TV-Broadcasting Competition and Advertising

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Abstract

We analyse the rivalry between two TV-channels competing both on the market for audience and the market for advertising. We identify the nature of TV-programs emerging from this competition, and the quantity of advertising that TV-viewers will have to attend at equilibrium. Finally, we examine how a government's regulation of this quantity will affect programs' selection by the channels.

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1 Introduction

Firms operating in the television industry lie at the interface between two markets. In the first one, they sell their audience and part of their broadcasting time to advertising companies (the advertising market). In the second they compete for increasing the size of their audiences by proposing attractive program-mixes to TV-viewers (the audience market). Two major links make these markets tightly interrelated. First, the larger the audience of a particular TV-channel, the more attractive this channel as a media support for the advertisers, and the higher their willingness to pay for having ad-spots inserted in its program. This simply reflects the fact that the impact of the advertising message increases with the size of the audience. Second, the higher the advertising rate of a channel (the ratio between *advertising* broadcasting time and *program* broadcasting time), the larger the number of its viewers who are willing to switch to competing programs with lower advertising rates. It is indeed recognized that most TV-viewers are *advertising-averse*, and incur a decrease in utility when watching the program of a channel whose advertising rate increases. Thus, the larger the advertising rate, the larger the audience loss, and the smaller the attractiveness of the channel in the advertising market![1]

Even if one expects TV-channels to select their program-mixes mainly by relying on the preferences of TV-viewers for the content of these programs, they cannot neglect the side-effects of their selection on their advertising revenues. For instance, a TV-channel can possibly compensate a loss in the audience resulting from an increase in its advertising rate by increasing the fraction of its broadcasting time devoted to sport, at the expense of the share devoted to culture. Since a more significant fraction of TV-viewers prefer sport to culture, this substitution could prevent some viewers to shift to a competing channel as a consequence of the advertising rate increase. Conversely, an increase in the advertising rate of a competitor gives more freedom to a

[1] With the exception of a report concerning the media consumption issued by IREP (1998), we did not find empirical evidence about the existence of advertising-aversion in the TV-viewers'population. Probably, this aversion is "country-specific", as the number of advertising spots which are visioned per week by an average TV-viewer considerably varies from country to country (in 1997, this number ranges from 773 in United-States to 194 in Germany). Nevertheless a recent issue of the French newspaper "Le Monde" (08-09-98) referring to consumers' attitudes with respect to advertising, points that in Europe, with the exception of British citizens, most individuals are significantly advertising averse (for instance 80 % of the population in Spain and Germany). Some authors consider that advertising can be regarded as a non-monetary cost to be borne by TV-viewers (Peltier (1999), Owen and Wildman (1992)).

given channel in the selection of its own program-mix, without running the risk of being penalized in terms of the "audimat". The interaction between the advertising and the audience markets is even magnified due to the fact that in several countries, the government imposes an upperlimit to the advertising rates which are allowed to broadcasting companies. No doubt that this advertising rate regulation may influence the nature of competition between TV-channels and, accordingly, the content of their programs.[2]

In this essay we propose a model capturing the major components of the interaction existing between the advertising and audience markets. Our main objective consists in analysing how the linkage between these markets shapes the TV-programs selected at equilibrium, when these programs follow from oligopolistic competition in the TV-broadcasting sector. This analysis is developed in the framework of a three-stage sequential game involving two companies, their TV-viewers and the advertising agencies buying ad-spots to be inserted in their programs. The players of the game are the TV-channels. A program consists of a mix of two characteristics in varying proportions. The first characteristic is "entertainment" (sports, varieties, ...) and the second "culture" (classic music, theater, movies, ...). The set of program-mixes – out of which channels select their program in the first stage – is represented by the unit interval, a particular value in this interval corresponding to a specific mix of entertainement and culture broadcasting time in the resulting proportions. TV-viewers are uniformly spread over this interval in terms of their programs' preferences: to each program-mix there corresponds a specific viewer for whom that mix is the "ideal" one. We assume that the utility of a specific TV-viewer for a program-mix selected by a channel decreases with the distance between his ideal mix and the selected program, taking into account the fact that he has the possibility of organizing his own "personal" mix by splitting his program attendance between the two channels to the best of his individual interest. Furthermore, we give formal content to the advertising aversion of TV-viewers by assuming that their utility decreases in proportion to the advertising rate they have to tolerate in a given channel when they watch this channel. After having chosen their programs in the first stage of the game, TV-companies select their advertising rates in the second-stage, taking into account the upper limit imposed by the government. In this game, the broadcaster revenue per unit of time obtains as the product of the advertising tariff times the advertising rate. But, due to the positive relationship

[2]In France, for instance, the broadcasting time devoted to advertising authorized by the so-called "Comité Supérieur de l'Audiovisuel" cannot exceed in average six minutes per hour, and twelve minutes in a given hour.

between the value of advertising and the size of the audience, one expects the advertising tariff to increase at equilibrium with the size of the audience. Accordingly, in the second stage-game, "broadcasting firms face a trade-off: either they capture large audiences by keeping the advertising rate low, or they stuff programs with advertising interruptions, thereby losing audience in favour of their competitor" (Vaglio, p. 35). Thus, the second stage-game in which advertising rates are decided, closely resembles the second stage-game of a spatial competition model in which, after having decided about their location, firms decide about their prices. There also, a low price increases the size of the market share while, conversely, a higher price tends to decrease it. Finally, in the third stage of the sequential game, TV-channels now decide non cooperatively about the advertising tariffs they will propose to advertisers. As expected, these tariffs reflect at equilibrium the sizes of the audiences, influencing accordingly the selection of advertising rates in the second stage-game. This, in turn, feeds back to the program-mix selection process performed by TV-companies in the first stage.

In the following, we identify the unique subgame perfect equilibrium of the sequential game described above. Furthermore, government's regulation is shown to reduce the diversity in the programs offered by the channels at equilibrium: the lower the upperlimit imposed by the government to the advertising rate, the weaker the diversity in the programs!

There are at least three papers closely related to the present analysis. Vaglio (1995) proposes a Hotelling-type model of the audience for TV-broadcasting which shares several properties with ours.[3] In particular, he also supposes that consumers are advertising-averse and notices that, due to this aversion, TV-channels face the dilemma of either making money with a low advertising rate in order to keep large audiences, or doing it with a high advertising rate, which entails audience losses. Also he tries to analyse how the fact that advertising rates are subject to government's regulation could affect the behaviour of TV-channels when selecting their programs. Unfortunately, Vaglio does not identify the equilibrium path of the sequential game, but supposes its existence as a solution of a system of first order conditions (see his assumption 3, p. 41). Of course, this reduces considerably the scope of his conclusions. A second paper related to the present approach is Gabszewicz, Laussel and Sonnac (1999). There we apply a similar methodology

[3]Other models based on spatial competition models à la Hotelling for representing the TV-industry also include Bowman (1975). See also Spence and Owen (1977) for a theoretical treatment of this industry, and Barnett and Greenberg (1971).

to the press industry, in order to analyse how press advertising influences the political message that editors of newspapers choose to display to their readers. We find that advertising induces the editors to moderate the political message they would have otherwise selected, in order to make their newspaper more attractive as a media support for the advertisers. The analysis differs however from the present one by two elements. First, receipts of newpapers' editors not only include advertising revenues, as for TV-channels, but also revenues resulting from newspapers' sales to the readers. Second, readers' advertising aversion is not as significant as the one observed in the TV-viewers' population, because newspapers' advertising has a more informational content than TV-advertising, which has mainly a persuasive purpose. Finally, Sonnac (1999) provides an analysis of the consequences of advertising-aversion in the monopoly case. Using a different model, she compares the advertising tariffs resulting from two alternative structures: private or public monopoly. She finds that the public firm always selects a higher advertising tariff than the tariff which would be proposed to advertisers by a private monopolist.

We present the model in Section 2. Section 3 is devoted to the equilibrium analysis of the sequential game and to the consequences of government's advertising rate regulation on TV-channels' equilibrium programs. Finally we examine in Section 4 how consumers' welfare is affected by the advertising rate regulation.

2 The model

We consider a model with two competing private TV-channels producing each, at a fixed cost F, a separate program which consists of a mix of entertainment (sports, varieties, ...) and culture (classic music, theater, movies, a.s.o.). The two companies also sell advertising time to announcers to promote their products or the products of their customers. For each channel, the total broadcasting time, programs plus advertising, is equal to T.

TV-viewers have varying tastes for the "program-mixes" offered by the channels. In order to represent this diversity among consumers' tastes for the programs, we assume that the set of mixes is the unit interval $[0, 1]$, with 0 corresponding to a "pure" entertainment program and 1 to a "pure" cultural program; a particular value, – say, t, – in the interval then corresponds to a mix representing a program with t % duration of entertainment program broadcasting and $(1-t)$ % of cultural program. In the following the parameters

a and b will denote the distance between the "extreme" mixes 0 and 1, and the mixes chosen by the TV-firms, 1 and 2, respectively: the "entertainment channel" (resp. "cultural" channel) accordingly chooses the mix represented by point a (resp. $1 - b$) as depicted in Figure 1.

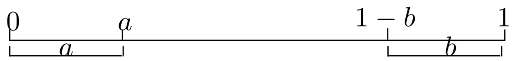

Figure 1: Channels' programs mixes

Once the TV-companies have selected their program-mix, they can decide to divert some proportion z_i of their total broadcasting time, T, in view of selling it to announcers for the promotion of their products. We define the *advertising rate* $v_i, i = 1, 2$, of channel i as the ratio between the share of the broadcasting time devoted by channel i to advertising and the share of the broadcasting time devoted by the same channel to programs, i.e. $v_i = \frac{z_i}{1-z_i}$. We assume that v_i cannot exceed a ceiling $\overline{v}$ imposed by the regulatory authority.

To each program-mix t, $t \in [0, 1]$, there corresponds a specific consumer t for whom that mix is the "ideal" one. We suppose that each consumer t is willing to attend *programs* during a fixed duration, which we denote by $\overline{Z}$: we call $\overline{Z}$ the *program attendance* of consumer t, namely the time spent by consumer t watching TV-channels *at the exclusion of ad-spots' interruptions.* We suppose that $\overline{Z} \leq \frac{T}{2}$. Similarly, we define the *TV-attendance* of consumer t as the total time spent by consumer t in front of his TV-set. Clearly the TV-attendance duration exceeds the program-attendance duration since each unit of program-attendance on channel i requires, in fact, $\frac{1}{1-z_i}$ units of TV-attendance. A fraction $\frac{z_i}{1-z_i} = v_i$ of this unit is indeed devoted to ad-spots' interruptions. We also denote by U the utility of viewer t if he would watch his ideal mix during the whole duration $\overline{Z}$, namely, without any ad spots' interruptions. Given the program-mixes a and $1-b$, as well as the advertising rates v_1 and v_2, selected by the channels, consumer t incurs a loss of utility with respect to U due to two reasons. First, at any fraction of time the channel he attends broadcasts *programs*, there is a utility-loss related to the discrepancy between the program-mix offered by that channel and consumer t-ideal mix; on the other hand, at any fraction of time the channel he attends broadcasts *advertising*, there is a utility loss related to the advertising-aversion of consumer t. Accordingly, in order to identify the *total* loss of utility of consumer

t, we have to consider it separately on the program-attendance fraction and on the advertising-attendance fraction of his TV-attendance. Concerning the program-attendance fraction, we shall assume that the farther the selected TV-mix of a particular channel from consumer t-ideal mix, the higher the disutility of this specific consumer for attending that channel's program. Notice however that, given the mix selections a and $1-b$, consumer t has always the possibility of superimposing to them his own "*personal*" mix, say λ, by splitting the program attendance $\overline{Z}$ between channel 1 in proportion λ and channel 2 in proportion $1-\lambda$, respectively, in which case he obtains the personal mix $\lambda a + (1-\lambda)(1-b)$. We assume that consumer t-disutility per unit of time for not getting his ideal mix increases as the square of the distance between his ideal mix, t, and the personal mix $\lambda a + (1-\lambda)(1-b)$. Then, the disutility related to the program-attendance fraction is measured by

$$\overline{Z} \cdot [\lambda a + (1-\lambda)(1-b) - t]^2. \tag{1}$$

Now let us consider the disutility related to the advertising-attendance fraction. First remember that, if channel i has diverted a proportion z_i of its total broadcasting time T to advertising, one unit of consumer t program-attendance on channel i requires, in fact, $\frac{1}{1-z_i}$ units of TV-attendance, since a fraction $\frac{z_i}{1-z_i} = v_i$ of this unit is devoted to advertising broadcasting. Accordingly, given a personal mix $\lambda a + (1-\lambda)(1-b)$, total duration of ad-spots interruptions to be incurred for obtaining the program-attendance $\overline{Z}$ is equal to

$$\overline{Z}(\lambda v_1 + (1-\lambda)v_2). \tag{2}$$

We shall suppose that consumer t-loss of utility related to the advertising-attendance fraction is measured by the duration of ad-spots interruptions, as given by (2). Consequently, consumer t-total disutility $U^-(\lambda, t)$ if he chooses the personal mix $\lambda a + (1-\lambda)(1-b)$ obtains as the sum of (1) and (2), namely

$$U^-(\lambda, t) = -\overline{Z}[\lambda a + (1-\lambda)(1-b) - t]^2 - \overline{Z}[\lambda v_1 - (1-\lambda)v_2]. \tag{3}$$

Now, it is easy to identify the proportion of program-attendance $\overline{Z}$ consumer t will devote to channel 1 — say $\lambda(t)$ — and channel 2 $(1-\lambda(t))$ when faced with advertising rates v_i, $i = 1, 2$. Minimizing $U^-(\lambda, t)$ with respect to λ, we obtain from the first-order necessary condition $\frac{\partial U^-}{\partial \lambda} = 0$,

$$\lambda(t) = \frac{-2(a+b-1)(1-b-t) - v_1 + v_2}{2(a+b-1)^2}, \quad \lambda(t) \in [0, 1]. \tag{4}$$

We notice that $\lambda(t) \geq 1$ whenever $t \leq t^-$, with t^- solution of the equation $\lambda(t) = 1$, namely

$$t^- = a + \frac{v_2 - v_1}{2(1-a-b)}. \tag{5}$$

Similarly, we obtain that $\lambda(t) \leq 0$ when $t \geq t^+$, with t^+ solution of the equation $\lambda(t) = 0$, or

$$t^+ = 1 - b + \frac{v_2 - v_1}{2(1-a-b)}. \tag{6}$$

Accordingly, given the TV-mix selections a and $1-b$ and the advertising rates v_i, all viewers in the interval $[0, t^-]$ prefer to devote their whole program-attendance $\overline{Z}$ to channel 1 while those in the interval $[t^+, 1]$ do the same with respect to channel 2. Only those consumers t for whom $\lambda(t)$ is interior to the interval $[0, 1]$ share effectively their program-attendance between the two channels, namely consumers in the interval $]t^-, t^+[$. It is easy to check that this interval is *always* of length equal to $1 - a - b$, irrespective of the values v_1 and v_2. Figure 2 depicts how TV-watchers are selecting their optimal personal mixes, assuming $v_2 > v_1$.

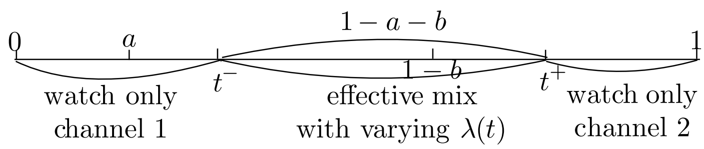

Figure 2: Consumers' optimal personal mixes

For further use, we denote by W_i the *total program-attendance on channel* i, namely, the total duration, over the population, devoted to attending *programs* on channel i. Since, as stated above, one unit of program-attendance requires $\frac{1}{1-z_i}$ units of TV-attendance due to ad-spots' interruptions, total population *TV-attendance on channel* i required by the total program-attendance W_i on channel i is equal to $\frac{W_i}{1-z_i}$. Finally the magnitude n_i defined by

$$n_i = \frac{W_i}{T(1-z_i)} \tag{7}$$

is called the *audience of channel* i, that is, total TV-attendance on channel i per unit of broadcasting time.

Now let us consider the advertising agencies which buy ad-spots units from TV-channels i at price s_i, $i = 1, 2$.[4] A particular type of advertiser is represented by a parameter θ, $\theta \in [0, 1]$, which expresses the intensity of his preferences for buying a spot from a channel. We suppose that the intensity θ for buying a spot from channel i, $i = 1, 2$, is multiplied by the audience n_i: the larger the audience, the higher the desirability of buying a spot from the corresponding channel, since the larger the number of potential consumers who will perceive the advertising message. Thus we assume that utility of buying a spot from channel i for an advertiser of type θ is measured by

$$n_i\theta - s_i, \qquad i = 1, 2; \theta \in [0, 1]. \tag{8}$$

In the following, we suppose that each advertising agency may select one among the three following possibilities: (i) to advertise in neither channel; (ii) to advertise in a single channel; (iii) to advertise in both. In the latter case we suppose that its utility is measured by

$$(n_1\theta - s_1) + (n_2\theta - s_2).^5 \tag{9}$$

We assume that the distribution of advertisers over the types θ is uniform and, for the sake of normalization, that there are $4k$ advertisers per type.

In the next section, we analyse the subgame perfect equilibrium of the following three-stage-game played by the TV-companies. In stage 1, each TV-channel selects the mix he will offer to TV-viewers: point a for channel 1, and $1 - b$ for channel 2. In the second stage, they select the advertising rates v_1 and v_2. Finally in the third stage, they choose the advertising tariffs s_1 and s_2 to be proposed to the advertising agencies.

[4] Thus, we suppose that the price of an ad-spot unit is constant over the period. If this price is determined by the audience corresponding to each unit of time, this assumption implies in turn that the number of TV-viewers is constant over these units. Of course, this assumption is not very realistic to the extent that TV-audience is known to have a peak-load during the so-called "prime-time" while it is lower during other units of time in the day. The extension of our analysis to peak-loads is beyond the purpose of the present paper; it could however be grounded on the approach followed by Boiteux (1954) for analysing peak-load pricing of electricity supply.

[5] This representation of advertisers' population and of their preferences is reminiscent of the wellknown model of *vertical product differentiation*, in which firms sell products which are differentiated by their quality (Mussa and Rosen (1978), Gabszewicz and Thisse (1979)). As in this model, if we assume for instance that $n_2 > n_1$, a spot in channel 2 gives a higher utility than a spot in channel 1 to all advertisers. However, it is generally assumed, in the vertical differentiation model, that consumers make mutually exclusive purchases, while it is assumed here that advertisers may also buy spots in *both* channels.

3 Equilibrium analysis

3.1 The advertising price game

As just stated at the end of the preceding section, the strategies of the editors in this third-stage-game are the tariffs s_1 and s_2. The mixes a and $1-b$ have been already selected in stage 1, while advertising rates $v_1(a,b)$ and $v_2(a,b)$ of the channels have been chosen in stage 2. To these correspond the audiences $n_1 = n_1(v_1, v_2)$ and $n_2 = n_2(v_1, v_2)$ of channels 1 and 2, respectively. Without loss of generality, let $n_2 \geq n_1$. Denote by $u(1,2)$, $u(1)$, $u(2)$ and $u(0)$ the utility levels corresponding to the alternatives consisting in buying at tariffs s_1 and s_2 spot units in both channels, in channel 1 only, in channel 2 only, or buying in neither channel, respectively. Using (8) and (9), it is easily checked that

$$\begin{aligned}
&(i) \quad u(1) \geq u(0) &&\Leftrightarrow \theta \geq \frac{s_1}{n_1}; u(2) \geq u(0) \Leftrightarrow \theta \geq \frac{s_2}{n_2}; && (10)\\
&(ii) \quad u(2) \geq u(1) &&\Leftrightarrow (n_2\theta - s_2) \geq (n_1\theta - s_1); && (11)\\
&(iii) \quad u(1,2) \geq u(2) &&\Leftrightarrow (n_2\theta - s_2) + (n_1\theta - s_1) \geq (n_2\theta - s_2) &&\\
& &&\Leftrightarrow \theta \geq \frac{s_1}{n_1}; && (12)\\
&(iv) \quad u(1,2) \geq u(1) &&\Leftrightarrow (n_2\theta - s_2) + (n_1\theta - s_1) \geq (n_1\theta - s_1) &&\\
& &&\Leftrightarrow \theta \geq \frac{s_2}{n_2}. && (13)
\end{aligned}$$

Furthermore, we notice that

$$\frac{s_2 - s_1}{n_2 - n_1} \geq \frac{s_1}{n_1} \Leftrightarrow \frac{s_2}{n_2} \geq \frac{s_1}{n_1}. \tag{14}$$

Finally, assume that[6]

$$\frac{s_1}{n_1} \leq \frac{s_2 - s_1}{n_2 - n_1}. \tag{15}$$

It is easy to see that (15) implies

$$\frac{s_2}{n_2} \leq \frac{s_2 - s_1}{n_2 - n_1} \tag{16}$$

for, if the reverse of (16) is assumed, we would obtain

$$\frac{s_1}{n_1} > \frac{s_2}{n_2},$$

[6]This assumption holds if there exist consumers who buy a spot in channel 1 only since then the set $\{\theta \mid u(1) \geq u(2) \text{ and } u(1) > u(0)\}$ is non empty.

an inequality which contradicts (14). Combining inequalities (14) and (16), we get

$$\frac{s_1}{n_1} \leq \frac{s_2}{n_2} \leq \frac{s_2 - s_1}{n_2 - n_1}.$$

These inequalities are depicted in Figure 3.

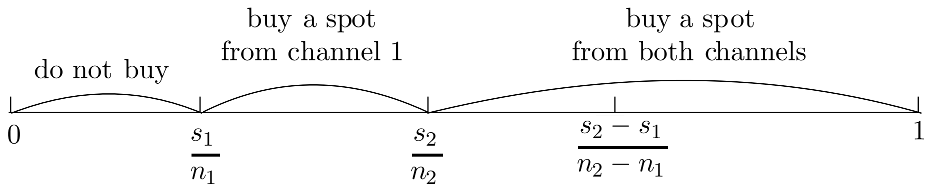

Figure 3: Advertisers' demands

The advertising agencies' types θ included in the interval $[\frac{s_2}{n_2}, 1]$ buy spot units from both channels since, according to (10) and (12), $u(1,2) > u(2) > u(0)$ *and* $u(1,2) > u(1)$; those in the interval $[\frac{s_1}{n_1}, \frac{s_2}{n_2}]$ buy spot units from channel 1 only since, for these advertising agencies θ, $u(1) > u(0)$, $u(1,2) < u(1)$ and $u(1) > u(2)$ (see (10), (13) and (11)). Consequently, demand to channel 1 is made of the advertising agencies' types θ in the interval $[\frac{s_1}{n_1}, 1]$ and demand to channel 2 of the advertising agencies' types θ in the interval $[\frac{s_2}{n_2}, 1]$. Accordingly, the demand function D_i for advertising spots per unit of time in channel i writes as

$$D_i(s_1, s_2) = 4k(1 - \frac{s_i}{n_i}), \quad i = 1, 2, \tag{17}$$

with corresponding advertising revenues $B_i(s_1, s_2)$ per unit of time

$$B_i(s_1, s_2) = 4k(1 - \frac{s_i}{n_i})s_i z_i, \quad i = 1, 2.^7 \tag{18}$$

We notice that the advertising revenues of TV-channel i do not depend on the tariff s_j selected by its rival j, and vice-versa, so that the game between

[7] The property that the advertising price game is degenerate implies that the equilibrium analysis would not be altered if the second and the third-stage game would be played simultaneously: it would not change anything in the equilibrium path at the first and second-stage games. This remark discards any robustness issue related to a problem of committment in the sequentiality of the game.

channels on the advertising market is "degenerate": each channel i behaves as a monopolist and selects independently its equilibrium tariff s_i^* as

$$s_i^* = \frac{n_i}{2}, i = 1, 2, \tag{19}$$

with corresponding instantaneous demand $D_i(s_1^*, s_2^*)$ equal to $2k$. Consequently, the advertising revenues $B_i(s_1^*, s_2^*)$ per unit of time are equal, at equilibrium, to kn_iz_i or, given the definition of n_i (see 7),

$$B_i(s_1^*, s_2^*) = \frac{kW_iz_i}{T(1 - z_i)} = \frac{kW_iv_i}{T}. \tag{20}$$

Of course the total program-attendance W_i on channel i depends on the advertising rate v_i selected by channel i, as well as the advertising rate v_j selected by channel j, $j \neq i$: these values determine, indeed, how TV-viewers allocate their program-attendance between the two channels in regard of their advertising aversion. The next section is precisely devoted to the second-stage game through which the advertising rates are determined, when the equilibrium advertising tariffs we have just derived are taken into account.

3.2 The advertising rate game

In the second-stage-game, TV-companies, – which have selected their program-mixes a and $1-b$ in the first-stage game – have to decide about the advertising rates v_i, $i = 1, 2$. Due to advertising aversion, these decisions will determine the partition of TV-viewers'attendance time between the two channels and, accordingly, how advertising revenues will be shared between them at the equilibrium tariffs s_i^* determined in the third-stage advertising game. First, remind that both channels are constrained to never select an advertising rate exceeding the upperlimt $\overline{v}$ imposed by the regulatory authority. Accordingly, we may restrict the strategies v_i in the advertising rate game to the set $[0, \overline{v}]$. In order to identify the total program-attendance $W_i(v_1, v_2)$ of channel i, we take into account the optimal personal mixes $\lambda(t)$ of consumers t as defined by (4). Given (v_1, v_2) all consumers in the interval $[0, t^-]$ watch only channel 1, while those in the interval $[t^+, 1]$ do the same with respect to channel 2, with t^- and t^+ defined by (5) and (6), respectively. On the other hand, total program-attendance devoted to channel 1, when integrated over those consumers who effectively split their program-attendance between the channels is given by

$$\overline{Z} \int_{t^-}^{t^+} \lambda(t)dt = \frac{\overline{Z}}{2}(1 - a - b).$$

In other words, both channels share equally between them the total program-attendance of those who split their program-attendance between the channels. From the above, we find that the total program-attendance $W_i(v_1, v_2)$ of channel i are given by

$$\begin{aligned} & W_1(v_1, v_2) \\ & \quad = 0, && \text{if } a + \frac{v_2 - v_1}{2(1-a-b)} + \frac{1-a-b}{2} < 0; \\ & \quad = \overline{Z}\left[a + \frac{v_2 - v_1}{2(1-a-b)} + \frac{1-a-b}{2}\right], && \text{if } 0 \leq a + \frac{v_2 - v_1}{2(1-a-b)} + \frac{1-a-b}{2} \leq 1; \\ & \quad = \overline{Z}, && \text{if } 1 < a + \frac{v_2 - v_1}{2(1-a-b)} + \frac{1-a-b}{2}, \end{aligned} \tag{21}$$

and

$$\begin{aligned} & W_2(v_1, v_2) \\ & \quad = 0, && \text{if } b + \frac{v_1 - v_2}{2(1-a-b)} + \frac{1-a-b}{2} < 0; \\ & \quad = \overline{Z}\left[b + \frac{v_2 - v_1}{2(1-a-b)} + \frac{1-a-b}{2}\right], && \text{if } 0 \leq b + \frac{v_1 - v_2}{2(1-a-b)} + \frac{1-a-b}{2} \leq 1; \\ & \quad = \overline{Z}, && \text{if } 1 < b + \frac{v_1 - v_2}{2(1-a-b)} + \frac{1-a-b}{2}, \end{aligned} \tag{22}$$

with corresponding advertising revenues over total broadcasting time equal to $k\left[W_i(v_1, v_2)\frac{v_i}{T}\right]T$ (see equation (20)). We denote this total advertising revenue of channel i by $\Pi_i(v_1, v_2)$, that is

$$\Pi_i(v_1, v_2) = kW_i(v_1, v_2) \cdot v_i,$$ [8]

$i = 1, 2$.

Taking into account the fact that the advertising rates v_i cannot exceed the upperlimit, $\overline{v}$, the reaction functions in the advertising rate game are easily derived as

$$\begin{aligned} v_1 &= Min\left[\overline{v}, \frac{1}{2}(v_2 + 1 - 2b + b^2 - a^2)\right]; \\ v_2 &= Min\left[\overline{v}, \frac{1}{2}(v_1 + 1 - 2a + a^2 - b^2)\right]. \end{aligned}$$

[8]We observe that the second stage-game described above is reminiscent of a spatial competition model with quadratic transportation costs in which the advertising rates v_i have been substituted to the usual price strategies (see d'Aspremont et al. (1979)). This is not surprising, since market shares here depend on the duration of advertising time, while they traditionally depend on prices.

Several parametric regions have then to be considered according as, given the couple (a, b), none, or both, or only one channel hits the upperlimit $\overline{v}$ when reacting optimally to the opponent's strategy. In the following R_i will denote the set of couples $(a, b) \in [0, 1]^2$ satisfying the inequalities which define region i.

Region 1: $(1-a-b)(1+\frac{a-b}{3}) \leq \overline{v}$ (A), $(1-a-b)(1+\frac{b-a}{3}) \leq \overline{v}$ (B). Adding up these two inequalities one obtains $(1-a-b) \leq \overline{v}$. Inequalities (A) and (B) are equivalent to the simpler condition $a \geq \max\left[-1+\sqrt{4-4b+b^2-3\overline{v}}, 2-\sqrt{1+2b+b^2+3\overline{v}}\right]$. If equilibrium advertising rates lie in region 1, they must satisfy the first order necessary conditions $\frac{\partial \Pi_i}{\partial v_i} = 0$, or

$$v_1 = (1-a-b)(1+\frac{a-b}{3}), \tag{23}$$

$$v_2 = (1-a-b)(1+\frac{b-a}{3}), \tag{24}$$

with corresponding equilibrium revenues $\Pi_1^*(a, b)$ and $\Pi_2^*(a, b)$ defined by

$$\Pi_1^*(a, b) = \frac{k}{18}(1-a-b)(a-b+3)^2; \tag{25}$$

$$\Pi_2^*(a, b) = \frac{k}{18}(1-a-b)(b-a+3)^2. \tag{26}$$

We notice that Π_1^* (resp. Π_2^*) is a strictly decreasing function of a (resp. b) for all $(a, b) \in R_1$.

Region 2: $\frac{1}{2}(\overline{v}+1-2b+b^2-a^2) \geq \overline{v}$ (C), $\frac{1}{2}(\overline{v}+1-2a+a^2-b^2) \geq \overline{v}$ (D). Adding up one obtains $(1-a-b) \geq \overline{v}$. Inequalities (C) and (D) are equivalent to the simpler condition $a \leq Min\left[1-\sqrt{b^2+\overline{v}}, \sqrt{1-2b+b^2-\overline{v}}\right]$. The equilibrium advertising rates are then given by $v_1 = v_2 = \overline{v}$, with corresponding equilibrium revenues

$$\Pi_1^*(a, b) = k\overline{v}\frac{1+a-b}{2},$$

$$\Pi_2^*(a, b) = k\overline{v}\frac{1+b-a}{2}.$$

Thus, in region 2, Π_1^* (resp. Π_2^*) is a strictly increasing function of a (resp. b) for all $(a, b) \in R_2$.

Region 3: $(1-a-b)(1+\frac{a-b}{3}) > \overline{v}$ (E) , $\frac{1}{2}(\overline{v}+1-2a+a^2-b^2) < \overline{v}$ (F) ($\Leftrightarrow \overline{v} > 1-2a+a^2-b^2$). These two inequalities together imply $a > b$. Inequalities (E) and (F) are equivalent to the simpler condition $1-\sqrt{b^2+\overline{v}} < a < -1+\sqrt{4-4b+b^2-3\overline{v}}$. Then equilibrium advertising rates are given by

$$\begin{aligned} v_1 &= \overline{v}, \\ v_2 &= \frac{1}{2}(1+\overline{v}-2a+a^2-b^2), \end{aligned}$$

with corresponding equilibrium revenues

$$\begin{aligned} \Pi_1^*(a,b) &= k\overline{v}\left(\frac{1+a-b}{2}+\frac{\frac{1}{2}(-\overline{v}+1-2a+a^2-b^2)}{2(1-a-b)}\right); \\ \Pi_2^*(a,b) &= \frac{k}{2}(\overline{v}+1-2a+a^2-b^2)\left(\frac{1+b-a}{2}-\frac{\frac{1}{2}(-\overline{v}+1-2a+a^2-b^2)}{2(1-a-b)}\right). \end{aligned}$$

It is shown in the appendix that, in region 3, $\frac{\partial\Pi_1^*}{\partial a} < 0$ and $\frac{\partial\Pi_2^*}{\partial b} > 0$.

Figure 4 depicts the reaction functions of TV-channels when the couple (a,b) belongs to Region 3.

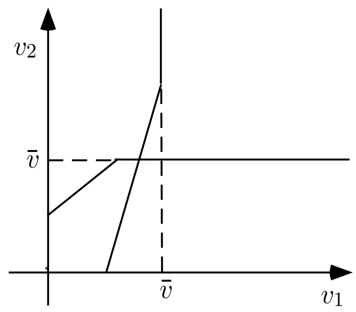

Figure 4: Reaction functions of TV-channels in Region 3

Region 4: $\frac{1}{2}(\overline{v}+1-2b+b^2-a^2) < \overline{v}$ (G), $(1-a-b)(1+\frac{b-a}{3}) > \overline{v}$ (H). These two inequalities together imply $a < b$. Inequalities (G) and (H) are equivalent to the simpler condition $\sqrt{1-2b+b^2-\overline{v}} < a < 2-\sqrt{1+2b+b^2+3\overline{v}}$. Then equilibrium advertising rates are given by

$$v_1 = \frac{1}{2}(1+\overline{v}-2b+b^2-a^2);$$
$$v_2 = \overline{v},$$

with corresponding equilibrium revenues

$$\Pi_1^*(a,b) = \frac{k}{2}(\overline{v}+1-2b+b^2-a^2)\left(\frac{1+a-b}{2}-\frac{\frac{1}{2}(-\overline{v}+1-2b+b^2-a^2)}{2(1-a-b)}\right);$$
$$\Pi_2^*(a,b) = k\overline{v}(\frac{1+b-a}{2}+\frac{\frac{1}{2}(-\overline{v}+1-2b+b^2-a^2)}{2(1-a-b)}.$$

It is shown in the appendix that, in region 4, we obtain $\frac{\partial \Pi_1^*}{\partial a} > 0$ and $\frac{\partial \Pi_2^*}{\partial b} < 0$.

3.3 The program-mix game

In the first-stage-game, TV-channels select the program-mix they will offer to their customers, taking into account the effects of their choice on ensuing competition in advertising rates and advertising tariffs. Payoffs in this game, as functions of a and b, are defined in the different regions as in the previous section. In order to obtain the reaction functions in the program-mix game it is necessary, for a given value of b (resp. a), to know precisely in which successive regions the equilibrium of the advertising rate game lies when a (resp. b) is increased from 0 to $1-b$ (resp. $1-a$). This is done in Lemma 1 below in which it appears that the value $\frac{1-\overline{v}}{2}$ plays a critical role: when $b \leq \frac{1-\overline{v}}{2}$, the equilibrium lies successively when a increases in regions 2, 3 and, finally, region 1; when $b \geq \frac{1-\overline{v}}{2}$, it lies successively in region 2 (possibly), region 4 and finally region 1.

Lemma 1 *(i) If* $b \leq \frac{1-\overline{v}}{2}$, $(a,b) \in R_2$ *for* $a \in [0, 1-\sqrt{b^2+\overline{v}}]$, $(a,b) \in R_3$ *for* $a \in (1-\sqrt{b^2+\overline{v}}, -1+\sqrt{4-4b+b^2-3\overline{v}})$ *and* $(a,b) \in R_1$ *for* $a \in [-1+\sqrt{4-4b+b^2-3\overline{v}}, 1-b]$.

(ii) If $b \geq \frac{1-\overline{v}}{2}$, $(a,b) \in R_2$ *for* $a \in \left[0, \sqrt{1-2b+b^2-\overline{v}}\right]$[9], $(a,b) \in R_4$ *for* $a \in \left(\sqrt{1-2b+b^2-\overline{v}}, 2-\sqrt{1+2b+b^2+3\overline{v}}\right)$ *and* $a \in R_1$ *for* $a \in \left[2-\sqrt{1+2b+b^2+3\overline{v}}, 1-b\right]$.

[9] Of course there is no $(a,b) \in R_2$ whenever $1-2b+b^2-\overline{v} < 0$ (i.e. $b > 1-\sqrt{\overline{v}}$).

Proof. See Appendix.

Similar results can be obtained for TV-channel 2, using the same argument: when $a \leq \frac{1-\overline{v}}{2}$ the equilibrium lies successively when b increases in regions 2, 4 and, finally, region 1; when $a \geq \frac{1-\overline{v}}{2}$ the equilibrium lies successively when b increases in regions 2 (possibly), 3 and, finally, region 1. The payoff of TV-channel 1 (resp. 2) being a monotone function of a (resp. b) within each region, its best reply-function in the program-mix game can now be easily identified and we can state the following:

Lemma 2 *(i) The best reply of TV-channel* 1 *to* $b \leq \frac{1-\overline{v}}{2}$ *is* $R^1(b) = 1 - \sqrt{b^2 + \overline{v}}$ *and its best reply to* $b \geq \frac{1-\overline{v}}{2}$ *is* $R^1(b) = Max[0, 2 - \sqrt{1 + 2b + b^2 + 3\overline{v}}]$;

(ii) the best reply of TV-channel 2 *to* $a \leq \frac{1-\overline{v}}{2}$ *is* $R^2(a) = 1 - \sqrt{a^2 + \overline{v}}$ *and its best reply to* $a \geq \frac{1-\overline{v}}{2}$ *is* $R^2(a) = Max[0, 2 - \sqrt{1 + 2a + a^2 + 3\overline{v}}]$.

Proof. See Appendix.

Now it is straightforward to check that the best reply functions obtained in Lemma 2 are continuous and intersect once and only once at the point $\left(\frac{1-\overline{v}}{2}, \frac{1-\overline{v}}{2}\right)$. Accordingly, we state

Proposition 1 *The unique Nash equilibrium of the program-mix game is*

$$(a^*, b^*) = \left(\frac{1-\overline{v}}{2}, \frac{1-\overline{v}}{2}\right).$$

It is easier to interpret Proposition 1 at the light of the variable $\hat{z} \overset{\text{def}}{=} \frac{\overline{v}}{1+\overline{v}}$, which corresponds to the proportion of total broadcasting time T which is allowed by the regulatory authority, given the advertising rate $\overline{v}$. First, notice that any regulation which would allow a proportion $\hat{z}$ of advertising broadcasting time exceeding $\frac{1}{2}$ is ineffective since it would imply that $\overline{v} \geq 1$, which in turn would imply corresponding equilibrium program-mixes $a^* = b^* \leq 0$: without regulation, no channel would in any case devote more than half of its total broadcasting time to advertising. In fact, this would be exactly the duration of advertising broadcasting which would realise without regulation. Indeed, in that case, the program-mixes selected at equilibrium by channel 1 and 2 would be $a^* = 0$ and $1 - b^* = 1$, respectively, entailing equilibrium advertising rates in the second-stage game $v_1^* = v_2^* = 1 - 2a^* = 1$, or $\hat{z} = \frac{1}{2}$. We may summarize the above in the following way. *Without advertising rate regulation, both channels would spontaneously select those program-mixes which* *entail maximal programs' diversity, and spend half of their total broadcasting time to ads' interruptions.*

At the other extreme, a regulatory agency which would simply forbid any advertising broadcasting would select $\hat{z} = 0$. Then $\overline{v} = 0$, so that the corresponding equilibrium program-mix would be $(a^*, b^*) = (\frac{1}{2}, \frac{1}{2})$: *when advertising is forbidden, both channels spontaneously select the same program-mix, with half of the broadcasting time devoted to entertainment and half to culture.* In fact, by selecting the upperlimit $\hat{z}$, a benevolent government can manipulate the program-mixes selected by TV-channels at equilibrium. The result of this manipulation is such that *the smaller the upperlimit $\hat{z}$, the smaller the diversity of the program-mixes proposed by TV-channels to their viewers.*

In the next section, we examine briefly how TV-viewers' welfare is affected by advertising rate regulation.

4 The advertising rate regulation

To start out, it is useful to examine first the welfare optimal program-mix selection, from the viewpoint of TV-viewers, in the case in which advertising would not exist. Then the loss in utility of TV-viewer t at a personal mix λ reduces to

$$U^-(\lambda, t) = -[\lambda a + (1-\lambda)(1-b) - t]^2,$$

when channel 1 (resp. channel 2) has selected the program-mix a (resp. $1-b$). The optimal mix of viewer t obtains from the first order condition as

$$\lambda(t) = \frac{1-b-t}{1-a-b}, \qquad \lambda(t) \in [0,1].$$

Accordingly, $\lambda(t) \geq 1$ whenever $t \leq t^-$, with t^- solution of the equation $\lambda(t) = 1$, i.e., $t^- = a$. Similarly, $\lambda(t) \leq 0$ whenever $t \geq t^+$, with $t^+ = 1-b$. Consequently, without advertising, all consumers with an ideal mix smaller than the program-mix a selected by channel 1 devote, at their optimal mix, their total TV-time attendance to this channel, and are accordingly unable to realise their ideal mix. A similar remark applies to TV-viewers with an ideal mix larger then $1-b$. Only those TV-viewers with an ideal mix located between a and $1-b$ can manage to let their optimal mix coincide with their ideal one. Accordingly, without advertising, *the total welfare loss of TV-viewers is* *minimal when both channels are fully specialized* ($a = b = 0$) *since all TV-viewers can then realise their ideal mix.*[10] Notice however that this conclusion is reached only when TV-channels' fixed costs F are not taken into account in total welfare evaluation. Without advertising revenues, some alternative ways to cover channels' fixed costs must then be considered, like a direct government's subsidy to the channels or a subscription fee imposed to TV-viewers.

Now let us come back to the case of TV-channels covering their fixed cost F thanks to advertising revenues, with a government's advertising rate regulation: TV-channels cannot select an advertising rate exceeding the upperlimit $\overline{v}$ imposed by the government.

In order to determine the value $\overline{v}$ to be selected by a benevolent government, we notice that the *net* utility of consumer t from watching TV is equal to $U - \hat{v}$ when $t \in [\frac{1-\hat{v}}{2}, \frac{1+\hat{v}}{2}], U - (\frac{1-\overline{v}}{2} - t)^2 - \overline{v}$ when $t \in [0, \frac{1-\overline{v}}{2}[$, and $U - (t - \frac{1+\overline{v}}{2})^2 - \overline{v}$ when $t \in [\frac{1+\overline{v}}{2}, 1]$. Integrating these values over $[0, 1]$, total consumers' net utility obtains as

$$\int_0^{\frac{1-\overline{v}}{2}} [U - (\frac{1-\overline{v}}{2} - t)^2 - \hat{v}]dt \; + \; (U - \overline{v})\overline{v} \; + \; \int_{\frac{1+\overline{v}}{2}}^1 [U - (t - \frac{1+\overline{v}}{2})^2 - \overline{v}]dt$$

which is equal to

$$U + \frac{1}{12}\overline{v}^3 - \frac{1}{4}\overline{v}^2 - \frac{3}{4}\overline{v} - \frac{1}{12}. \tag{27}$$

It is easily checked that the expression (27) is a decreasing function of $\overline{v}$ when $\overline{v} \in [0, 1]$. *Accordingly, while without advertising total consumers' utility is maximal when channels programs' diversity is maximal* ($a = b = 0$)*, now it decreases with this diversity when advertising is introduced.* The reason is that an increase in the authorized advertising rate $\overline{v}$, although it entails an increase in the diversity of the equilibrium program-mixes, also increases the total consumers' disutility resulting from advertising aversion. It turns out that the "advertising aversion effect" dominates the "diversity effect" since the *net* effect of an increase in $\overline{v}$ consists in decreasing total consumers' utility. From the above, it follows that *a benevolent government will select the*

[10]This has to be contrasted with the welfare solution in the Hotelling's spatial competition model with quadratic transportation costs. Then locations minimizing consumers' welfare losses correspond to the quartiles ($a = \frac{1}{4}; 1 - b = \frac{3}{4}$). The difference between the two welfare solutions follows from the fact that TV-viewers may "mix the program-mixes" proposed by the channels.

smallest value of $\overline{v}$ which is compatible with the condition that TV-channels' revenues at equilibrium exceed their fixed cost, F.

Total advertising revenues of channel i are equal, at equilibrium, to $\frac{k\overline{v}}{2}$ (see (20)). Consequently, in order to cover the fixed costs F and to maximise total TV-viewers' welfare, the ceiling $\overline{v}$ must satisfy

$$\overline{v} = \min_{v}\{v \mid \frac{kv}{2} \geq F\},$$

or

$$\overline{v} = \frac{2F}{k},$$

which implies

$$\hat{z} = \frac{2F}{k(1-2F)}.$$ [11]

Finally, it is worth noticing that another way for obtaining an endogenous determination of the maximal advertising rate $\overline{v}$, is simply to suppose that citizens vote on $\overline{v}$. If we assume that the population of TV-viewers coincides with the population of voters, and that the revenues of TV-channels, accrue to less than half of the population, the fact that all TV-viewers favour the lowest value for $\overline{v}$ (see (27)) implies that voting will yield again the same outcome for $\overline{v}$ as the one selected by a benevolent government.

[11] Thus we must assume that $F \leq \frac{k}{2(k+1)}$.

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Appendix

Properties of $\Pi_1^*(a,b)$ and $\Pi_2^*(a,b)$ in Regions 3 and 4

(a) for any $(a,b) \in R_3$, we obtain

$\frac{\partial \Pi_1^*}{\partial a} = \frac{1}{4}\overline{v}\frac{(-1+a+b)^2-\overline{v}}{(-1+a+b)^2}$;

$\frac{\partial^2 \Pi_1^*}{\partial a^2} = 2\frac{\overline{v}^2}{(-1+a+b)^3} < 0.$

From condition (F) $\overline{v} > 1-2a+a^2-b^2$ and hence, since $1-2a+a^2-b^2 \geq (1-a-b)^2$, we conclude that $\overline{v} > (1-a-b)^2$ and, then, that $\frac{\partial \Pi_1^*}{\partial a} < 0$. Note that a similar argument shows that, for any $(a,b) \in R_4$, $\frac{\partial \Pi_2^*}{\partial b} < 0$. ∎

(b) for any $(a,b) \in R_4$, we obtain

$\frac{\partial \Pi_1^*}{\partial a} = \frac{1}{8}\left(\overline{v}+1-2b+b^2-a^2\right)\frac{-4a+3a^2+4ab+1-2b+b^2+\overline{v}}{(-1+a+b)^2}$

Hence, from the positivity of the equilibrium value of v_1,

$$\begin{aligned} sign(\frac{\partial \Pi_1^*}{\partial a}) &= sign(-4a+3a^2+4ab+1-2b+b^2+\overline{v}) \\ &= sign\left(-4a(1-a-b)+1-2b+b^2-a^2+\overline{v}\right) \end{aligned}$$

On the other hand, from condition (G),[12]

$$\begin{aligned} &-4a(1-a-b)+1-2b+b^2-a^2+\overline{v} \\ &> -4a(1-a-b)+2(1-2b+b^2-a^2) \\ &> 2(1-a-b)(1-2a) \end{aligned}$$

On the other hand , from $1-a-b \geq 0$ and $b > a$ (in Region 4), $1-2a > 0$. We can now conclude that $\frac{\partial \Pi_1^*}{\partial a} > 0$. Note that a similar argument shows that, for any $(a,b) \in R_3$, $\frac{\partial \Pi_2^*}{\partial b} > 0$. ∎

Proof of Lemma 1

(i) $F(b) = [1-\sqrt{b^2+\overline{v}}]-\sqrt{1-2b+b^2-\overline{v}}$ is such that $F(0)=0$ iff $b = \frac{1-\overline{v}}{2}$ and $F_b'(\frac{1-\overline{v}}{2}) > 0$: hence $b \leq \frac{1-\overline{v}}{2}$ and $a \leq 1-\sqrt{b^2+\overline{v}} \Rightarrow a \leq \sqrt{1-2b+b^2-\overline{v}} \Longrightarrow (a,b) \in R_2$;

[12]Notice that the last inequality is equivalent to $a(a-1) < b(b-1)$ which holds always true since $1 \geq b > a$.

$H(b) = 1 - \sqrt{b^2 + \overline{v}} - (-1 + \sqrt{4 - 4b + b^2 - 3\overline{v}})$ is such that $H(0) = 0$ iff $b = \frac{1-\overline{v}}{2}$ and $H_b^{'}(\frac{1-\overline{v}}{2}) > 0$: for every $b < \frac{1-\overline{v}}{2}$ there exists a non-void interval $(1 - \sqrt{b^2 + \overline{v}}, -1 + \sqrt{4 - 4b + b^2 - 3\overline{v}})$ of values of a such that $(a, b) \in R_3$;

$K(b) = -1 + \sqrt{4 - 4b + b^2 - 3\overline{v}} - (2 - \sqrt{1 + 2b + b^2 + 3\overline{v}})$ is such that $K(0) = 0$ iff $b = \frac{1-\overline{v}}{2}$ and $K_b^{'}(\frac{1-\overline{v}}{2}) < 0$: hence $b \leq \frac{1-\overline{v}}{2}$ and $a \geq -1 + \sqrt{4 - 4b + b^2 - 3\overline{v}} \Rightarrow a \geq 2 - \sqrt{1 + 2b + b^2 + 3\overline{v}} \Longrightarrow (a, b) \in R_1$;

(ii) given the properties of $F(b)$, $b \geq \frac{1-\overline{v}}{2}$ and $a \leq \sqrt{1 - 2b + b^2 - \overline{v}} \Rightarrow a \leq 1 - \sqrt{b^2 + \overline{v}} \Longrightarrow (a, b) \in R_2$;

$J(b) = \sqrt{1 - 2b + b^2 - \overline{v}} - (2 - \sqrt{1 + 2b + b^2 + 3\overline{v}})$ is such that $J(0) = 0$ iff $b = \frac{1-\overline{v}}{2}$ and $J_b^{'}(\frac{1-\overline{v}}{2}) < 0$: for every $b > \frac{1-\overline{v}}{2}$ there exists a non-void interval $\left(\sqrt{1 - 2b + b^2 - \overline{v}}, 2 - \sqrt{1 + 2b + b^2 + 3\overline{v}}\right)$ of values of a such that $(a, b) \in R_4$;

given the properties of $K(b)$, $b \geq \frac{1-\overline{v}}{2}$ and $a \geq 2 - \sqrt{1 + 2b + b^2 + 3\overline{v}} \Rightarrow a \geq -1 + \sqrt{4 - 4b + b^2 - 3\overline{v}} \Longrightarrow (a, b) \in R_1$. ■

Proof of Lemma 2

(i) if $b \leq \frac{1-\overline{v}}{2}$ we know from Lemma 1 that, as a increases from 0, (a, b) belongs succesively to R_2, where Π_1^* is increasing in a, and then to R_3 and R_1 where it is decreasing in a : it follows that the best reply to any such b is $a = 1 - \sqrt{b^2 + \overline{v}}$;

if $b \geq \frac{1-\overline{v}}{2}$ as a increases from 0, (a, b) belongs successively to R_2, to R_4, where Π_1^* is increasing in a, and then to R_1, where it is decreasing in a, if $b \leq 1 - \sqrt{v}$, or only to R_4 and then to R_1 when $b > 1 - \sqrt{v}$: it follows that the best reply to any such b is $a = Max[0, 2 - \sqrt{1 + 2a + a^2 + 3\overline{v}}]$.

(ii) the best reply function of TV-channel 2 is derived in the same way. We leave the proof to the reader. ■