tax structure for given relative values of the economic parameters considered here. First, for relatively low competition pressure (high degrees of differentiation) which means δ large or if the fixed cost is relatively large, the government uses only τ, the tax on the subscription fee. This is quite natural as the more δ is large, the more A is large and then the more is large the fiscal revenue.

Second, consider all the possible values of the tax on the subscription fee and the equilibrium outcomes associated, for each level $\widetilde{\tau}$ there exists an individual quantity $\widetilde{q}$. Moreover, $(1-\widetilde{\tau})^2\delta$ and $\frac{c}{\eta-1}\widetilde{q}$ are continuous and decrease with $\widetilde{\tau}$. Suppose now that $\frac{c}{\eta-1}\widetilde{q}(\widetilde{\tau}=0) > f + \delta$, the fixed fee and the degree of differentiation are relatively small[26], then it appears that the government only use the tax on the usage fee for small values of $\widetilde{\tau}$, or equivalently for high values of $\widetilde{q}$ (see the figure).

A large cost of public funds leads to higher tax levels because the gap between marginal cost and the usage fee is less costly. Then, the higher the cost of public funds is the more likely the tax on the usage fee is preferred by the government.

Then, depending of the values of economic parameters (in particular λ, f and δ) it may be useful to use a combination of the two taxes or just one of the two.

6 Preferences for tariffs

In this section, we want to address the following question: do agents prefer the use of some kind of tariffs given a particular tax system. More precisely, we compare in this section the use of two-part tariffs and the use of a non linear tariff $P(q)$, like the ones used in Cheung (1998). This tariff $P(q)$ corresponds to prepaid telephone cards and are used for mobile phones (for a fixed monthly fee you

[26] It might be the case that for all the values of $\widetilde{\tau}$, we have that $\frac{c}{\eta-1}\widetilde{q}(\widetilde{\tau}) \geqslant f + \delta$. Then government chooses to only use the tax on the usage fee.

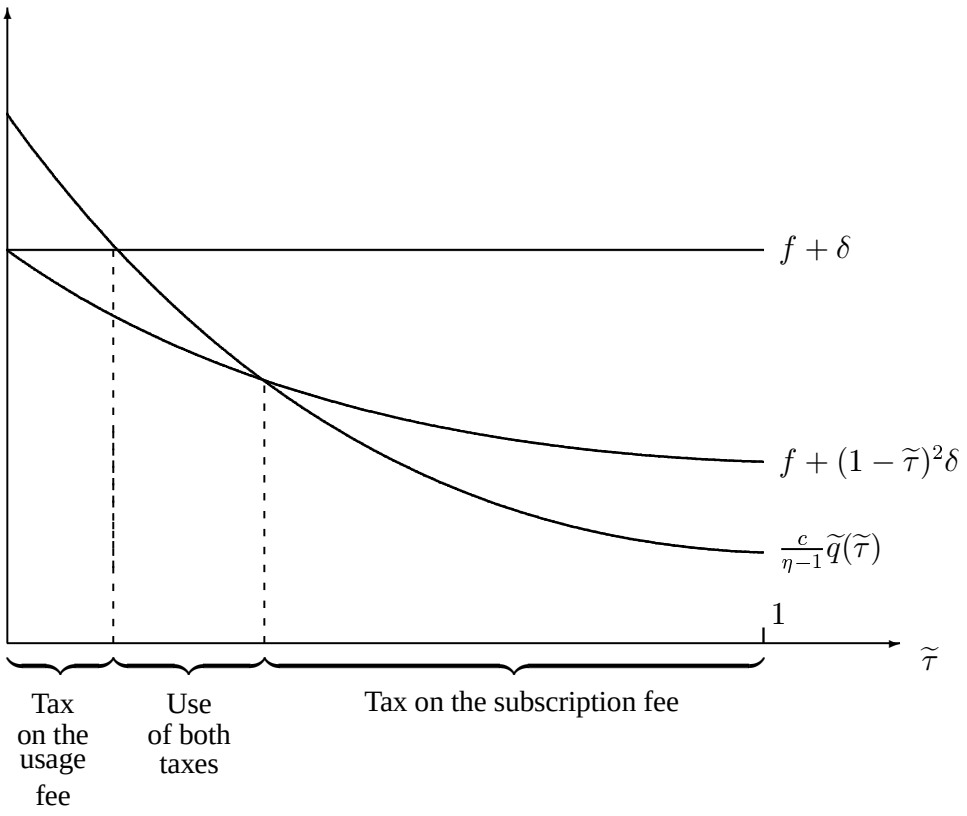

Figure 1: Choice of instruments

have a fixed amount of calls with). In this model, firms cannot discriminate, then it is sufficient for firms to only propose one price associated to a unique quantity (rather than a price function). We suppose that the government has the same instruments than in Cheung (1998), which are the only ones that are consistent with our hypothesis on the observable variables by the government. The government can then use a specific tax $t_s \geqslant 0$ and an *ad valorem* one on P, T_v. The profit (per consumer) of firm i is then

$$\pi_i = (1-T_v)P_i - (c+t_s)q_i - f.$$

As in section 3, we consider a competitive market in which firms choose an individual quantity and a net surplus $w_i = u(q_i) - P_i$. Market shares are $\alpha_i = \alpha(w_i, w_j) = \frac{1}{2} + \frac{1}{2\delta}(w_i - w_j)$, then the objective function of firm i is

$$\max_{w_i, q_i} \alpha(w_i, w_j)\left[(1-T_v)(u(q_i) - w_i) - (c+t_s)q_i - f\right].$$

First order conditions give for q_i: $u\prime(q^*) = \frac{c+t_s}{1-T_v}$ and for w_i

$$\frac{1}{2\delta}\left[(1-T_v)(u(q_i) - w_i) - (c+t_s)q_i - f\right] - (1-T_v)\alpha_i = 0. \tag{12}$$

The following proposition could then be established:

Proposition 5 *For a given tax T_v and a specific tax t_s, the equilibrium is symmetric and unique. Moreover the equilibrium individual quantity q^* is such that $u'(q^*) = \frac{c+t_s}{1-T_v}$, and the price is*

$$P^* = \frac{(c+t_s)q^* + \delta + f}{1-T_v},$$

Finally, equilibrium profits are independent of the level of taxes and are equal to $\Pi^* = \frac{\delta}{2}$.

Profit is independent of the taxation when firm use a tariff $P(q)$. Then, firms are indifferent between a specific tax or an *ad valorem* one and consumers support all the burden of taxation. But, firms are not indifferent between the two types of tariff studied (except when the tax on the subscription fee is equal to zero). Then, even if customer's demand function is known and the same for all connected consumers, firms obtain less profits with a two-part tariff. Yet, without taxation, firms cannot do better than using two-part tariffs, as argued for instance by Laffont, Rey and Tirole (1998). Then, the choice of a particular tariff is affected by the fiscal regime. For the government, this means that it should take into account these reactions when it designs its fiscal policy[27]: the structure of the tariff is endogenous. This result raises the question of the "completeness of the fiscal policy": is it possible for a government to design a fiscal policy for each possible price structure ?

We now turn to the comparison between the taxes from the government's point of view.

Proposition 6 *Suppose the specific tax is restricted to be non-negative. For any $T_v \in]-1,1[$, when public funds are costly the optimal rate of specific taxation is zero when firms use non linear prices $P(q)$.*

This results extend the one due to Cheung (1998) with a monopoly. The government has a preference for the *ad valorem* taxation when firms use a tariff $P(q)$. But, does the government have preferences for a type of tariff as firms do ?

Due to the hypothesis made on the observable variables, the government is restrained in the set of the instruments it can use, in particular the use of non linear taxes. Then, the main advantage of a two-part tariff is that it enables the government to tax at different rates the transfers between consumers and firms. Moreover, the losses due to the taxation are shared between consumers and firms when these latter use two-part tariffs. But, the disadvantage of such tariffs is that, contrary to a tariff $P(q)$, the marginal utility (equal to marginal the usage fee) is different from the perceived marginal cost. Then, it appears that there is no clear preference for a tariff from the government's point of view.

Proposition 7 *The government may prefer the use of two-part tariffs rather than the use of more general non linear tariff.*

If it could, the government may sometimes wished to forbid the use of some particular pricing behavior. For instance, in network industries, it is often the case that universal service obligations exist. Yet, these obligations are sometimes financed by taxes on the goods sold by the competitors of the firm in charge of these obligations[28]. In that case, in order to have the least possible welfare losses, the government may prohibit the use of particular tariffs.

[27] For instance, a policy that consist in the taxation of all transfers between consumers and firms at the same rate, would to lead to a strategic choice of a particular tarif by the firms.

[28] See, for instance, Choné-Flochel-Perrot (2000) for more details.

7 Final remarks

When firms use two-part tariffs, a tax on the subscription fee is not neutral in terms of social welfare. Hence, even if the subscription fee is not costly itself, its taxation is. We show that an *ad valorem* taxation on the usage fee dominates a specific taxation and that the dominance between taxation on the usage fee and taxation on the subscription fee depends on economic parameters like the degree of differentiation between the goods. Moreover, we pointed out that the choice of a particular tariff by firms is affected by the taxation system.

The first natural development of this work is to study the balance between these taxes for other specification of imperfect competition between firms, or other types of tariff. But there exist many other ways to extend this model.

We should introduce heterogeneity in consumers behaviors and asymmetric information between consumers and producers. The comparison between different type of commodity taxes might be affected by price discrimination. Indeed, when their tariffs, firms will propose different usage fees which are not affected in the same way by the different type of taxation. Then, the superiority of the *ad valorem* tax (on the usage fee) could be alterate.

One should also consider a setting where there are two separate markets: one for the subscription and one for the use (camera and films or car and gas). In this setting, the use and the choice between the instruments could be different.

Finally, the question of which type of taxes to use should be studied when governments have other goals than the social welfare, for instance universal service consideration or market coverage could be the subject of future researches.

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Appendix

Proof. [proof of proposition 1] First, we have to show that there does not exist equilibrium where one firm serves all the market ($\alpha_i = 1$).

Suppose that firm 1 serves all the market, its profit is then

$$\Pi_1 = (1-\tau)A_1 + p^* q^* - f - cq^* \geqslant 0$$

But, the profit per consumer is positive and the same for each individual as, first, there is no price discrimination and second as consumers have the same demand function. Then, per consumer profit ($\pi_1 = (1-\tau)A_1 + pq - f - cq$) is positive whatever the localization may be, in particular, the consumers who are the more distant from firm 1. Firm 2 could, by using the following tariff $A_1 + p^* q^*$, obtain a positive profit because consumers close to 1 will prefer this tariff. Then, firm could not serve all the market. The equilibrium is then such that both firms have strictly positive market shares.

The proof of the existence, uniqueness and symmetry is an easy extension of the proof of propositions 1 and 7 in Laffont, Rey and Tirole (1998). Its presentation will then be short. The first order condition with respect to w_i is

$$\frac{1}{2\delta}\left[(1-t_v)p_i^{1-\eta} - (c+t_s)p_i^{-\eta} + (1-\tau)(v(p_i) - w_i) - f\right] - (1-\tau)\alpha_i = 0 \tag{13}$$

Taking into account the first order condition with respect to the usage fee,condition 13 defines a reaction function $w_i = w_i^r(w_j)$. The slope of this function is

$$\frac{dw_i^r}{dw_j} = \frac{1-\tau}{2-\tau}$$

As τ is such that

$$-1 \leqslant \tau \leqslant 1,$$

the slope of this function is positive and less than 1. Then, the equilibrium, if it exists, is unique and symmetric.

We now turn to second order conditions.

First, the profit function of a firm could be written as

$$\widehat{\Pi}(v_i) = \alpha(w_i, w_j)\left[(1-t_v)\widehat{p}_i(v_i)\widehat{q}(v_i) - (c+t_s)\widehat{q}(v_i) + (1-\tau)(v_i - w_i) - f\right]$$

where

$$\widehat{p}_i(v_i) \equiv \left[(\eta-1)\, v_i\right]^{-\frac{1}{\eta-1}}$$
$$\widehat{q}(v_i) \equiv q(\widehat{p}_i(v_i)) \equiv \left[(\eta-1)\, v_i\right]^{\frac{\eta}{\eta-1}}.$$

We have that

$$\frac{\partial \widehat{\Pi}}{\partial v} = \alpha - (\eta-1)\,(1-t_v)\left[\frac{p^M - \widehat{p}(v)}{\widehat{p}(v)}\right] - (1-\tau)$$

where $p^M = \frac{c+t_s}{1-t_v}\frac{\eta}{\eta-1}$, which is the price choose by a linear pricing monopoly (the price that solves $\max_p \left[(1-t_v)p - c - t_s\right] q(p)$). Then, we get that

$$\frac{\partial^2 \widehat{\Pi}}{\partial v^2} = \alpha\left\{-(1-t_v)(\eta-1)\frac{p^M}{\left[\widehat{p}(v)\right]^2 \widehat{q}(v)}\right\} < 0.$$

Firm i's best response entails $p_i = \frac{\eta(c+t_s)}{\eta(1-t_v)+t_v-\tau} = p^*$, so keeping w_j (and p_j) fixed, firm i's profit if it chooses to offer w_i is:

$$\widetilde{\Pi}_i(w_i) = \left(\frac{1}{2} + \frac{1}{2\delta}(w_i - w_j)\right)\left\{\left[(1-t_v)p^* - (c+t_s)\right]q_i(p^*) + (1-\tau)(v_i(p^*) - w_i) - f\right\}$$

First order condition of this function is

$$\frac{d\widetilde{\Pi}}{dw_i} = \frac{1}{2\delta}\left\{\left[(1-t_v)p^* - (c+t_s)\right]q_i(p^*) + (1-\tau)(v_i(p^*) - w_i) - f\right\} - (1-\tau)\left(\frac{1}{2} + \frac{1}{2\delta}(w_i - w_j)\right)$$

then we have that the second order condition is

$$\frac{d^2\widetilde{\Pi}}{dw_i^2} = -(1-\tau)\frac{1}{\delta} < 0.$$

This function is strictly concave and so the candidate equilibrium is indeed an equilibrium.

Marginal effect of taxes on equilibrium fixed fee.

$$\frac{\partial A^*}{\partial t_s} = \frac{(\eta-1)(\tau-t_v)q^*}{(1-\tau)(\eta(1-t_v)+t_v-\tau)} \tag{14}$$

$$\frac{\partial A^*}{\partial t_v} = \frac{((\eta-1)(\tau-t_v)+1-\tau)\eta q^*}{(1-\tau)(\eta(1-t_v)+t_v-\tau)^2} \tag{15}$$

$$\frac{\partial A^*}{\partial \tau} = \frac{1}{(1-\tau)^2}\left[f - (c+t_s)q^*\frac{(t_v^2(\eta-1)+\eta(1-\tau)+(\eta-1)\tau^2 - t_v(\eta+(\eta-2)\tau)}{(1-\tau)(\eta(1-t_v)+t_v-\tau)^2}\right] \tag{16}$$

■

If $\tau > t_v$ then $\frac{\partial A^*}{\partial t_s}$ and $\frac{\partial A^*}{\partial t_v}$ are positive. The sign of the derivative with respect to the tax on the fixed fee is more difficult to determine.

Proof. Consider a tax system (τ, t_v, t_s) and the following shift in this tax system

$$dt_v = -\frac{\eta}{p(\eta-1)}dts > 0. \tag{17}$$

This shift leaves the usage fee unchanged, $dp = 0$. Using (7)the impact on the subscription is

$$dA = -\frac{1}{1-\tau}\left[(1-t_v)pqdt_v - qdt_s\right].$$

which gives with (17)

$$\begin{aligned} dA &= \left[\frac{q}{1-\tau} - \frac{\eta q}{(1-\tau)(\eta-1)}\right]dt_s \\ &= \frac{-1}{(1-\tau)(\eta-1)}qdt_s. \end{aligned}$$

Then, $dA < 0$ if $dts > 0$. As all consumers choose to be connected ($\overline{u}$ large enough), we can focus on the per consumer fiscal revenue $r = \tau A + t_v pq + t_s q$. The effect of the shift is then

$$dr = \tau dA + pqdt_v + qdt_s$$

using (17) we obtain

$$dr = qdt_s - \frac{\tau}{(1-\tau)(\eta-1)}qdt_s - \frac{\eta}{(1-\tau)(\eta-1)}qdt_s$$
$$= \frac{1}{(1-\tau)(\eta-1)}qdt_s = -dA$$

Then, $dr > 0$ if $dts > 0$. By increasing the *ad valorem* taxation and decreasing the specific one keeping the usage fee constant, the government can increase the fiscal revenue but increasing also the subscription fee. ■

Proof. We suppose here that $t_s \geqslant 0$. Social welfare per consumer is from (3) $sw = I + u(q) + \overline{u} - \delta x - T_1(q) + \pi_1 + (1+\lambda)r$. The "q-shift" from a specific to an *ad valorem*taxation on the usage fee affects this welfare in the following way (remember that the profit of a firm is only affected by a change in the level of the tax on the subscription fee) $dsw = -dA + (1+\lambda)dr > 0$ as long as λ is strictly positive. ■

Proof. If only one type of tax is used, we note it with $\widetilde{-}$. The *ad valorem* tax on the subscription fee is noted by $\widetilde{\tau}$ and the *ad valorem* tax on the usage fee $\widetilde{t_v}$. Consider a tax system (τ, t_v) and the following change $d\tau = -(\eta-1)dt_v$. From (9) and (10) which describes the marginal effect of taxes on the usage fee we have $dp = 0$.

Moreover

$$\begin{array}{c} dA = -\frac{1}{1-\tau}(-pq + (1-t_v)q\frac{\partial p}{\partial t_v} + (1-t_v)\frac{\partial q}{\partial p}\frac{\partial p}{\partial t_v})dt_v \\ -\frac{1}{(1-\tau)^2}((1-t_v)pq - cq - f)d\tau \\ -\frac{1}{1-\tau}((1-t_v)\frac{\partial p}{\partial \tau}q + (1-t_v)\frac{\partial q}{\partial p}\frac{\partial p}{\partial \tau})d\tau \end{array}$$

which, knowing that $dp = 0$, gives

$$dA = \frac{pq}{1-\tau}dt_v - \frac{(1-t_v)pq - cq - f}{(1-\tau)^2}d\tau$$

the social welfare associated to a consumer is

$$sw = U(q) - cq - f + \lambda(\tau A + t_v pq)$$

and the variation of this welfare following the shift in the tax system is

$$dsw = \lambda\tau dA + \lambda pq dt_v + \lambda A d\tau.$$

then

$$dsw = \lambda\left(\frac{\tau}{1-\tau}pq + pq\right)dt_v +$$
$$\lambda\left(-\frac{\tau}{(1-\tau)^2}\left((1-t_v)pq - cq - f\right) + \delta - \frac{1}{1-\tau}\left((1-t_v)pq - cq - f\right)\right)d\tau$$

which finally gives

$$dsw = \frac{\lambda pq}{1-\tau}dt_v + \frac{\lambda}{(1-\tau)^2}\left((1-\tau)^2\delta - (1-t_v)pq + cq + f\right)d\tau.$$

By using the fact that $d\tau = -(\eta - 1)dt_v$ we obtain

$$dsw = \frac{(\eta-1)\lambda}{(1-\tau)^2}\left(\frac{(1-t_v)\eta + t_v - \tau}{\eta - 1}pq - cq - f - (1-\tau)^2\delta\right)dt_v.$$

Then, as $p = \frac{\eta c}{(1-t_v)\eta + t_v - \tau}$, we get that

$$dsw = \frac{(\eta-1)\lambda}{(1-\tau)^2}\left(\frac{c}{\eta - 1}q - f - (1-\tau)^2\delta\right)dt_v.$$

Consider that the case where the tax on the subscription fee is solely used as the benchmark. This tax $\widetilde{\tau}$ leads to an equilibrium usage fee $\widetilde{p}$ ($\widetilde{q}$ is the associated individual quantity).

If $\frac{c}{\eta-1}\overline{q} \leqslant f + (1-\widetilde{\tau})^2\delta$, then $dsw \leqslant 0$, which is true for all $\tau \leqslant \widetilde{\tau}$. Any combinations (τ, t_v) that lead to a usage fee $\widetilde{p}$, decrease the welfare as compare to the one obtain with $\widetilde{\tau}$.

If $\frac{c}{\eta-1}\overline{q} \geqslant f + \delta$, then $dsw \geqslant 0$ when $\tau = 0$, which is also true for $\tau \geqslant 0$. Then, any combinations (t_v, τ) that lead to a usage fee $\widetilde{p}$ decrease the welfare as compare to the one obtain with $\widetilde{t_v}$.

If $\frac{c}{\eta-1}\overline{q} > f + (1-\widetilde{\tau})^2\delta$ and $\frac{c}{\eta-1}\overline{q} < f + \delta$, then $t_v = 0$, $dsw > 0$ and for $\tau = 0$, $dsw < 0$. Then, it exists combinations (with strictly positive levels) that lead to a usage fee $\widetilde{p}$ and give a higher welfare than the one obtain with $\widetilde{\tau}$ or $\widetilde{t_v}$. ■

Proof. The proof (for the part on uniqueness, existence and symmetry) follows the same path and logic as the proof of proposition 1. First order conditions give(voir relation (12)):

$$\begin{aligned} u'(q^*) &= \frac{c + t_s}{1 - T_v} \\ P_i(q^*) &= \frac{(c+t_s)q^* + f}{1 - T_v} - 2\delta\alpha_i \end{aligned}$$

As the equilibrium is unique, each market share is equal to one half, the equilibrium price is then

$$P^* = \frac{(c+t_s)q^* + \delta + f}{1 - T_v},$$

profit of firm i is such that

$$\begin{aligned} \Pi_i &= \frac{1}{2}[(1-T_v)\frac{(c+t_s)q_i + \delta + f}{1-T_v} - (c+t_s)q_i - f] \\ &= \frac{\delta}{2} \end{aligned}$$

■

Proof. This proof is not as proofs of proposition 2 and 3. We show, here, that for a given individual consumption level, the social welfare associated is greater with wholly *ad valorem* taxation, T_v', than with a combination (T_v, t_s) that leads to the same consumption level. All the consumers are identical and connected, then we can focus on social welfare per consumer. The variation of social welfare is just the variation of fiscal revenue:

$$\Delta sw = \lambda(T_v'P' - t_s q - T_v P),$$

where $P = \delta + \frac{1}{1-T_v}\left[(c+t_s)q + f\right]$. This two taxation system leads to the same level of consumption if

$$t_s = \frac{(T_v' - T_v)c}{(1-T_v')}.$$

We have that $T_v' > T_v$ as $t_s > 0$. The variation is then equal to

$$\Delta sw = (T_v' - T_v)\left[\delta + \frac{1}{(1-T_v')(1-T_v)}f\right] > 0,$$

it is better to use the *ad valorem* tax on P. ■

Proof. We want to compare the level of social welfare (per consumer) obtained with a tariff $P(q)$ and an*ad valorem* tax T_v on P, with the one obtained with a two-part tariff $A + pq$ a tax on the subscription fee τ and an *ad valorem* tax on the usage fee t_v. All taxes are positive. In order to do this comparison, we suppose that the individual consumption level is the same whatever the tariff may be, We denote by $\overline{q}$ this level. Tax system should then be such that (from (1))

$$\frac{\eta c}{\eta(1-t_v) + t_v - \tau} = \frac{c}{1-T_v}. \tag{18}$$

The difference between the two level of social welfare is simply the difference of fiscal revenues

$$\Delta sw = T_v P - \tau A - t_v p\overline{q}$$

which gives after computations, taking into account (18)

$$\Delta sw = \frac{(\eta-1)t_v + \tau}{(1-t_v)\eta + t_v - \tau}(f+\delta) - \tau\delta - \frac{\tau}{1-\tau}f$$

$$+\frac{(\tau - t_v)c\overline{q}}{(1-\tau)\left((1-t_v)\eta + t_v - \tau\right)} \tag{19}$$

Suppose that the government, for the consumption level $\overline{q}$, prefers to only use the tax on the subscription fee. Then, from proposition 3 we have $\frac{c}{\eta-1}\overline{q} - f - (1-\overline{\tau})^2\delta \leqslant 0$, moreover as $t_v = 0$ we have

$$\Delta sw = \frac{\tau}{(1-\tau)(\eta-\tau)}\left[c\overline{q} - (\eta-1)f - (\eta-1)\delta + \tau(\eta-\tau)\delta\right].$$

Then $\Delta sw \leqslant 0$ if $\eta \geqslant 2$. In that case the government prefers two-part tariffs.

If for this given level $\overline{q}$ it is better to use the tax on the usage fee then from proposition 3, we have that $\frac{c}{\eta-1}\overline{q} - f - \delta \geqslant 0$. Then, as $\tau = 0$ and from (18) we have

$$\Delta sw = \frac{t_v}{(1-t_v)\eta + t_v}\left((\eta-1)(f+\delta) - c\overline{q}\right) \leqslant 0,$$

the government prefers two-part tariffs. ■