Strategic Multilateral Exchange and Taxes

Jean J. Gabszewicz[1] and Lisa Grazzini*[2]

[1] CORE, Université Catholique de Louvain, Belgium. E-mail: gabszewicz@core.ucl.ac.be
[2] CORE, Université Catholique de Louvain and Dipartimento di Scienze economiche, Università di Firenze. E-mail: grazzini@core.ucl.ac.be.

Abstract. This contribution investigates the effectiveness and welfare implications of fiscal policies in a context of multilateral trade, when traders behave strategically. The present approach deals simultaneously with two aspects of fiscal policies: collecting resources for redistributive purposes and correcting distortions related to imperfectly competitive behaviour.

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1 Introduction

Because it provides the most simple version of markets' interaction, the exchange model constitutes a natural starting point for exploring specific issues in general equilibrium theory. Professor Hildenbrand himself has not disregarded to use this model extensively in his elegant contributions to core theory; thus we hope he will forgive us to use it as well in our own contribution to the Festschrift prepared in his honour! This contribution investigates the effectiveness and welfare implications of fiscal policies in a context of multilateral trade, when traders behave strategically.

One of the objectives pursued when taxing economic agents is to collect resources for redistributive purposes. With imperfect competition, taxation can also serve the purpose of correcting distortions generated by the market mechanism. The present approach deals simultaneously with these two facets of fiscal policies, in an exchange economy in which some agents – the "inside agents" – participate to exchange while the remaining ones – the "outside agents"– are excluded from trade, simply because they do not own any initial resources: this constitutes a stylized formulation of more elaborate redistributive schemes. In the present context, the redistributive purpose of taxation consists accordingly in transferring to the outside agents initial resources levied on inside agents, in order to guarantee them some minimal survival resources. The first question we raise is whether these transfers can allow to

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reach a Pareto-optimal overall-allocation, providing preassigned utility levels to outside agents, and taking into account the fact that inside agents play a strategic market game when exchanging goods among them. Here the second facet of fiscal policy enters into the picture. Indeed, strategic behaviour generally affects trade in a way which destroys Pareto-efficiency, complicating thereby the possibility of reaching a Pareto-optimal outcome via transfers. Nevertheless, we show that there exist lump-sum taxes and transfers reaching a Pareto-optimal overall-allocation of goods among inside and outside agents, which simultaneously constitutes a Nash equilibrium outcome in the strategic market game played by the inside agents. This result is reminiscent of the second welfare theorem, but this time applies to strategic trade.

The significance of this result should however be tempered due to the well-known drawbacks of lump-sum taxes: informational requirements needed to make them effective and non-anonymity, since they can require to be personalized on the basis of the characteristics of each agent. It is thus natural to examine whether alternative methods of taxation could not be effective in reaching a Pareto-optimal outcome while avoiding the drawbacks we have just referred to. Leaving aside the informational problem of identifying these characteristics, we examine then whether non-anonymity can be somewhat relaxed by imposing taxes affecting each good in the same proportion, as it is in particular the case for commodity taxation, which bears on transactions, and as if it would be the case if taxes would bear on initial endowments with the same tax rate per good for all agents (endowment taxation). Using a simple example, we show that such fiscal policies are not sufficient tools to reach an overall Pareto-optimal allocation: trade distortions due to imperfect competition cannot be wiped out via such methods. Finally, we propose an alternative method combining anonymous endowment taxation and transfers among inside agents, proportional to the amount sent for exchange, and test it on our particular example. We show that this method is effective both in resorbing trade distortions due to strategic behaviour, and in providing an overall-allocation among inside and outside agents which is Pareto-optimal.

The above analysis uses a strategic market game formulation in order to describe the outcome resulting from strategic interaction among the inside agents in the exchange economy (Shapley (1976), Shapley and Shubik (1977), Gabszewicz and Michel (1997)). This formulation can be viewed as an extension of the Cournot oligopoly model into two directions: first, it is cast into a general equilibrium context and, second, *all* participants to exchange behave strategically using quantities as strategies. To the best of our knowledge, the only paper using strategic market games in order to explore fiscal policy measures in a context of market power is Gabszewicz and Grazzini (1999).[1] This paper concentrates on the second facet of fiscal policy measures re-

[1] This analysis relies on the non-cooperative concept of Nash equilibrium, and should be distinguished from the cooperative framework used in other contexts

ferred above. Namely, it raises the question whether taxing strategic agents could induce them to restore a competitive outcome while continuing to exert their market power. Through a series of examples of *bilateral oligopolies*, introduced in Cordella and Gabszewicz (1998) and Bloch and Ferrer (2000), we show that taxes combined with transfers proportional to the amounts of goods sent for exchange are sufficient instruments to resorb the distortions due to strategic behaviour and restore competitive exchange. The present contribution can be viewed as an extension of the latter paper since, beyond the problem of counterdistortionary measures, it also deals with the issue of redistributing resources not only among agents directly participating to exchange, but also in favor of some agents who are excluded from trade.

Two earlier contributions to the problem of taxing market power in a general equilibrium set-up are those of Myles (1989) and Guesnerie and Laffont (1978). Myles (1989) extends the theory of optimal commodity taxation to a general equilibrium model with imperfect competition. His analysis is focused on the Ramsey problem of choosing commodity tax rates to maximise welfare subject to a given level of public revenue when markets operate in a non competitive framework. Guesnerie and Laffont (1978) study optimal taxation when the objective of the government is to restore first best Pareto efficiency when this first best is destroyed by the non competitive behaviour of a monopolist. Although the issues analysed in the above contributions are close to our own subject, they differ from ours in several respects. First, their analysis is cast into a model inspired from Gabszewicz and Vial (1972), which constitutes a much more general set-up than the one considered here. This model involves a productive sector with firms behaving noncompetitively while consumers are price-takers. However, a difficulty arises with this formulation, because taxes and transfers are not invariant with respect to the normalization rule used to normalize prices, while this difficulty disappears in our approach. Furthermore, only some agents behave strategically: in particular, the Guesnerie-Laffont's paper considers only the case of a monopolistic agent.

The next section provides a general definition of a strategic market game and studies the possibility of reaching an overall Pareto-optimal allocation when the outcome of this game depends on lump-sum taxes on inside agents to be transferred to the outside agents. Sections 3, 4 and 5 are devoted to analyse the possibility of obtaining a similar result using more anonymous tax and transfer schemes, like commodity taxation (section 3), endowment taxation (section 4) and endowment taxation combined with incentive transfers (section 5). Section 6 provides a short conclusion.

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in which taxation and strategic power have been analysed, like in Aumann and Kurz (1977).

2 Strategic Market Games and Pareto-Optimality

In this section, we first define a general class of strategic market games as follows. Consider an exchange economy involving $n+m$ agents i, $i = 1, ..., n+m$, and l goods h, $h = 1, ..., l$. The initial ownership of these goods is fully concentrated in the hands of the first n agents, ω_i, $\omega_i \in R^l_+$, $\omega_i \neq 0$, denoting the *initial endowment* of agent i, $i = 1, ..., n$. By assumption, we set $\omega_i = 0$, for agents i, $i = n+1, ..., n+m$: only agents i, $i = 1, ..., n$, named hereafter the *inside agents*, participate to exchange and are accordingly the *players* in the strategic market game. Agents i, $i = n+1, ..., n+m$, are excluded from trade and we call them the *outside agents*. We denote by U_i, $i = 1, ..., n+m$, the *utility function* of agent i, representing his preference relation among the commodity bundles x, $x \in R^l_+$. Standard assumptions on U_i, -continuity, strict monotonicity and strict quasi-concavity-, are made from the outset. An *allocation* is a n-tuple of commodity bundles $(x_1, ..., x_i, ..., x_n)$ such that $\sum_{i=1}^n x_i = \sum_{i=1}^n \omega_i$. The strategy set S_i of agent i, $i = 1, ..., n$, is defined by $S_i = \{y \mid 0 \leq y \leq \omega_i\}$. Given an n-tuple of strategies $(y_1, ..., y_i, ..., y_n)$, a price vector $p(y_1, ..., y_i, ..., y_n) \in R^l_+$ obtains as a result of some price formation mechanism (an example is considered below). To this price vector is associated an allocation assigning the bundle $z_i(p(y_1, ..., y_i, ..., y_n))$ to player i, $i = 1, ..., n$, which specifies the commodity bundle he obtains at the vector of strategies $(y_1, ..., y_i, ..., y_n)$. The *payoffs* V_i of the strategic market game are defined as

$$V_i(y_1, ..., y_i, ..., y_n) = U_i(z_i(p(y_1, ..., y_i, ..., y_n))).$$

The particular vector of strategies $(y_1, ..., y_i, ..., y_n) = (0, ..., 0, ..., 0)$ corresponds to a situation in which no player sends any strictly positive amount of some commodity to the market for trade. In that case, no trade can occur and the utility level enjoyed by agent i at this vector of strategies is given by the utility level reached at his initial endowment, i.e.

$$V_i(0, ..., 0, ..., 0) = U_i(\omega_i). \tag{1}$$

Furthermore, when all players, but one, -say player i-, select as strategy the null vector, no trade can occur either, so that the utility level reached by player i cannot exceed in that case the level $U_i(\omega_i)$. Thus, we set, for all $i = 1, ..., n$, and all $y_i \neq 0$,

$$U_i(0, ..., y_i, ..., 0) \leq U_i(\omega_i). \tag{2}$$

A *strategic equilibrium* is a Nash equilibrium of the strategic market game we have just defined, namely, a n-tuple of strategies $(y_1^*, ..., y_i^*, ..., y_n^*)$, $y_i^* \in S_i$, such that, $\forall i = 1, ..., n$, $\forall y_i \in S_i$

$$U_i(z_i(p(y_i^*, y_{-i}^*))) \geq U_i(z_i(p(y_i, y_{-i}^*))),$$

with y^*_{-i} denoting the $(n-1)$-tuple of strategies y^*_k, $k \neq i$.

To illustrate the above definition of a strategic market game, we consider the price formation mechanism proposed by Gabszewicz and Michel (1997), which will be used below in the examples analysed in the following sections.[2] Let each agent i, $i = 1, ..., n$, choose a strategy $y_i \in S_i$ and consider a price vector p, $p \neq 0$. Given the choice y_i, the income of agent i (i.e. the value of resources sent to the market) is equal to $p \cdot y_i$, where $p \cdot y_i$ denotes the scalar product of p and y_i. If trader i, endowed with income $p \cdot y_i$, chooses to buy a commodity bundle x, he reaches a utility level equal to $U_i(\omega_i - y_i + x)$: $\omega_i - y_i$ is the bundle of goods still remaining available for consumption given the choice of strategy y_i, and x is the vector of purchases performed at the price system p. Accordingly, given p and y_i, the choice of x is dictated by the solution to the program

$$\operatorname*{Max}_{x} \qquad U_i(\omega_i - y_i + x) \quad s.t. \quad p \cdot x \leq p \cdot y_i \quad and \quad x \geq 0.$$

Denote by $x_i(p, y_i)$ the solution to this problem: by monotonicity, continuity and strict quasi-concavity of U_i, this solution exists and is unique.

Let $p(y_1, ..., y_i, ..., y_n)$ be a price system which clears all markets in the exchange economy in which traders i, $i = 1, ..., n$, are endowed with initial holdings y_i, i.e. $p(y_1, ..., y_i, ..., y_n)$ solves

$$\sum_{i=1}^{n} x_i(p, y_i) = \sum_{i=1}^{n} y_i \tag{3}$$

(standard assumptions guarantee the existence of a price system $p(y_1, ..., y_i, ..., y_n)$ for any n-tuple of strategies $(y_1, ..., y_i, ..., y_n)$, $y_i \in S_i$, at which trade occurs; we shall *assume* that this price system is unique). The payoff V_i of the game is then defined by

$$V_i(y_1, ..., y_i, ..., y_n) = U_i(z_i(p(y_1, ..., y_i, ..., y_n))),$$

with $z_i(p(y_1, ..., y_i, ..., y_n)) = \omega_i - y_i + x_i(p(y_1, ..., y_i, ..., y_n))$. Furthermore, if all players i select the strategy $y_i = 0$, no exchange can occur and the price system $p(y_1, ..., y_i, ..., y_n)$ is not defined, so that all traders remain with their initial endowments and reach utility levels $V_i(0, ..., 0, ..., 0) = U_i(\omega_i)$. If one of these traders would deviate from 0 to some alternative strategy $y_i \neq 0$, while the remaining traders stick to the strategy 0, no trade can occur either, and the utility level of player i is given by $V_i(0, ..., y_i, ..., 0) = U_i(\omega_i - y_i) < U_i(\omega_i)$, where strict inequality follows from the monotonicity assumption.

[2] Alternative mechanisms have been proposed in the literature on strategic market games; see, for example, Dubey and Shubik (1978), Amir, Sahi, Shubik and Yao (1990).

Accordingly, for the price mechanism we have just described, the payoffs V_i satisfy equations (1) and (2).

In the following we are interested in identifying tax and transfer schemes through which outside agents i, $i = n+1, ..., n+m$, obtain a share of the resources initially owned by inside agents who participate directly to the exchange process described in the above strategic market game. The redistributive purpose pursued with such schemes consists in providing agents, who are initially deprived from any resource, with some amount of the various commodities, thereby allowing them to survive, in spite of the fact that they are unable to participate directly to trade. Formally, we assume that these "survival" levels are represented by utility levels -say $\overline{U}_i$-, for the outside agents i, $i = n+1, .., n+m$. These utility levels constrain accordingly the set of transfers and corresponding taxes which can be used in order to meet these preassigned levels. In this spirit, we define utility levels $\overline{U}_i$, $i = n+1, .., n+m$, as *feasible* if there exists lump-sum taxes t_i, $t_i \in R^l_+$, and transfers s_i, $s_i \in R^l_+$, $i = n+1, .., n+m$, such that

$$(i)\ \ t_i \leq \omega_i, \quad i = 1, ..., n;$$

$$(ii)\ \ \textstyle\sum_{i=n+1}^{n+m} s_i = \sum_{i=1}^{n} t_i,$$

$$(iii)\ U_i(s_i) \geq \overline{U}_i, \quad i = n+1, ..., n+m.$$

Given taxes t_i, a *post-tax allocation* among inside agents is a n-tuple of commodity bundles $(x_1, ..., x_i, ..., x_n)$ such that

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} (\omega_i - t_i).$$

Now we are in a position to state the following

Proposition 1. *Given any preassigned feasible utility levels $\overline{U}_i$ for outside agents i, $i = n+1, ..., n+m$, there exist lump-sum taxes t_i on inside agents' endowments ω_i, $i = 1, ..., n$, and transfers s_i from inside agents to outside agents, such that*
(i) the overall-allocation of goods $(\omega_1 - t_1, ..., \omega_n - t_n; s_{n+1}, ..., s_{n+m})$ among inside and outside agents is Pareto-optimal, and outside agents obtain, at that overall-allocation, commodity bundles providing utility levels $\overline{U}_i$, $i = n+1, ..., n+m$;
(ii) the post-tax allocation $(\omega_1 - t_1, ..., \omega_n - t_n)$ among inside agents is the unique Nash equilibrium of the strategic market game in which the strategy sets of inside agents are defined by $S_i = \{y_i \mid 0 \leq y_i \leq \omega_i - t_i\}$.

Proof. In the exchange economy with $(n+m)$ agents made of the inside agents $i = 1, ..., n$ and the outside agents $i = n+1, ..., n+m$, a *(n+m)*-tuple of commodity bundles $(x_1^*, ..., x_n^*; x_{n+1}^*, ..., x_{n+m}^*)$ is Pareto-optimal if it solves the problem

$$\begin{aligned} &\max_{x_1,\dots,x_{n+m}} \textstyle\sum_{i=1}^{n+m} U_i(x_i) \qquad s.t. \\ &(i) \qquad \textstyle\sum_{i=1}^{n+m} x_i = \sum_{i=1}^{n} \omega_i \\ &(ii) \qquad x_i \geq 0, \qquad i = 1, ..., n+m. \end{aligned} \tag{4}$$

Furthermore, since the preassigned utility levels $\overline{U}_i$, $i = n+1, ..., n+m$, are feasible for lump-sum taxes t_i and transfers s_i, the *(n+m)*-tuple of commodity bundles $(x_1^*, ..., x_n^*; x_{n+1}^*, ..., x_{n+m}^*)$ can be chosen eventually in order to solve the problem

$$\begin{aligned} &\max_{x_1,\dots,x_{n+m}} \textstyle\sum_{i=1}^{n} U_i(x_i) \qquad s.t. \\ &(i) \qquad \textstyle\sum_{i=n+1}^{n+m} s_i = \sum_{i=1}^{n} t_i \\ &(ii) \qquad x_i = \omega_i - t_i \geq 0; \quad i = 1, ..., n; \\ &(iii) \qquad x_i = s_i \geq 0; \quad i = n+1, ..., n+m; \\ &(iv) \qquad U_i(x_i) = U_i(s_i) \geq \overline{U}_i, \qquad i = n+1, ..., n+m. \end{aligned} \tag{5}$$

Since the function $\sum_{i=1}^{n} U_i(x_i)$ is continuous as a sum of continuous functions, and the set of vectors satisfying (i), (ii), (iii) and (iv) is non-empty and compact, there exists a *(n+m)*-tuple of commodity bundles $(x_1^*, , ..., x_n^*; x_{n+1}^*, ..., x_{n+m}^*) = (\omega_1 - t_1^*, ..., \omega_n - t_n^*;\ s_{n+1}^*, ..., s_{n+m}^*)$ which is a solution to problem (5).

Now consider the exchange economy consisting of the n inside traders with initial endowments $\omega_i - t_i^*$, $i = 1, ..., n$. The allocation $(\omega_1 - t_1^*, ..., \omega_n - t_n^*)$ of the resources $\sum_{i=1}^{n} (\omega_i - t_i^*)$ among inside agents is also Pareto-optimal in the subeconomy consisting of these inside agents. If some agent $i \in \{1, ..., n\}$ uses a strategy y_i which differs from the null vector, he reaches a utility level $V_i(y_i, y_{-i})$, with $y_{-i} = (0, ..., 0)$, smaller or equal to $V_i(0, ..., 0)$, in the subeconomy consisting of n inside agents (see (2)). Accordingly, no unilateral deviation from the allocation $(\omega_1 - t_1^*, ..., \omega_n - t_n^*)$ can increase the utility of an inside agent, so that $(\omega_1 - t_1^*, ..., \omega_n - t_n^*)$ is a Nash equilibrium in the strategic market game in which the set of strategies S_i of agent i is given by $S_i = \{y_i \mid 0 \leq y_i \leq \omega_i - t_i^*\}$, $i = 1, ..., n$.

Finally, the uniqueness of equilibrium follows from the fact that any alternative strategic equilibrium would generate an allocation which is Pareto dominated by the allocation $(\omega_1 - t_1^*, ..., \omega_n - t_n^*)$. Accordingly, at least one inside agent can deviate from the strategy he has selected at this alternative Nash equilibrium by playing the strategy 0, and obtain a higher payoff, a contradiction. ∎

Proposition 1 is simply a restatement of the second welfare theorem formulated for our particular setting with some agents excluded from trade and the remaining ones behaving strategically while, in the usual formulation of this theorem, the Pareto-optimal allocation is viewed as "sustained" by competitive prices. The proof of this proposition takes advantage of the fact that the initial allocation of an exchange economy always appears as the outcome of a Nash equilibrium in the associated market game (the so-called *trivial equilibrium*, see Dubey and Shubik (1978)).

The usual criticisms against lump-sum taxes and transfers needed to sustain a particular Pareto-optimal allocation as a competitive outcome, applies as well in our attempt to sustain such an allocation as a strategic equilibrium. In particular, these taxes and transfers need to be personalized since they vary across agents depending on the quantity of resources required to achieve the particular Pareto-optimal allocation. Thus, it is natural to wonder whether it is possible to design taxes which are not linked to individual characteristics, but rather to the commodities themselves, like in the case of commodity taxation. Such taxes appear as more *anonymous*, to the extent that they are imposed on all consumers in the same manner for the same good. In an exchange economy, one can think of two types of taxes satisfying this anonymity criterion. The first one consists in imposing a tax t^h on good h, to be collected on transactions concerning good h: this is *commodity taxation*. The second one consists in imposing a tax t^h on the $h^{th}-$component of ω_i, independently of i, before transactions take place: this is *endowment taxation*. In the next two sections, we examine whether using such tax instruments could be sufficient to perform the job which we have just seen to be feasible with personalized lump-sum taxes and transfers.

3 Commodity Taxation

In this section, we construct a particular example of an exchange economy and show that commodity taxation is not a sufficient instrument in order to realize a Pareto-optimal allocation when inside agents behave strategically. Consider an exchange economy with two goods, 1 and 2, and consisting of $n+1$ agents, namely n inside agents and one outside agent. The inside agents fall into two *types*, with $\frac{n}{2}$ agents of each type. All agents i, $i = 1, ..., n+1$, have the same utility function U, which is defined by

$$U(x^1, x^2) = x^1 x^2. \tag{6}$$

Initial endowments are defined by

$$\omega_i = (1, 0), \qquad i = 1, ..., \frac{n}{2} \tag{7}$$

$$\omega_i = (0, 1), \qquad i = \frac{n}{2} + 1, ..., n \tag{8}$$

and

$$\omega_i = (0,0), \qquad i = n+1. \tag{9}$$

To the exchange economy consisting of the n inside agents, we may associate a strategic market game Γ as follows. For all agents, the *strategy set* S_i consists of the $[0,1]$ - interval. For an agent $i \in \{1,...,\frac{n}{2}\}$, we denote by q_i a strategy in $[0,1]$: q_i is to be interpreted as the quantity of good 1 that agent i sends to the market for trade. Similarly, b_i, $b_i \in [0,1]$, denotes a strategy of agent i, $i \in \{\frac{n}{2}+1,...,n\}$. Given an n-tuple of strategies $(q,b) = (q_1,...,q_{\frac{n}{2}}; b_{\frac{n}{2}+1},...,b_n)$, the resulting allocation of goods obtains as

$$(1-q_i, pq_i)$$

for an agent i in $\{1,...,\frac{n}{2}\}$, and

$$(\frac{b_i}{p}, 1-b_i)$$

for an agent i in $\{\frac{n}{2}+1,...,n\}$, with p denoting the price of good 1 expressed in units of good 2 (good 2 serves as numeraire). Using the price mechanism defined above by equation (3), this price follows from the equality of demand and supply of good 1, i.e. it satisfies the equation:

$$\sum_{k=\frac{n}{2}+1}^{n} b_k = p \sum_{k=1}^{\frac{n}{2}} q_k;$$

or

$$p = \frac{\sum_{k=\frac{n}{2}+1}^{n} b_k}{\sum_{k=1}^{\frac{n}{2}} q_k}.$$

Accordingly, given an n-tuple (q,b) of strategies, the *payoffs* in the strategic market game Γ obtain as

$$V_i(q,b) = U(1-q_i, \frac{\sum_{k=\frac{n}{2}+1}^{n} b_k}{\sum_{k=1}^{\frac{n}{2}} q_k} q_i),$$

for $i = 1,...,\frac{n}{2}$, and

$$V_i(q,b) = U(\frac{\sum_{k=1}^{\frac{n}{2}} q_k}{\sum_{k=\frac{n}{2}+1}^{n} b_k} b_i, 1-b_i),$$

for $i = \frac{n}{2}+1,...,n$.

At the vector of strategies $(\hat{q},\hat{b})$ defined by

$$(\hat{q},\hat{b}) = (0,\ldots,0;0,\ldots,0),$$

payoffs are defined by

$$V_i(\hat{q},\hat{b}) = U(\omega_i) = 0, \qquad i = 1, ..., n.$$

A *strategic equilibrium* is a n-tuple of strategies $(q_1^*, ..., q_{\frac{n}{2}}^*; b_{\frac{n}{2}+1}^*, ..., b_n^*)$ such that no trader has an advantage to deviate unilaterally from his choice. As noticed above, it is a general property of strategic market games that the vector of strategies $(\hat{q},\hat{b})$ where each trader bids and supplies nothing is always a strategic equilibrium (the so-called *trivial equilibrium*). As shown in Gabszewicz and Michel (1997) where this example is introduced, all traders of the same type must adopt the same strategy at equilibrium and, apart from the trivial one, *the vector of strategies* (q^*, b^*) *defined by*

$$(q^*, b^*) = \left(\frac{n-2}{2(n-1)}, \frac{n-2}{2(n-1)}\right) \tag{10}$$

is the only strategic equilibrium of the market game Γ.[3]

Now suppose that commodity taxes t^h, $h = 1, 2$, are levied on transactions in order to provide the outside agent $n+1$ with some amount of the two goods and realize thereby a feasible utility level $\overline{U}$ for him. More specifically, suppose that, *when exchange takes place*, a per unit tax t^1, $0 < t^1 < 1$, is levied on the supply of good 1 and a per unit tax t^2, $0 < t^2 < 1$, is levied on the supply of good 2, giving rise to a total tax product equal to $t^1 \sum_{k=1}^{\frac{n}{2}} q_k$ units of good 1, and $t^2 \sum_{k=\frac{n}{2}+1}^{n} b_k$ units of good 2. Furthermore, suppose that, *after trade has occurred* at an n-tuple of strategies $(q_1, ..., q_{\frac{n}{2}}; b_{\frac{n}{2}+1}, ..., b_n)$, the total product of the tax $t^1 \sum_{k=1}^{\frac{n}{2}} q_k$ in good 1 and $t^2 \sum_{k=\frac{n}{2}+1}^{n} b_k$ in good 2 is transferred to the outside agent $n+1$. Consequently, this commodity tax and transfer scheme generates a new strategic market game Γ' as follows. Given a commodity tax t^1, $0 < t^1 < 1$, and a commodity tax t^2, $0 < t^2 < 1$, the strategy set of inside agents i, $i = 1, ..., n$, is the interval $[0, 1]$. Furthermore, the initial endowments are still $\omega_i = (1, 0)$, $i \in \left\{1, ..., \frac{n}{2}\right\}$ and $\omega_i = (0, 1)$, $i \in \left\{\frac{n}{2}+1, ..., n\right\}$. However, the price $p(q_1, ..., q_{\frac{n}{2}}; b_{\frac{n}{2}+1}, ..., b_n)$ now obtains as

$$p(q_1, ..., q_{\frac{n}{2}}; b_{\frac{n}{2}+1}, ..., b_n) = \frac{(1-t^2)\sum_{k=\frac{n}{2}+1}^{n} b_k}{(1-t^1)\sum_{k=1}^{\frac{n}{2}} q_k},$$

and the resulting post-tax allocation of goods in the game Γ' is given by

$$x_i = \left(1 - q_i, \frac{(1-t^2)\sum_{k=\frac{n}{2}+1}^{n} b_k}{(1-t^1)\sum_{k=1}^{\frac{n}{2}} q_k} q_i (1-t^1)\right), \qquad i = 1, ..., \frac{n}{2} \tag{11}$$

[3] It is easy to see that the strategic equilibrium converges to the unique competitive equilibrium of the exchange economy when $n \to \infty$. This simply reflects the fact that competition is restored when the number of traders increases without limit.

$$x_i = \left(\frac{(1-t^1)\sum_{k=1}^{\frac{n}{2}} q_k}{(1-t^2)\sum_{k=\frac{n}{2}+1}^{n} b_k} b_i (1-t^2), 1-b_i \right). \qquad i = \frac{n}{2}+1, ..., n \quad (12)$$

It gives rise to utility levels

$$V_i(q,b) = (1-q_i)\left(\frac{(1-t^2)\sum_{k=\frac{n}{2}+1}^{n} b_k}{\sum_{k=1}^{\frac{n}{2}} q_k} q_i \right), \qquad i = 1, ..., \frac{n}{2} \qquad (13)$$

and

$$V_i(q,b) = \left(\frac{(1-t^1)\sum_{k=1}^{\frac{n}{2}} q_k}{\sum_{k=\frac{n}{2}+1}^{n} b_k} b_i \right)(1-b_i), \qquad i = \frac{n}{2}+1, ..., n. \qquad (14)$$

In the following proposition, we show that the commodity taxes and transfers scheme defined above does not allow to obtain a Pareto-optimal overall-allocation, with the outside agent obtaining the preassigned utility level $\overline{U}$ at the outcome of the strategic market game Γ'.

Proposition 2. *Given any utility level $\overline{U}$ for the outside agent $n+1$, there do not exist commodity taxes t^h, $h = 1,2$ such that, when the product of these taxes is transferred to agent $n+1$, (i) the overall-allocation resulting from this transfer and from the strategic equilibrium of the game Γ' is Pareto-optimal and (ii) the utility of the outside agent after transfer is at least equal to $\overline{U}$.*

Proof. Suppose, contrary to proposition 2, that there exist commodity taxes t^h, $h = 1, 2$, for which requirements *(i)* and *(ii)* are met. First, we show that, even with these taxes t^h, $h = 1, 2$, the vector of strategies (q^*, b^*) of the game Γ, as obtained in (10), remains the unique interior Nash equilibrium of the strategic market game Γ'. The first order conditions which must be satisfied at an interior equilibrium are

$$\frac{\partial V_i}{\partial q_i} = (1-q_i)\left(\frac{\sum_{k=\frac{n}{2}+1}^{n} b_k \left(\sum_{k=1}^{\frac{n}{2}} q_k - q_i\right)}{\left(\sum_{k=1}^{\frac{n}{2}} q_k\right)^2} \right) - \frac{\sum_{k=\frac{n}{2}+1}^{n} b_k}{\sum_{k=1}^{\frac{n}{2}} q_k} q_i = 0, \quad i = 1, ..., \tfrac{n}{2}$$

$$\frac{\partial V_i}{\partial b_i} = (1-b_i)\left(\frac{\sum_{k=1}^{\frac{n}{2}} q_k \left(\sum_{k=\frac{n}{2}+1}^{n} b_k - b_i\right)}{\left(\sum_{k=\frac{n}{2}+1}^{n} b_k\right)^2} \right) - \frac{\sum_{k=1}^{\frac{n}{2}} q_k}{\sum_{k=\frac{n}{2}+1}^{n} b_k} b_i = 0, \quad i = \tfrac{n}{2}+1, ..., n.$$

Using the fact that all traders of the same type must adopt the same strategy at equilibrium,[4] we may denote by q (resp. b) the supply of inside agents $i \in \left\{1, ..., \frac{n}{2}\right\}$ (resp. $i \in \left\{\frac{n}{2}+1, ..., n\right\}$) on the market. Using this property, the first order conditions which must be satisfied at an interior strategic equilibrium can be rewritten as

$$\frac{(n-2)(1-q)}{nq} - 1 = 0, \qquad i = 1, ..., \frac{n}{2} \qquad (15)$$

[4] For a formal proof, see Gabszewicz and Grazzini (1999), Appendix, p.493.

$$\frac{(n-2)(1-b)}{nb}-1=0, \qquad i=\frac{n}{2}+1,...,n, \tag{16}$$

so that the interior equilibrium is given by $(q^*,b^*)=\left(\frac{n-2}{2(n-1)},\frac{n-2}{2(n-1)}\right)$ (simple calculations show that this vector of strategies is the unique interior equilibrium). Furthermore, notice that, at the strategic equilibrium, we have

$$p(q^*,b^*)=\frac{1-t^2}{1-t^1}. \tag{17}$$

Substituting the equilibrium values q^* and b^* into the payoffs V_i (see (13) and (14)), we obtain

$$V_i(q^*,b^*)=\frac{n(n-2)(1-t^2)}{4(n-1)^2}, \qquad i=1,...,\frac{n}{2} \tag{18}$$

and

$$V_i(q^*,b^*)=\frac{n(n-2)(1-t^1)}{4(n-1)^2}, \qquad i=\frac{n}{2}+1,...,n. \tag{19}$$

Any Pareto-optimal allocation, which would follow from commodity taxes t^1 and t^2 and provide agent $n+1$ with utility level $\overline{U}$, must solve the problem

$$\operatorname*{Max}_{t^1,t^2} \textstyle\sum_{i=1}^{n} V_i(q^*,b^*) \qquad s.t.$$

$$U\left(\tfrac{n}{2}t^1q^*,\tfrac{n}{2}t^2b^*\right)=\overline{U},$$

or

$$\begin{gathered}\operatorname*{Max}_{t^1,t^2} \tfrac{n}{2}\cdot\tfrac{n(n-2)(1-t^2)}{4(n-1)^2}+\tfrac{n}{2}\cdot\tfrac{n(n-2)(1-t^1)}{4(n-1)^2} \qquad s.t. \\ \left(t^1\tfrac{n}{2}\tfrac{(n-2)}{2(n-1)}\right)\left(t^2\tfrac{n}{2}\tfrac{(n-2)}{2(n-1)}\right)=\overline{U}.\end{gathered} \tag{20}$$

Simple calculations show that the unique solution to this problem is $t^1=t^2=t^*=\frac{4(n-1)\sqrt{\overline{U}}}{n(n-2)}$. Substituting the value t^* into (11) and (12) reveals that the marginal rate of substitution between good 1 and good 2 is equal to $1-\frac{2(1+2\sqrt{\overline{U}})}{n}+\frac{4\sqrt{\overline{U}}}{n^2}$ for the inside agents i, $i=1,...,\frac{n}{2}$, to $1/\left(1-\frac{2(1+2\sqrt{\overline{U}})}{n}+\frac{4\sqrt{\overline{U}}}{n^2}\right)$ for the inside agents i, $i=\frac{n}{2}+1,...,n$, and to 1 for the outside agent $2n+1$. Consequently, these marginal rates of substitution vary across agents, so that the resulting overall-allocation cannot be Pareto-optimal, a contradiction to our initial assumption. ∎

Having reached this negative outcome, we examine in the next section whether endowment taxation cannot realize the objective which was unsuccessfully pursued with commodity taxation.

4 Endowment Taxation

Consider now the following tax and transfer scheme. Firstly, *before exchange takes place*, taxes are introduced which consist in collecting, for each good, and each inside agent, a share t^h, $h = 1, 2$, of the amount of commodity h he owns initially: we call this tax scheme *endowment taxation*. Secondly, *after trade has occurred*, the product of these taxes is redistributed in favor of the outside agent who gets accordingly $\frac{n}{2}t^h$ of good $h = 1, 2$.

More specifically, consider again the same example of exchange economy as in section 3. The endowment tax and transfer scheme we have just considered generates a new strategic market game Γ'' as follows. Since, after taxation, traders are deprived from a share of their initial endowment, they start the game Γ'' with strategy sets S_i equal to the interval $\left[0, 1 - t^1\right]$ for agents $i = 1, ..., \frac{n}{2}$, and to the interval $\left[0, 1 - t^2\right]$ for $i = \frac{n}{2} + 1, ..., n$. Given any n-tuple of strategies $(q_1, ..., q_{\frac{n}{2}}; b_{\frac{n}{2}+1}, ..., b_n)$ the resulting price is

$$p(q_1, ..., q_{\frac{n}{2}}; b_{\frac{n}{2}+1}, ..., b_n) = \frac{\sum_{k=\frac{n}{2}+1}^{n} b_k}{\sum_{k=1}^{\frac{n}{2}} q_k},$$

giving rise to the post-tax allocation of goods

$$x_i = \left(1 - t^1 - q_i, \frac{\sum_{k=\frac{n}{2}+1}^{n} b_k}{\sum_{k=1}^{\frac{n}{2}} q_k} q_i\right), \qquad i = 1, ..., \frac{n}{2} \tag{21}$$

$$x_i = \left(\frac{\sum_{k=1}^{\frac{n}{2}} q_k}{\sum_{k=\frac{n}{2}+1}^{n} b_k} b_i, 1 - t^2 - b_i\right). \qquad i = \frac{n}{2} + 1, ..., n \tag{22}$$

These outcomes generate utility levels

$$V_i(q, b) = (1 - t^1 - q_i)\left(\frac{\sum_{k=\frac{n}{2}+1}^{n} b_k}{\sum_{k=1}^{\frac{n}{2}} q_k} q_i\right), \qquad i = 1, ..., \frac{n}{2} \tag{23}$$

and

$$V_i(q, b) = \left(\frac{\sum_{k=1}^{\frac{n}{2}} q_k}{\sum_{k=\frac{n}{2}+1}^{n} b_k} b_i\right)(1 - t^2 - b_i), \qquad i = \frac{n}{2} + 1, ..., n. \tag{24}$$

In the following proposition, we show that the endowment tax and transfer scheme we consider above does not allow either to obtain a Pareto-optimal overall-allocation, with the outside agent obtaining the preassigned utility level $\overline{U}$, at the outcome of the strategic market game Γ''.

Proposition 3. *Given any utility level $\overline{U}$ for the outside agent $n + 1$, there do not exist endowment taxes t^h, $h = 1, 2$ such that, when the product of these taxes is transferred to agent $n+1$, (i) the overall-allocation resulting from this transfer and from the strategic equilibrium of the game Γ'' is Pareto-optimal and (ii) the utility of the outside agent after transfer is at least equal to $\overline{U}$.*

Proof. Suppose, contrary to proposition 3, that there exist endowment taxes t^h, $h = 1, 2$, for which requirements *(i)* and *(ii)* are met. First, we show that (q^*, b^*) defined by

$$(q^*, b^*) = \left(\frac{(1-t^1)(n-2)}{2(n-1)}, \frac{(1-t^2)(n-2)}{2(n-1)} \right), \tag{25}$$

is the unique interior strategic equilibrium of the market game Γ''. Using the fact that all traders of the same type must adopt the same strategy at equilibrium, we may denote by q (resp. b) the supply of inside agents $i \in \left\{1, ..., \frac{n}{2}\right\}$ (resp. $i \in \left\{\frac{n}{2}+1, ..., n\right\}$) on the market. The first order conditions which must be satisfied at an interior equilibrium can then be written as

$$\frac{(1-t^1-q)(n-2)}{nq} - 1 = 0, \qquad i = 1, ..., \frac{n}{2} \tag{26}$$

$$\frac{(1-t^2-b)(n-2)}{nb} - 1 = 0, \qquad i = \frac{n}{2}+1, ..., n. \tag{27}$$

Simple calculations show that the vector of strategies (q^*, b^*) in (25) is the unique interior equilibrium. Furthermore, notice that, at the strategic equilibrium, we have

$$p(q^*, b^*) = \frac{1-t^2}{1-t^1}. \tag{28}$$

Substituting the equilibrium values q^* and b^* into the payoffs V_i (see (23) and (24)), we obtain

$$V_i(q^*, b^*) = \frac{n(n-2)(1-t^1)(1-t^2)}{4(n-1)^2}, \qquad i = 1, ..., n. \tag{29}$$

Any Pareto-optimal allocation which would follow from endowment taxes t^1 and t^2 and provide agent $n+1$ with utility level $\overline{U}$ must solve the problem

$$\underset{t^1,t^2}{\text{Max}} \sum_{i=1}^{n} V_i(q^*, b^*) \qquad s.t.$$

$$U\left(\tfrac{n}{2}t^1, \tfrac{n}{2}t^2\right) = \overline{U},$$

or

$$\underset{t^1,t^2}{\text{Max}}\; n \cdot \tfrac{n(n-2)(1-t^1)(1-t^2)}{4(n-1)^2} \qquad s.t.$$
$$\left(\tfrac{n}{2}t^1\right)\left(\tfrac{n}{2}t^2\right) = \overline{U}. \tag{30}$$

Simple calculations show that the unique solution to this problem is $t^1 = t^2 = t^* = \frac{2\sqrt{\overline{u}}}{n}$. Substituting the optimal value t^* into (21) and (22) reveals that the marginal rate of substitution between good 1 and good 2 is equal to $\frac{n-2}{n}$ for the inside agents i, $i = 1, ..., \frac{n}{2}$, to $\frac{n}{n-2}$ for the inside agents i, $i = \frac{n}{2} + 1, ..., n$, and to 1 for the outside agent $2n + 1$. Consequently, these marginal rates of substitution vary across agents, so that the resulting overall-allocation cannot be Pareto-optimal, a contradiction. ∎

The negative results shown in section 3 and 4 both follow from the same cause: the tax schemes proposed are not powerful enough to wipe out the distortions introduced by the strategic behaviour of inside agents. Optimal taxes corresponding to these schemes can only reach a "second best" because they are unable to manipulate sufficiently the game in order to neutralize the market power of inside agents.

In the next section, we propose a tax and transfer scheme relying, as in this section, on endowment taxation but combined with a more elaborate system of transfers: these will not only go from the inside agents to the outside one, but also take place among the inside agents themselves. As it will be shown, when applied to our example, this method will be proved to be a powerful instrument both as a counterdistortionary measure and as an efficient redistributive tool.

5 Endowment Taxation and Incentive Transfers

In this section, we start by defining a tax and transfer scheme which is specific to the exchange economy we have considered in the two previous sections. The aim of this tax and transfer scheme is not only to insure a preassigned utility level to the outside agent, but also to resorb the distortion resulting from the strategic behaviour of inside agents. First, *before exchange takes place*, suppose that endowment taxes are levied, as in the preceding section, on the goods owned initially by inside agents. Furthermore, *after trade has occurred*, a share of the product of these taxes is redistributed among inside agents, while the remaining part is transferred to the outside one. More specifically, the share received by inside agents is redistributed among them *in a manner assigning to them a quantity of the good they do not own initially, which is proportional to the amount of the good they own initially and supply for exchange*. In other words, the transfer received by each inside agent, in units of the good which is not initially owned, increases as he raises the amount of the initially owned good he sends to the market. Furthermore, the transfer received by the outside agent is required to assign a commodity bundle providing him the utility level $\overline{U}$.

Clearly, the imposition of this tax and transfer scheme allows a manipulation of the market game Γ defined above. On the one hand, the strategy set of inside agents is no longer the unit interval $[0, 1]$, but an interval strictly included in it. On the other hand, transfers among inside agents of the share of the tax product they receive reshape their payoffs in the game since these transfers have now to be added to the outcomes already obtained from strategic exchange. In the following, we show that there exists a tax and transfer scheme which manipulates the game in such a way that the allocation corresponding to the interior strategic equilibrium of this game is a competitive allocation in the subeconomy consisting of the inside agents. Furthermore, the transfer obtained by the agent $n+1$ generates a utility level $\overline{U}$, and the overall-allocation is Pareto-optimal.

In the following, we suppose that $\overline{U} < \frac{n^2}{4}$.[5] Let a tax t^h, $t^h = t = \tau + \frac{2\sqrt{\overline{U}}}{n}$, be levied on both goods $h = 1, 2$. Furthermore, assume that, after trade has occurred at an n-tuple of strategies $(q_1, ..., q_{\frac{n}{2}}; b_{\frac{n}{2}+1}, ..., b_n)$, a share $\frac{\tau}{\frac{1}{2}-\tau-\frac{\sqrt{\overline{U}}}{n}}$, $0 < \tau < \frac{1}{4} - \frac{\sqrt{\overline{U}}}{2n}$, of the total tax product in good 1 (resp. good 2) is transferred to each agent $i \in \{\frac{n}{2}+1, ..., n\}$ (resp. $i \in \{1, ..., \frac{n}{2}\}$), while the remaining share of the tax product in each good $(\sqrt{\overline{U}})$ is redistributed to the outside agent. This tax and transfers scheme generates a new strategic market game Γ''' as follows. The strategy set of all agents is the interval $[0, 1-t]$. The post-tax allocation of the game Γ''' at a vector of strategies $(q_1, ..., q_{\frac{n}{2}}; b_{\frac{n}{2}+1}, ..., b_n)$ now obtain as

$$x_i = \left(1 - \tau - \frac{2\sqrt{\overline{U}}}{n} - q_i, \frac{\sum_{k=\frac{n}{2}+1}^{n} b_k}{\sum_{k=1}^{\frac{n}{2}} q_k} q_i + \frac{\tau}{\frac{1}{2} - \tau - \frac{\sqrt{\overline{U}}}{n}} q_i\right), \quad i = 1, ..., \frac{n}{2} \tag{31}$$

$$x_i = \left(\frac{\sum_{k=1}^{\frac{n}{2}} q_k}{\sum_{k=\frac{n}{2}+1}^{n} b_k} b_i + \frac{\tau}{\frac{1}{2} - \tau - \frac{\sqrt{\overline{U}}}{n}} b_i, 1 - \tau - \frac{2\sqrt{\overline{U}}}{n} - b_i\right), \quad i = \frac{n}{2}+1, ..., n \tag{32}$$

giving rise to utility levels

$$V_i(q,b) = (1-\tau-\frac{2\sqrt{\overline{U}}}{n}-q_i)\left(\frac{\sum_{k=\frac{n}{2}+1}^{n} b_k}{\sum_{k=1}^{\frac{n}{2}} q_k} q_i + \frac{\tau}{\frac{1}{2} - \tau - \frac{\sqrt{\overline{U}}}{n}} q_i\right), \quad i = 1, ..., \frac{n}{2} \tag{33}$$

and

$$V_i(q,b) = \left(\frac{\sum_{k=1}^{\frac{n}{2}} q_k}{\sum_{k=\frac{n}{2}+1}^{n} b_k} b_i + \frac{\tau}{\frac{1}{2} - \tau - \frac{\sqrt{\overline{U}}}{n}} b_i\right)(1-\tau-\frac{2\sqrt{\overline{U}}}{n}-b_i), \quad i = \frac{n}{2}+1, ..., n. \tag{34}$$

Proposition 4. *Given a feasible preassigned utility level $\overline{U}$ for the outside agent $n+1$, there exists endowment taxes t^h, $h = 1, 2$, and incentive transfers of a share of the resulting tax product among inside agents such that, (i)*

[5] This assumption is equivalent to assume that the utility level $\overline{U}$ is feasible.

when the remaining share of this product is transferred to the outside agent, the overall-allocation resulting from this transfer and from the strategic equilibrium of the game Γ''' is Pareto-optimal and (ii) the utility of the outside agent at the commodity bundle obtained from the transfer is equal to $\overline{U}$.

Proof. First, take out, from the initial endowment of the inside agent i, an amount equal to $\frac{2\sqrt{\overline{U}}}{n}$ from the good he owns initially, and transfer it to the outside agent. Then the latter gets the bundle $(\sqrt{\overline{U}}, \sqrt{\overline{U}})$ and obtains accordingly a utility level equal to $\sqrt{\overline{U}} \cdot \sqrt{\overline{U}} = \overline{U}$. Furthermore, impose also, on each inside agent i's endowment a levy τ on the good he owns initially, with

$$\tau = \frac{n - 2\sqrt{\overline{U}}}{n(n+2)}. \tag{35}$$

Using the fact that all inside agents of the same type must adopt the same strategy at equilibrium, we denote by q (resp. b) the strategy inside agents $i \in \left\{1, ..., \frac{n}{2}\right\}$ (resp. $i \in \left\{\frac{n}{2} + 1, ..., n\right\}$) use at such an equilibrium. Using this property, the first order conditions which must be satisfied at an interior strategic equilibrium can be written as

$$(1 - \tau - \frac{2\sqrt{\overline{U}}}{n} - q)\left(\frac{n-2}{n}\frac{b}{q} + \frac{\tau}{\frac{1}{2} - \tau - \frac{\sqrt{\overline{U}}}{n}}\right) - \left(b + \frac{\tau}{\frac{1}{2} - \tau - \frac{\sqrt{\overline{U}}}{n}} q\right) = 0,$$

$$(1 - \tau - \frac{2\sqrt{\overline{U}}}{n} - b)\left(\frac{n-2}{n}\frac{q}{b} + \frac{\tau}{\frac{1}{2} - \tau - \frac{\sqrt{\overline{U}}}{n}}\right) - \left(q + \frac{\tau}{\frac{1}{2} - \tau - \frac{\sqrt{\overline{U}}}{n}} b\right) = 0,$$

with τ as defined in (35). The only solution of the above system is given by (q^*, b^*) with

$$q^* = b^* = \frac{n - 2\sqrt{\overline{U}}}{2(n+2)},$$

which is accordingly the unique interior Nash equilibrium of the game Γ'''. Furthermore, it is easily checked that the transfers among inside agents are feasible at equilibrium. Substituting these values into (31) and (32), we obtain

$$x_i^* = \left(\frac{1}{2} - \frac{\sqrt{\overline{U}}}{n}, \frac{1}{2} - \frac{\sqrt{\overline{U}}}{n}\right), \qquad i = 1, ..., n.$$

Accordingly, the common marginal rate of substitution between the two goods, at the post-tax allocation, is equal to one for all inside agents which is also the value of the marginal rate of substitution of the outside agent $n+1$ at the commodity bundle $(\sqrt{\overline{U}}, \sqrt{\overline{U}})$ he gets from the transfer. We conclude that the overall-allocation is Pareto-optimal. ∎

6 Concluding Remarks

Arriving at the end of this paper, we are aware that it is certainly a rather difficult piece of work that the patient reader had to swallow. It is so because it combines the well-known intricacies encountered when analysing strategic multilateral trade, with the arcane mysteries of optimal taxation. Furthermore, our analysis is cast into the framework of exchange economies which, by their very nature, are never met in real life. This gives to our paper a flavour of abstraction which is not in line with current academic fashion. In spite of these drawbacks, we hope that reading our paper has provided its readers, and in particular Professor Hildenbrand himself, with some enjoyable time and stimulating ideas, inasmuch as they were sufficiently patient to tolerate us until its end!

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