The Importance of Being Small: Size Effects in International Trade*

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Abstract

Market size and transport costs are important ingredients of international trade. We propose to look at these issues from a different perspective. Using a Hotelling duopoly model with quadratic transport costs, we analyze the welfare effects of international trade between two countries which differ only in size. Our results indicate that in most cases free trade will lead to a decrease in prices. Furthermore, the firm of the small country will benefit from market expansion. Finally, the model predicts that the small country benefits from a move towards free trade whereas the large country may be hurt by the opening to trade.

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1 Introduction

It is our contention that geography matters for trade as well as for its welfare implications through various variables such as the geographical size of countries and the dispersion of consumers over national territories, and not only through population differences and transport cost between dimensionless countries. In this paper, we follow Lösch (1940) who proposed to think about countries "in all their diversity and expanse rather than mere points" (p.314 of the English translation). To this end, we take into account the spatial extent of countries as well as transportation costs due to the geographical dispersion of consumers within countries. In this way, *countries are no longer modeled as points separated by a given distance but have a geographical extent*. This implies that some consumers are closer to foreign firms than to domestic firms. In addition, countries may differ in their geographical sizes, thus introducing spatial asymmetries in trade. This has not only the advantage of being more realistic but, as will be seen below, this leads to major implications for the welfare effects of free trade.

The impact of size has been studied in different contexts. In neoclassical trade theory, the size of countries plays a role in determining the welfare implications of trade. While supply-side elements determine the pattern of trade, country size influences the distribution of trade gains between countries. Indeed, the terms of trade determine the gains: the more these terms of trade differ from the autarky relative prices, the more the country gains. The relative prices under free trade depend on demand conditions also. The larger the country, the larger its demand and the closer the terms of trade are to its autarky relative prices. Thus arises the possibility that the difference in size is so important that the terms of trade will be equal to the large country's autarky relative prices. In such a case, only the small country gains from free trade (Takayama, 1972). In new trade theories, countries exchange differentiated varieties and trade allows them to have access to a broader range of varieties. When scale economies are combined with transportation costs, both countries benefit from trade. However, since the price index is lower in the large country, it gains more from free trade than the small country (Krugman, 1980). Finally, Becker (1994) has recently argued that small nations may specialize in producing goods and services for the world market that have too small outlets for large nations (see also Alesina and Spolaore, 1997).

An alternative way of looking at market size is as follows. Assume two countries whose geographical sizes differ. Assume further that the population of each country is dispersed, each consumer having a specific, personal address in her country. In such a context, production may be geographically concentrated in one place of each country, but bringing the good to the consumers' locations entails different costs. Using a simple partial equilibrium setting à la Hotelling, our objectives are then as follows. First, we want to determine how the difference in geographical size affects the pattern of trade. Specifically, it is shown that there is always trade from the small to the large country. Second, we want to investigate the spatial distribution of gains/losses associated with the opening to trade. In particular, we will see that *the small country always gains from free trade but the large country always loses.* This asymmetry finds its origin in the expansion of the small country market share resulting from the invasion of the large country by the output of the small one. Hence, the relative size of countries affects differently the impact of trade on welfare. This confirms Lösch's conjecture in that *national borders prevent the emergence of an efficient system of market areas.* However, free trade does not lead to the socially optimal market division since the new market areas still depend on the autarky locational configuration, reflecting indirectly the influence of formerly existing borders. Furthermore, within the large country, some consumers are better off under free trade but others are worse off, suggesting a conflict of interest between different groups of citizens of the large country. Finally, firms' profits are affected differently by free trade in the small and the large country. Hence, the divergence between the interests among the economic agents located in the large country might explain the emergence of lobbying against free trade within some large nations.

Clearly, the foregoing results are obtained in the context of a very particular model and may depend very much on the geography that is considered. However, our setting is standard in spatial oligopoly theory and the results are sufficiently provocative to invite further research. In this respect, a related question has been investigated by Ohsawa (1999) in the case of commodity tax competition between two countries of different sizes. Using a spatial competition model, Ohsawa shows that the small country gets more than its size-proportional share of total revenues. This suggests that the small country is the relative winner in fiscal competition as it is the case in our international trade setting.

The paper is organized as follows. The model is presented in section 2. In section 3, we show the existence of an equilibrium and characterize this equilibrium in terms of the fundamental parameters of the economy. The welfare implications are discussed in section 4. Section 5 concludes. Before proceeding, it is worth noting that a similar setting has been used by Shachmurove and Spiegel (1995). However, as shown in Appendix A, their results suffer from one major shortcoming: the equilibrium they focus on does not exist.

2 A Spatial Model of International Trade

Consider two countries: one small and one large country. In the tradition of spatial models à la Hotelling, each country is represented by a linear segment. The first country has a length, or geographical size, s and the second country a length L; the two countries are adjacent and colinear as shown in Figure 1.

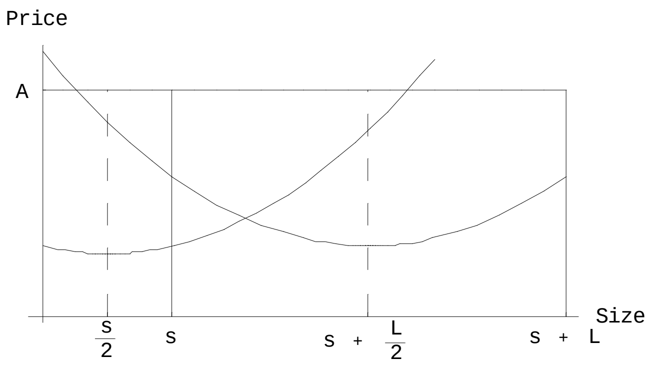

Figure 1: The model

For the labels "small" and "large" to be appropriate, these two parameters must be such that $L > s$. Without loss of generality, we may set $s = 1$. This implies that L will be both the geographical size of the large country as well as a size dissimilarity index. The population of each country is uniformly distributed along the two segments, each consumer having a specific location along the segment corresponding to her country. The population density is equal to one in both countries. There are two goods in the economy. The first one is available as an endowment to each consumer and can be traded at zero cost across locations and countries; it is used as the numéraire. The second good is produced at constant and equal marginal costs in the capital of each country by a single firm operating under increasing returns to scale; fixed production costs are supposed to be large enough for each firm to be a natural monopoly in autarky while marginal costs are normalized to zero. As in Alesina and Spolaore (1997), the capital is located in the center of the country. This location allows each firm to maximize its profit in autarky. It is chosen once-and-for-all because firms show a strong spatial inertia, especially when they are located in major urban centers generating strong economies of agglomeration (Fujita and Thisse, 1996).

Each firm quotes a single mill price to which the transport cost is added. As is well known, a pure strategy price equilibrium exists regardless of firms' locations when transport costs are quadratic rather than linear (d'Aspremont *et al.*, 1979). For a consumer located at a and buying from a firm at $b = 1/2$ (resp. $1 + L/2$) at the mill price p_b, the full price paid by this consumer is defined as follows:

$$p_b(a) = p_b + t(a - b)^2$$

where $t > 0$ is constant and identical across consumers and firms. Therefore, the full price varies across consumers. Without loss of generality, we may choose the unit of the numéraire for t to be equal to one. Each consumer buys at most one unit of the good produced in either capital and has the same reservation price A for this good. Whether or not the consumer decides to consume depends on the comparison between the consumer's reservation price A and the lower full price. More precisely, a consumer at a buys one unit of the good from the firm at b with the lower full price if and only if $p_b(a) \leq A$; otherwise she does not buy the good in question. As a result, in autarky the market boundaries of a firm's market are given by the boundary of the country or by the location of the consumer for whom the full price is equal to her reservation price.

As already mentioned, we assume that firms first operate in autarky. In this case, the equilibrium prices depend only upon the parameter values of the country each firm belongs to; they are independent of the behavior of the firm in the other country. The results are similar for the small and the large countries. Therefore, we state the results below for a country of size l, keeping in mind that $l = 1$ for the small country and $l = L$ for the large country.

Since there is one firm located at $l/2$, it is readily verified that the firm's profit function is defined as follows:

$$\pi(p) = \begin{cases} pl & \text{if} \quad 0 \leq p < A - l^2/4 \\ 2p\sqrt{A-p} & \text{if} \quad A - l^2/4 \leq p < A \end{cases} \tag{1}$$

Proposition 1 *If $A < 3l^2/4$, then the price equilibrium is $2A/3$; otherwise the price equilibrium is $A - l^2/4$.*

Proof. It is shown in Appendix B.1 that the profit function (1) is quasi-concave. Hence, the profit-maximizing price either is at the kink $p = A - l^2/4$ or belongs to the segment of the profit function given by $2p\sqrt{A-p}$. If the RHS derivative of the profit function at $A - l^2/4$ is negative or zero, the maximum occurs at this point; if it is positive, the maximum belongs to the second segment of the profit function. Differentiating $2p\sqrt{A-p}$ yields

$$2\sqrt{(A-p)} - \frac{p}{\sqrt{(A-p)}}$$

Evaluating this expression at $A - l^2/4$, we obtain

$$l - \frac{A - l^2/4}{l/2}$$

which is positive if and only if $A < 3l^2/4$. It is then easy to see that the profit-maximizing price is obtained by applying the first order condition to $2p\sqrt{A-p}$ and is given by $2A/3$. ■

Our aim is now to determine what happens when the two countries trade with one another. Since both markets may or may not be totally served in free trade, it is convenient to suppose that both markets are served in autarky. The above proposition then shows that a market of size l is entirely served if and only if $A > 3l^2/4$. Since $L > 1$, the condition for the large country is more restrictive and is given by:

$$A \geq 3L^2/4 \tag{2}$$

It is called the *full market condition* under autarky. In what follows, we restrict ourselves to the domain of parameter values for which (2) holds. The corresponding mill prices are $p_L^a = A - L^2/4$ for the large country and $p_s^a = A - 1/4$ for the small one.

3 Equilibrium under Free Trade

In this section, we analyze what happens once borders are opened to trade between the two countries with *zero tariffs*. Hence, consumers are free to purchase from either firm at the same mill price, regardless of their locations, but must bear the corresponding transport costs. We define such a situation as free trade.

Depending on the values for the reservation price A and for the dissimilarity between country sizes L, there will be different types of equilibria. We first study the properties of the profit functions and show the existence of the price equilibrium. We then characterize the different types of equilibria. The welfare changes associated with each of these equilibria will be discussed in the subsequent section.

3.1 Existence of a price equilibrium under free trade

The expressions of the demand functions vary with the price interval considered. Specifically, there are two critical prices at which both demands exhibit kinks. First, for each firm i, there is a price p_i^a such that the outside border of its home market is reached. Clearly, this price arises at the profit-maximizing price in autarky. Given the rival's price, there is a second critical price p_i^x such that the consumer indifferent between the two firms pays exactly her reservation price A. Its value varies with the price set by the other firm: if the latter is higher (lower) than the other country's autarky price, then the critical price is lower (higher) than its own autarky price. When the rival firm sets its autarky price, the two critical prices are identical ($p_i^x = p_i^a$). It is readily verified that each profit function is continuous over the whole range of prices, but it is not differentiable at the two critical prices p_i^a and p_i^x.

Since the marginal consumer is given by

$$x_m = \frac{p_L - p_s}{L+1} + \frac{L+3}{4}$$

the profit functions of the two firms are as follows:

$$\begin{aligned} \pi_s &= p_s \left[Min\left\{v, x_m\right\} - Max\left\{0, w\right\} \right] \\ \pi_L &= p_L \left[Min\left\{L+1, y\right\} - Max\left\{z, x_m\right\} \right] \end{aligned}$$

where we define

$$
\begin{aligned}
v &= \frac{1}{2} + \sqrt{A - p_s} \\
w &= \frac{1}{2} - \sqrt{A - p_s} \\
y &= 1 + \frac{L}{2} + \sqrt{A - p_L} \\
z &= 1 + \frac{L}{2} - \sqrt{A - p_L}
\end{aligned} \tag{3}
$$

Proposition 2 *Assume two countries of sizes* 1 *and* L*, with each country having one firm located respectively at* $1/2$ *and* $1 + L/2$*. Assume further that consumers are uniformly located over the integrated market with a unique reservation price* A *and quadratic transport costs. Then, for any* (A, L) *a non-cooperative Nash equilibrium in prices exists.*

Proof. (i) The strategy space is given by $[0, A]$, which it is compact and convex. (ii) Both profit functions, defined in (2), are continuous both in p_s and p_L.

(iii) The functional form of each profit function depends on the price set by the rival. The rival's price belongs to one of the following three intervals: it is below, equal to, or above the autarky price of the rival's country. We show in Appendix B.2 that the resulting profit function is quasi-concave on $[0, A]$. ■

3.2 Characterization of the free trade equilibria

For each firm, there are three possible price intervals under free trade: the firm quotes a price lower, equal or higher than its autarky profit-maximizing price. If the equilibrium price is lower or equal to the autarky price, all consumers belonging to the domestic market will be served. On the contrary, if it sets a price higher than its autarky price, the consumers located near the outside border will not be served. As for the consumers located close to the common border, there is still the possibility for some of them to be served by the foreign firm. This means that we must consider nine possible outcomes. In what follows, we show that only four of them arise in equilibrium.

Lemma 1 *If a firm sets a price higher or equal to its autarky price, then it is optimal for the other firm to set a price lower than or equal to its own autarky price:*

if $p_j^* \geq p_j^a$, then $p_i^* \leq p_i^a$, $i, j = s, L$ and $i \neq j$

Proof. When a firm sets a price equal to or higher than its autarky price, the rival firm can make profits at least as high as what it would have done in autarky. Since the rival firm chooses to serve its whole domestic market in autarky, it is optimal for it to set a price lower than or equal to its autarky price. ■

This result enables us to eliminate all the configurations in which both firms select prices above their respective autarky prices as well as the cases in which one firm quotes a price higher than its autarky price while the rival firm sets its autarky price.

Lemma 2 *If the equilibrium price of the firm located in the large country is lower than its autarky price, then the same holds for the firm located in the small country.*

Proof. We want to eliminate the two price configurations for which the price set in the large country is lower than its autarky price (it can be written as $A - L^2/4 - k$ where $k > 0$), while the price charged in the small country is equal to or higher than its autarky price.

The first price configuration is such that the price in the small country is given by $A - 1/4$. Then, a necessary condition for this configuration to be an equilibrium is:

$$\left.\frac{\partial \pi_L^-}{\partial p_L}(p_s, p_L)\right|_{(p_s^a, p_L^a)} < 0$$

or, equivalently,

$$A > \frac{1}{4}\left(5L^2 + 4L\right) \tag{4}$$

The autarky price will be an equilibrium for the small country firm if

$$\left.\frac{\partial \pi_s^-}{\partial p_s}(p_s, p_L)\right|_{(p_s^a, p_L^a - k)} > 0$$

which is amounts to

$$A < \frac{1}{12}\left(5L^2 + 12L + 10\right) \tag{5}$$

It is obvious that the conditions (4) and (5) are not compatible, which implies that we can eliminate this price configuration as a possible equilibrium.

The second price configuration is such that the price of the small country firm is now higher than its autarky price while the price of the large country firm is lower than its own autarky price, that is, $A-L^2/4-k$. Since the price of the small country firm is higher than the autarky price, a necessary condition for $A-L^2/4-k$ to be the best reply for the large country firm can be written as follows:

$$\left.\frac{\partial \pi_L^+}{\partial p_L}(p_s,p_L)\right|_{(p_L^a,p_s^a+k)} < 0 \quad \text{for any } k>0$$

which amounts to

$$A > \frac{5L^2}{4}$$

Similarly, the price for the large country firm being lower than its autarky price, a price for the small country firm higher than its autarky price is the best reply if

$$\left.\frac{\partial \pi_s^+}{\partial p_L}(p_s,p_L)\right|_{(p_s^a,p_L^a-k)} > 0$$

or,

$$A < \frac{5L+6-4k}{4\,(L+2)}$$

Again, the two conditions on the parameter values are not compatible and we can eliminate this price configuration as a possible equilibrium. ■

The two preceding lemmas imply that *the highest possible equilibrium price for the small country firm is the autarky price.* This in turn means that all consumers residing in the small country always are served in equilibrium. Furthermore, they also imply that five price configurations can be eliminated. We now show that the space of admissible parameter values (A,L) can be partitioned into four subsets, each associated with a different price equilibrium. These four domains are defined as follows.

$$\mathbf{R}_1 = \{(A,L)\in R_+^2 \mid A < K_1 \equiv \frac{1}{4}\,(4L+5) \text{ and } A > \frac{3L^2}{4}\}$$

$$\mathbf{R}_2 = \{(A,L)\in R_+^2 \mid A > K_1,\ \ A > \frac{3L^2}{4} \text{ and } A < K_2 \equiv \frac{12L^3+14L^2+5L}{4\,(5L+2)}\}$$

$$\mathbf{R}_3 = \{(A,L)\in R_+^2 \mid A > K_1,\ \ A > K_2,\ \ A > \frac{3L^2}{4} \text{ and } A < K_2 \equiv \frac{10L^2+12L+5}{12}\}$$

$$\mathbf{R}_4 = \{(A, L) \in R_+^2 \mid A > K_3\}$$

Furthermore, the LHS and RHS derivatives of the profit functions at the autarky prices will be used below:

$$\frac{\partial \pi_s^-}{\partial p_s}(p_s^a, p_L) = \frac{p_L - 2p_s^a}{L+1} + \frac{L+3}{4}$$

$$\frac{\partial \pi_s^+}{\partial p_s}(p_s^a, p_L) = \frac{L+1}{4} + \frac{p_L - 2p_s^a}{L+1} + \sqrt{A - p_s^a} - \frac{p_s^a}{2\sqrt{A - p_s^a}}$$

$$\frac{\partial \pi_L^-}{\partial p_L}(p_s, p_L^a) = -\frac{2p_L^a - p_s}{L+1} + \frac{3L+1}{4}$$

$$\begin{aligned}\frac{\partial \pi_L^+}{\partial p_L}(p_s, p_L^a) &= 1 + \frac{L}{2} + \sqrt{A - p_L^a} - \frac{p_L^a - p_s + \frac{1}{4}(L+1)(L+3)}{L+1} \\ &\quad - p_L^a\left(\frac{1}{2\sqrt{A - p_L^a}} + \frac{1}{L+1}\right)\end{aligned}$$

These expressions are now used to determine the four parameter domains.

Proposition 3 *If $(A, L) \in \mathbf{R}_1$, then the price equilibrium is the autarky configuration $p_s^* = A - 1/4$ and $p_L^* = A - L^2/4$. If $(A, L) \in \mathbf{R}_2$, then the equilibrium price for the small country firm is below its autarky price while the equilibrium price of the large country firm is above its autarky price; they are implicitly defined as the solution to $\partial \pi_s^- / \partial p_s = 0$ and $\partial \pi_L^+ / \partial p_L = 0$. If $(A, L) \in \mathbf{R}_3$, then $p_s^* = (4A + 4L + 3)/8$ and $p_L^* = A - L^2/4$. If $(A, L) \in \mathbf{R}_4$, then $p_s^* = (L+1)(5L+7)/12$ and $p_L^* = (L+1)(7L+5)/12$.*

Proof. Since the profit functions are quasi-concave, a price equilibrium exists for any admissible pair (A, L). In order to determine this, it is sufficient to sign the partial derivatives of the profit functions evaluated at the autarky prices. In doing so, each parameter domain is defined by the four partial derivatives.

R$_1$. $\left(A-1/4, A-L^2/4\right)$ is an equilibrium if and only if the following four conditions are met:

Small country :

$$\frac{\partial\pi_s^-}{\partial p_s}(p_s,p_L)\bigg|_{(A-1/4,A-L^2/4)}>0,\ \frac{\partial\pi_s^+}{\partial p_s}(p_s,p_L)\bigg|_{(A-1/4,A-L^2/4)}<0,$$

Large country:

$$\frac{\partial\pi_L^-}{\partial p_L}(p_s,p_L)\bigg|_{(A-1/4,A-L^2/4)}>0,\ \frac{\partial\pi_L^+}{\partial p_L}(p_s,p_L)\bigg|_{(A-1/4,A-L^2/4)}<0$$

The only domain for which these four conditions are simultaneously verified is when $A<(4L+5)/4$.

R$_3$. Here the price equilibrium is given by $[(4A+4L+3)/8, A-L^2/4]$ and the four conditions that define the corresponding parameter domain are as follows:

Small country :

$$\frac{\partial\pi_s^-}{\partial p_s}(p_s,p_L)\bigg|_{(A-1/4,A-L^2/4)}>0,\ \frac{\partial\pi_s^+}{\partial p_s}(p_s,p_L)\bigg|_{(A-1/4,A-L^2/4)}<0,$$

Large country:

$$\frac{\partial\pi_L^-}{\partial p_L}(p_s,p_L)\bigg|_{[(4A+4L+3)/8,A-L^2/4]}>0,\ \frac{\partial\pi_L^+}{\partial p_L}(p_s,p_L)\bigg|_{[(4A+4L+3)/8,A-L^2/4]}<0$$

which are satisfied if the inequalities defining **R**$_3$ are met.

R$_4$. The price equilibrium $[(L+1)(5L+7)/12, (L+1)(7L+5)/12]$ occurs in the domain characterized by the following first order conditions.

Small country :

$$\frac{\partial\pi_s^-}{\partial p_s}(p_s,p_L)\bigg|_{(A-1/4,(L+1)(7L+5)/12)}<0,\ \frac{\partial\pi_s^+}{\partial p_s}(p_s,p_L)\bigg|_{(A-1/4,(L+1)(7L+5)/12)}<0$$

Large country:

$$\frac{\partial\pi_L^-}{\partial p_L}(p_s,p_L)\bigg|_{((L+1)(5L+7)/12,A-L^2/4)}<0,\ \frac{\partial\pi_L^+}{\partial p_L}(p_s,p_L)\bigg|_{((L+1)(5L+7)/12,\ A-L^2/4)}<0.$$

The only domain in which these four conditions are satisfied is the domain defined by $A>\left(10L^2+12L+5\right)/12$.

R$_2$. In this case, there is no explicit solution to the first order conditions. The corresponding parameter domain is obtained as the domain not covered by the other price equilibria. The equilibrium prices are such that $p_s^* < A - 1/4$ and $p_L^* > A - L^2/4$ because this is the only remaining configuration out of the nine ones identified in the foregoing. ■

This result has several important implications. One of them is that the small country never imports.

Corollary 1 *In equilibrium, the marginal consumer is located either at the border between the two countries or in the large country.*

Proof. The marginal consumer is given by :

$$x_m(p_s, p_L) = \frac{p_L - p_s}{L+1} + \frac{L+3}{4}$$

For parameter values belonging to $\mathbf{R}_1$, $\mathbf{R}_2$ or $\mathbf{R}_3$, the price equilibrium can be characterized in the following way:

$$p_s^* = A - \frac{1}{4} - k \text{ and } p_L^* = A - \frac{L^2}{4} + k'$$

where $k \in [0, A - 1/4]$ and $k' \in \left[0, L^2/4\right]$. This means that the marginal consumer can be written as:

$$x_m(p_s^*,\ p_L^*) = 1 + \frac{k + k'}{L+1}$$

Because of the restrictions imposed on k and k', we have $k + k' \geq 0$. In other words, $x_m(p_s^*, p_L^*) \geq 1$.

If (A, L) belongs to $\mathbf{R}_4$, then the equilibrium prices are given by $p_s^* = (L+1)(5L+7)/12$ and $p_L^* = (L+1)(7L+5)/12$ so that $x_m(p_s^*, p_L^*)$ can be computed as :

$$x_m(p_s^*,\ p_L^*) = \frac{5L+7}{12}$$

which is larger than one since $L > 1$. ■

3.3 Parameter domains

In the previous subsection, we have shown that four price equilibria can emerge and that the type of price equilibrium that occurs depends on the parameter values for A and L. In Figure 1, we represent the conditions defining the four domains.

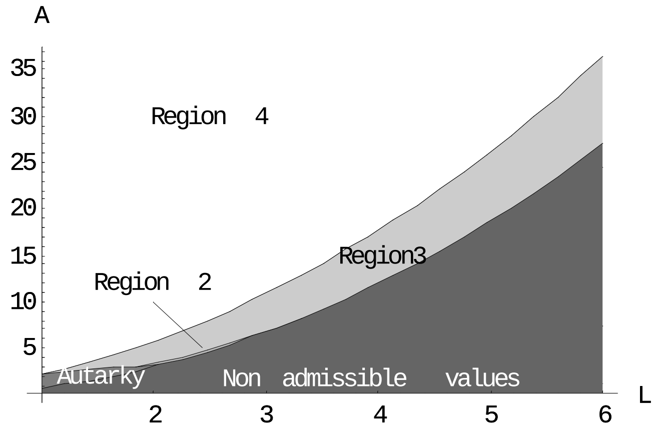

Figure 2: Parameter domains

The dark grey colored domain in the "south-east" is the parameter domain where A and L do not satisfy the full market condition (2). The "white" north-east domain is the free trade area in which both firms quote prices lower than their respective autarky prices. The lowest south-west domain corresponds to the parameter values for which firms, even though they have the possibility to sell in the other country, prefer to remain at the autarky equilibrium. The larger of the two central domains are the parameter values that lead to an equilibrium in which the large country firm is still at the autarky price whereas the small country firm charges a price lower than its autarky price. Finally, we have the small parameter domain $\mathbf{R}_2$ in which the small country firm quotes a price lower than its autarky price and the large country firm a price higher than its autarky price. Thus, in all domains but $\mathbf{R}_1$ where firms behave as in autarky, *the small country firm exports toward the large country.* By contrast, *the large country firm never sells in the small country.* When there is trade, the small country has an advantage due to the fact that some consumers of the large country are closer to the foreign firm than to their domestic firm.

Depending on the relative sizes of the two countries (i.e. on L), the sequence of market equilibria vastly differs as the reservation price A rises, that is, as consumers become richer and richer. First, if the two countries are similar enough in that $1 < L < 2.3$, both firms choose to sell at the autarky prices when A is low enough so that opening the border does not change anything. This is because each firm prices at the kink of its own demand function and this configuration holds for a whole range of values of A as in Salop (1979): no firm has an incentive to invade its rival's territory. When A rises, the market moves into $\mathbf{R}_3$ in which the large country firm sticks to its autarky price which corresponds to a kink in its demand function, while the small country firm reduces its price and exports to the foreign consumers located near the common border. In this case, the large country firm has no incentive to invade the foreign market, but the small country firm has an incentive. A further increase in A leads the economy into $\mathbf{R}_4$ in which both firms decrease their prices but only the small country firm exports because its price fall is larger than that of its rival. In these two domains, free trade lead to a price drop ,first, by the small country firm and, then, both firms.

Second, when $2.3 < L < 2.9$ and A is small, the economy is in $\mathbf{R}_2$. This means that *the small country firm decreases its price with respect to its autarky price while the large country firm increases its own and stops supplying the consumers located near its outside border.* Despite the competitive pressure exercised by the rival, the large country firm finds it profitable to charge a price higher than its autarky price on a smaller market segment because its demand becomes "much less" elastic over the corresponding domain of A. As A increases, one moves first into $\mathbf{R}_3$ and then into $\mathbf{R}_4$. In these successive moves, the price of the large country firm decreases with respect to its autarky price. This sequence of prices is reminiscent of Salop (1979) who observes that the market price first increases with the willingness-to-pay and then, becomes independent of the reservation price when A keeps rising. This is due to kinks in the profit functions that give rise, both in his and in our case, to nonmonotone reaction functions.

Last, when $L > 2.9$ we start in $\mathbf{R}_3$ and then enter $\mathbf{R}_4$ as A rises. Thus, in all cases, we observe a tendency towards a lowering of both equilibrium prices as the willingness-to-pay increases.

4 Welfare Changes under Free Trade

We have shown that four different price equilibria may arise, depending on the parameters A and L. Each case leads to changes in profits and consumer surplus relative to those obtained under autarky. In this section, the outcomes for each of these cases are discussed. The autarky outcome takes place when $(A, L) \in \mathbf{R}_1$. In this case, it is obvious that free trade does not lead to any welfare change. For this reason, we will not discuss further this particular equilibrium and the results stated below refer only the equilibria corresponding to domains $\mathbf{R}_2$ (where p_L is higher than p_L^a), $\mathbf{R}_3$ (where p_L is exactly equal to the autarky price, and p_s lower than its autarky price) and $\mathbf{R}_4$ where both prices are below the autarky prices.

4.1 The small country

Proposition 4 *If the large country firm does not set a price lower than the autarky price, the profit of the small country firm increases under free trade. If both countries set prices lower than their autarky prices, the profit of the small country firm increases provided that the reservation price is not too high.*

Proof. $\mathbf{R}_2$. We know that the price of the large country firm is higher than the autarky price, while the opposite holds for the small country firm. The free trade price for the large country firm can then be written as follows:

$$p_L = A - L^2/4 + k'$$

where $k' > 0$. The corresponding free trade price for the small country firm can be computed by using the reaction function for the small country firm:

$$p_s = \frac{1}{8}\left(3 + 4A + 4L + 4k'\right)$$

Based on this information, the difference in profits between the two market situations is:

$$\Delta\pi_s = \frac{16A^2 + 16(k')^2 - 8A(4L - 4k' + 5) + 8k(4L + 3) + (4L + 5)^2}{64(L + 1)}$$

Since $\dfrac{\partial \Delta\pi_s}{\partial k'} > 0$, then $\Delta\pi_s$ will be positive for any $k' > 0$ if $\Delta\pi_s$ is positive when $k' = 0$.

When $k' = 0$, we have:

$$\Delta\pi_s = \frac{16A^2 - 8A(4L + 5) + (4L + 5)^2}{64(L + 1)}$$

which is positive for any admissible A and L because the numerator is a quadratic form in A which has a single root at $(4L + 5)/4$ (see the definition of $\mathbf{R}_2$).

R$_3$. The equilibrium price for the small country firm is the same as the one computed for **R**$_2$ when $k' = 0$, that is:

$$p_s = \frac{1}{8}(3 + 4A + 4L)$$

This means that the difference in profits can be written as follows:

$$\Delta\pi_s = \frac{(4L + 5 - 4A)^2}{64(L + 1)}$$

which is positive for any A and L.

R$_4$. The difference in profits is:

$$\Delta\pi_s = \frac{1}{144}(L + 1)(5L + 7)^2 - A + \frac{1}{4}$$

This expression is positive as long as the reservation price A is lower than A' given by:

$$A' \equiv \frac{1}{144}\left(25L^3 + 95L^2 + 119L + 85\right)$$

■

Thus, the small country firm does not benefit from free trade once the willingness-to -pay of consumers is sufficiently high for all consumers to buy and price competition to be fierce.

Proposition 5 *All consumers of the small country are better off under free trade.*

Proof. The consumers of the small country will be better off if the following two conditions are met: (i) they are all served at the free trade equilibrium; (ii) each consumer pays a price lower than the autarky price. The consumers in the vicinity of 0 are still served because the price is lower than p_s^a and the consumers in the neighborhood of 1 are also served because $x_m(p_s^*,\ p_L^*) > 1$. ■

Proposition 6 *The total welfare in the small country is higher under free trade than under autarky.*

Proof. The loss made by the firm on the domestic market generated by the price fall is exactly matched by the gain made by the domestic consumers. The aggregate welfare change in the small country is positive because the corresponding producer expands its market segment by selling to some consumers of the large country. ■

Thus, one may expect *small countries to be more inclined in promoting free trade agreements.*

4.2 The large country

Proposition 7 *The profit of the large country firm always decrease under free trade.*

Proof. $\mathbf{R}_2$. Recall that in this case the equilibrium prices can be written as $p_s^* = (3 + 4A + 4L + 4k')/8$ and $p_L^* = A - L^2/4 + k'$. For the large country, the difference in profits is:

$$\Delta\pi_b = (p_L^a + k')[y - x_m(p_s^*, p_L^*)] - p_L^a L$$

with $k' > 0$. Let

$$\Delta\pi_b^{'} \equiv (p_L^a + k')2\sqrt{A - p_L^a - k'} - p_L^a L$$

It is obvious that $\Delta\pi_b^{'} > \Delta\pi_b$. For any $k' > 0$, it is then easy to show that $\Delta\pi_b^{'}$ is negative. This implies that $\Delta\pi_b < 0$.

$\mathbf{R}_3$ **and** $\mathbf{R}_4$. The large country firm's equilibrium price is lower than or equal to the autarky price. To show that the profit change is negative, it is sufficient to observe that the marginal consumer is located in the large country (Corollary 1). ■

This suggests that *the large country firm would always lobby against free trade* while this is not necessarily true for the small country firm.

Proposition 8 *In* $\mathbf{R}_3$ *and* $\mathbf{R}_4$*, the consumer surplus increases in the large country under free trade.*

Proof. $\mathbf{R}_3$. Recalling that $p_s^* = (3 + 4A + 4L)/8$ and $p_L^* = A - L^2/4$, we see that the change in consumer surplus for the large country is:

$$\Delta CS_L = \frac{(4L + 5 - 4A)^2}{128(L + 1)}$$

which is positive for any A and L.

$\mathbf{R}_4$. The change in consumer surplus is now:

$$\Delta CS_L = \frac{1}{288}[-215L^3 - 313L^2 + (-145 + 288A)L + 25]$$

This expression is positive for any A which larger than A' where A' is given by

$$A' \equiv -\frac{[-215L^3 - 313L^2 + (-145 + 288A)L + 25]}{288L}$$

This condition on the reservation price is satisfied since we are in $\mathbf{R}_4$, thus implying that the consumer surplus change is positive. ■

Hence, each consumer of the large country is better off under free trade when the reservation price is sufficiently high for free trade to yield a market outcome in which both firms price below their autarky prices. However, we cannot preclude the possibility for many consumers of the large country to be worse off under free trade. Indeed, in $\mathbf{R}_2$ the consumer surplus change in the large country may be positive or negative. All the consumers of the large country are not affected in the same way by a move towards free trade. More precisely, *consumers of the large country can be partitioned into four different groups according to the way they are influenced by trade.* The first group of consumers buys from the small country firm and gains in comparison to the autarky situation. The second group also buys from the small country firm but loses in comparison to their situation in autarky. The third group continues to buy from the large country firm and, therefore, loses with respect to autarky. The last group of consumers loses because they no longer buy the good.

Proposition 9 *The total welfare change in the large country is negative when it engages in free trade.*

Proof. $\mathbf{R}_2$. To show that the overall welfare change is negative for the large country, it is sufficient to show that the positive surplus change that certain consumers gain by purchasing from the small country firm at a lower price does not compensate the losses incurred by the large country firm because these same consumers no longer buy from the domestic firm. Indeed, the gain made by the large country firm resulting from a higher price is simply a transfer from the corresponding consumers to this firm. Recall that in this case the equilibrium prices can be written as $p_s^* = (3 + 4A + 4L + 4k')/8$ and $p_L^* = A - L^2/4 + k'$. Denote the positive consumer surplus change by ΔCS_L and let $\Delta CS'_L \equiv (A - P'_s)[x_m(p_s^*, p_L^*) - 1]$ where P'_s is the full price of the small country firm evaluated at the border. The price P'_s is given by $A/2 + (L + k' + 5/4)/2$. It is obvious that $\Delta CS_L < \Delta CS'_L$. The total welfare change will be negative if $(A - P'_s)[x_m(p_s^*, p_L^*) - 1] < p_L^a [x_m(p_s^*, p_L^*) - 1]$. This condition can be rewritten as $A > 2L^2/4 - (L + k' + 5/4)$, which always holds in $\mathbf{R}_2$.

$\mathbf{R}_3$. For the corresponding price equilibrium, the total welfare change is given by:

$$\Delta W_L = \frac{-16L^3 - 4L^2 + 40L + 25 - 48A^2 + 8A(2L^2 + 4L + 5)}{128(L+1)}$$

This expression is negative for any $A > (4L + 5)/4$, which holds in $\mathbf{R}_3$.

$\mathbf{R}_4$. The overall change in welfare in the large country is now given by:

$$\Delta W_L = -\frac{5}{96}\left(3L^3 + 5L^2 - 3L - 5\right)$$

which is independent of A and negative for any $L > 1$. ■

This suggests that large countries, unlike small ones, should be less inclined towards free trade.

The comparison of global welfare under autarky and free trade shows that global welfare never decreases when the entire integrated market is served. This is clear in domains $\mathbf{R}_1$, $\mathbf{R}_3$ and $\mathbf{R}_4$ in which the integrated market is supplied. Indeed, Corollary 1 implies that total transport costs borne by consumers are lower than (or equal to) those prevailing in autarky since the marginal consumer is (weakly) closer to the midpoint between the two firms. On the other hand, in domain $\mathbf{R}_2$ some consumers residing in the large country no longer buy under free trade, thus making the comparison especially hard because we do not have a closed form solution in this domain.

5 Conclusions

We have seen that the small country always gains from trade at the aggregate level. Going over to free trade lowers the equilibrium price prevailing in this country. This constitutes a gain for the local consumers and is known as the pro-competitive effect of free trade. The same price reduction negatively affects the producer's profits earned from home sales. But this negative effect is compensated by an output expansion associated with exports, so that *the total welfare in the small country is unambiguously improved by the opening to trade.* By contrast, *total welfare is always negatively affected by trade in the large country.* The reason is that the negative impact of the pro-competitive effect does not compensate the loss made by the local producer. More precisely, this effect is not compensated here by an output expansion effect, the large country always having a disadvantage in the "border region". Finally, global welfare is in general higher under free trade than under autarky, due to the better allocation of consumers between the two firms.

Trade always flows from the small to the large country. But even if the large country does not export to the small one, there is still a downward, competitive pressure on its prices. Indeed, the highest price that it can quote is its autarky price, and this occurs for a small region of parameter values. For the large country, the situation is different. Free trade does not seem to have the same impact on prices. We have even a parameter domain in which trade leads to an increase in the price set by the domestic firm. For another domain of parameters, the firm of the large country still posts its autarky price.

All together, these results suggest that *geographical size matters for the nature of trade and the intensity of international competition*. Although our model is in the Hotelling tradition, we hope that it will serve as a prototype for a more general setting in which firms located in the main urban centers of two distinct countries compete altogether under free trade. Clearly, more work is called for here. Among possible extensions, it would be worth investigating the case of positive, but not prohibitive, tariffs. Equally interesting would be to study the impact of the geography (shape, relative position and transport networks) of the two countries on the distribution of gains associated with free trade.

Finally, let us make it clear that we would be the last to believe that the conclusions of this paper should imply that large countries always loose from international trade. Many other factors should be taken into account and may well reverse our results. However, we believe that the integration of more geographical variables in international trade theory is susceptible to give rise to other unsuspected forces.

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A The case of linear transport costs

When linear transport costs are used, certain conditions on the firms' locations must be met in order for a Nash price equilibrium in pure strategies to exist. The main proof and explanation of the argument can be found in d'Aspremont *et al.* (1979) and therefore will not be repeated here. In d'Aspremont *et al.* (1979, p.1146), the necessary and sufficient conditions for a price equilibrium in a market of length l with firms located at a and $l-b$, are given as:

1. $\left(l + \frac{a-b}{3}\right)^2 \geq \frac{4}{3} l \left(a + 2b\right),$
2. $\left(l + \frac{b-a}{3}\right)^2 \geq \frac{4}{3} l \left(b + 2a\right)$

In our model, because $l = L + 1$ and the firms are located at $a = 1/2$ and $l - b = L/2 + 1$, the first condition becomes:

$$\left[L + 1 + \left(\frac{\frac{1}{2} - \frac{L}{2}}{3}\right)\right]^2 \geq \frac{4}{3}(L+1)\left(L + \frac{1}{2}\right)$$

Rearranging the terms yields the following condition:

$$-23L^2 - 2L + 25 \geq 0$$

It is obvious that this condition is never satisfied for any $L > 1$, so that a pure strategy equilibrium does not exist[1].

B Quasi-concavity of the profit functions

Lemma 3 *Assume $A > 3l^2/4$. Then, the first derivative of $2p\sqrt{A-p}$ evaluated at $A - l^2/4$ or at any higher price is negative.*

Proof. The first derivative of $2p\sqrt{A-p}$ is given by

$$2\sqrt{A-p} - \frac{p}{\sqrt{A-p}}$$

This expression evaluated at $A - l^2/4$ gives us:

$$l - \frac{A - \frac{l^2}{4}}{\frac{l}{2}}$$

which is negative since $A > 3l^2/4$.

The second order derivative is given by:

$$-\frac{2}{\sqrt{A-p}} - \frac{1}{2}\frac{p}{\left(\sqrt{A-p}\right)^3}$$

which is negative for any A and p. Consequently, the first derivative is negative when it is evaluated at any price higher than or equal to $A - l^2/4$. ■

Lemma 4 *If the profit function can be written as $p(s-t)$, then the second order derivative of the profit function with respect to its own price is negative if $s' \leq 0, t' \geq 0, s" \leq 0$, and $t" \geq 0$.*

[1] For an analysis of the existence problem in a Hotelling model where the reservation price is binding, see Economides (1984).

B.1 Quasi-concavity in autarky

Lemma 5 *Consider a country in autarky. Then, the profit function of a firm located at the center of the country is quasi-concave in its price.*

Proof. If the market size is l, the profit function is given by:

$$\pi(p) = \begin{cases} pl & \text{if} \quad 0 \leq p \leq A - \frac{l^2}{4} \\ 2p\sqrt{A-p} & \text{if} \quad A - \frac{l^2}{4} < p \leq A \end{cases}$$

This function is continuous but not differentiable at $A - l^2/4$. To show quasi-concavity, it is sufficient to prove that the partial derivative of the second segment of π is always negative. This is shown in Lemma 3. ■

B.2 Quasi-concavity in free trade

Proposition 10 *A firm located at* $1 + L/2$ *and competing with a firm located at* $1/2$ *has a profit function which is quasi-concave in its price.*

Proof. The functional form of the profit function of the large country firm depends on the price quoted by the small country firm. Three cases may arise: (a) $p_s < p_s^a$;(b) $p_s = p_s^a$; (c) $p_s > p_s^a$.

The profit function is given by:

$$\pi_L(p_s, p_L) = \begin{cases} p_L(L + 1 - x) & \text{if} \quad 0 \leq p_L \leq pc_L^1 \\ p_L(\alpha - \beta) & \text{if} \quad pc_L^1 \leq p_L \leq pc_L^2 \\ p_L(2\sqrt{A - p_L}) & \text{if} \quad pc_L^2 \leq p_L \leq A \end{cases}$$

$pc_L^i = p_L^a$ and $pc_L^j = p_L^x$. If $p_s < p_s^a$, then $i = 1$ and $j = 2$; if $p_s > p_s^a$, then $i = 2$ and $j = 1$.

The profit function is quasi-concave when the following conditions are satisfied: (i) if the LHS derivative of the profit evaluated at pc_L^1 is negative, the RHS derivative is also negative; (ii) if the RHS derivative of the profit evaluated at pc_L^1 is negative, the LHS derivative evaluated at pc_L^2 is also negative, (iii) the LHS derivative evaluated at pc_L^2 is negative, the RHS derivative is also negative.

(i) It is sufficient to show that $[p_L(L + 1 - x)]' > [p_L(\alpha - \beta)]'$ holds. If $p_s < p_s^a$, then $(\alpha, \beta) = (z, x)$. Evaluated at $p_L = p_L^a$, this condition reduces to $-p_L z' > 0$ which is always true since $z' < 0$. If $p_s > p_s^a$, then we have $(\alpha, \beta) = (L+1, y)$. Evaluated at $p_L = p_L^x$, this condition becomes $p_L^x > A - (L+1)^2/4$ which is always satisfied since otherwise the two full price curves would intersect on the left of $1/2$.

(ii) It is sufficient to show that the second derivative is negative for $pc_L^1 \leq p_L \leq pc_L^2$. If the RHS first derivative at pc_L^1 is negative, the LHS first derivative at pc_L^2 is negative. Indeed, by Lemma 4, if $p_s < p_s^a$ then $(s,t) = (z, x)$ with $z' < 0, x' > 0, z" < 0$, and $x" = 0$; if $p_s > p_s^a$ then $(s,t) = (L+1, z)$ with $(L+1)' = 0, z' > 0, (L+1)^" = 0$, and $z" > 0$.

(iii) It is not necessary to prove the desired relationship because the RHS derivative evaluated at pc_L^2 is always negative. This is proved by using Lemma 3.

It remains to discuss the case where $p_s = p_s^a$. Then, $pc_L^1 = pc_L^2 = p_L^a$ so that the second segment of the profit function vanishes. To show quasi-concavity, it is sufficient to prove that the RHS derivative is always negative at the critical price, which holds by Lemma 3. ■

Proposition 11 *A firm located at $1/2$ and competing with a firm located at $1 + L/2$ has a profit function which is quasi-concave in its price.*

Proof. As in the above proof, three cases may arise: (a) $p_L < p_L^a$; (b) $p_L = p_L^a$; (c) $p_L > p_L^a$

$$\pi_s(p_s, p_L) = \begin{cases} p_s x & \text{if} \quad 0 \leq p_s \leq pc_s^1 \\ p_s(\alpha - \beta) & \text{if} \quad pc_s^1 \leq p_s \leq pc_s^2 \\ p_s(2\sqrt{A - p_s}) & \text{if} \quad pc_s^2 \leq p_s \leq A \end{cases}$$

(i) It is sufficient to show that $(p_s x)' > [p_s(\alpha - \beta)]'$ holds. If $p_L < p_L^a$, then $(\alpha, \beta) = (x, w)$. The condition evaluated at $p_s = p_s^a$ is $-p_s w' < 0$, which is always true because $w' > 0$. If $p_L > p_L^a$, then $(\alpha, \beta) = (v, 0)$ and the condition becomes $(p_s x)' > (p_s v)'$ where p_s is evaluated at p_s^x. This reduces to $x' > v'$ or $p_s^x > A - (L+1)^2/4$. Again, this condition is always satisfied since otherwise the two full price curves would intersect on the right of $L/2 + 1$.

(ii) This is proved by showing that the second order derivative is negative. Because the first price is lower than the second price, if it is negative at that price it will negative also for any price higher. If $p_L < p_L^a$ then, using Lemma 4 and setting $(s,t) = (x, w)$, we have the sign sequence $(< 0, > 0, 0, > 0)$. On the contrary, if $p_L > p_L^a$ then, using again Lemma 4 with $(s,t) = (v, 0)$ we have $(< 0, 0, < 0, 0)$.

(iii) In both cases, this is proved in Lemma 3.

If $p_L = p_L^a$ then as in the case of the large country, $pc_s^1 = pc_s^2 = p_s^a$ and the second segment of the profit function disappears. Again, a sufficient condition for the profit function to be quasi-concave is that the last partial derivative is always negative (Lemma 3). ■