

CORE DISCUSSION PAPER
2004/7

**INVARIANTS IN THE RIEMANNIAN
GEOMETRY OF CONVEX SETS**

Roland HILDEBRAND¹

April 2004

Abstract

In this contribution we study some aspects of the Riemannian geometry induced on a convex set by a barrier function of the set. Using Noether's theorem, we link the symmetries of the set to invariants of the geodesic flow. This allows to lower the dimension of the differential system defining the geodesics and gives insights in the structure of the geodesic flow, specifically on the configuration of geodesic submanifolds. We use the developed apparatus to completely integrate the geodesic equations for the convex hulls of the sphere, the paraboloid, the hyperboloid and the standard symmetric cones and to obtain explicit formulae for the geodesics on these sets.

Keywords: Riemannian geometry, barrier function, convexity, symmetry.

JEL Classification: C69.

¹Laboratoire de Modélisation et Calcul, IMAG, Grenoble, France.
E-mail: roland.hildebrand@imag.fr

This paper presents research results of the Belgian Program on Interuniversity Poles of Attraction initiated by the Belgian State, Prime Minister's Office, Science Policy Programming. The scientific responsibility is assumed by the author.