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**THE RELATION OF TIME INDEXED FORMULATIONS OF
SINGLE MACHINE SCHEDULING PROBLEMS TO THE
NODE PACKING PROBLEM**

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Abstract

The relation of time indexed formulations of nonpreemptive single machine scheduling problems to the node packing problem is formally established and then used to provide simple and intuitive alternate proofs of validity and maximality for previously known results on the facial structure of the scheduling problem. Previous work on the facial structure has focused on describing the convex hull of the set of feasible partial schedules, i.e. schedules in which not all jobs have to be started. The equivalence between the characteristic vectors of this set and those of the set of feasible node packings in a graph whose structure is determined by the parameters of the scheduling problem is established. The main contribution of this paper is to show that the facet inducing inequalities for the convex hull of the set of feasible partial schedules that have integral coefficients and right hand side 1 or 2 are the maximal clique inequalities and the maximally and sequentially lifted 5-hole inequalities of the convex hull of the set of feasible node packings in this graph respectively.

Keywords: scheduling, node packing, polyhedral methods, facet defining graphs, lifted valid inequalities, facet inducing inequalities.

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