Game Theory and the Environment: Old Models, New Solution Concepts*

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Abstract

We survey some basic game theoretic models to analyse environmental problems. We also discuss some new game theoretic solution concepts such as (coalition-proof) correlated equilibrium and the theory of social situations, which have never been used in environmental economics and suggest that these concepts should be used to analyse environmental problems.

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1. INTRODUCTION

In many real life situations, we make decisions to achieve certain outcomes; in many of such situations however, we also have to interact with other decision makers and as a consequence, the outcome depends not only on what we choose but also on what others do. A *game* is essentially such an interactive decision making process. *Game theory* is a tool to model and thereby analyse situations involving interactions, and possibly, cooperation, between several rational and intelligent decision makers. Game theory, not surprisingly, has been extremely useful as a modelling tool in many areas of social sciences. A lot of environmental problems also involve interactive decision making and hence game theory can be easily applied to model, analyse and solve, at least theoretically, these problems. For example, a pollutant emitted by a firm (or a country) pollutes the global atmosphere; hence, the decisions of several firms (countries) about their emission levels can easily be modelled as a game.

In game theory, we make a broad distinction between non-cooperative and cooperative situations (games). In most situations, decision makers (players) make decisions (play the game) only for themselves without any form of cooperation among themselves. Such games are called non-cooperative games. Some of these games are played simultaneously in which the players make decisions simultaneously. Some games however are played over time. To formulate simultaneously played game, we use *normal* or *strategic* forms while we have *extensive forms* to model games played over time. Non-cooperative solution concepts are used to analyse such games. The most popular concept is *Nash equilibrium*. A Nash equilibrium is a strategy profile consisting of one strategy for each player, such that no player has any incentive to deviate unilaterally from the profile; it is therefore a profile in which everyone is doing their best given others. Several refinements (*perfect*, *proper* for normal form, *subgame perfect*, *sequential* for extensive form games) of this basic concept have been extensively used to analyse models.

In many game theoretic situations, cooperation, in the sense of forming coalitions and doing something together, is an essential feature. In fact, in the global pollution example, the countries may very well form coalitions and act together. In the cooperative branch of game theory, every possible coalition of players is assumed to have a *value*. The cooperative solutions suggest how the total value of the grand coalition can be split among all the players in a satisfactory way. The *stable set* and the *core* are the two most popular concepts.

Within the restrictions of non-cooperative games, we may allow unlimited pre-play communication among the players; the players may be free to discuss their strategies but not allowed to make binding commitments. Thus, the players can form betraying coalitions and hence the stability of any outcome is always threatened by potential deviations by any coalition. In such an environment, Nash best-response is certainly a requirement for self-enforceability but not, in general, sufficient. Considerations of this sort have motivated the notion of *strong Nash equilibrium* (Aumann, 1959) which in fact is the first *refinement* of Nash equilibrium and it requires stability against deviations by every conceivable coalition. Later, Bernheim, Peleg and Whinston (1987) provided a stronger definition of self-enforceability and labelled the class of *self-enforcing* agreements *coalition-proof*.

The objective of this particular study is twofold. First we explicitly present two very simple but fundamental models of environmental games. These two basic models analyse two environmental problems as games by using several well-known solution concepts. These models are enough to capture and summarise the primary contributions of game theory to environmental economics. In the second part of this survey we discuss several new game theoretic solution concepts. These concepts have not yet been used in the environmental literature; however, if applied, these concepts could certainly provide a better understanding of any environmental problem like the ones we describe in the first part. We are however just proposing this new agenda of research and leaving it for the researchers to pursue.

Let us briefly describe the two models we discuss in the paper. First, consider two (or a few) neighbouring countries who are emitting a pollutant. The pollutant obviously affects the global atmosphere. Suppose now the countries are thinking of reducing the level of emission. A reduction in the emission level would reduce the global pollution and hence both countries would benefit. However, such an abatement also involves a cost. Thus reducing emission reduces the environmental damage cost but increases the abatement cost. So the countries have to consider the total cost of their emissions. This is a classic environmental problem which can easily be treated as a game between these countries and therefore game theoretic solution concepts can be applied to analyse this problem (Endres and Finus, 1998).

Environmental problems such as transfrontier pollution are often multilateral, and they exhibit public goods (rather, public bads) characteristics in the sense that whenever pollution is generated, it affects all the agents in the economy (countries). Thus one can deal with such a problem by modelling an economy with several agents whose productive activities generate multilateral negative externalities. The externalities are additive because what is received by any agent (country) is simply the sum of what is emitted by all the generators. One can easily model such an economy as a game (Chander and Tulkens, 1997).

Obviously, the above two models are not the only game theoretic models to analyse environmental problems. Readers are particularly encouraged to see two excellent texts edited by Pethig (1992) and by Hanley and Folmer (1998) that collect a series of brilliant papers.

The main purpose of this paper however is to point out some new directions, some new concepts of game theory which have not yet been used so far and would definitely be extremely useful as tools to analyse environmental games. The first is *correlation*. When the players can *correlate* their strategies in a non-cooperative game, a new self-enforcing agreement emerges out which we call *correlated equilibrium* (Aumann, 1974, 1987). Another possible version of correlation is due to Moulin and Vial (1978). Second, within a non-cooperative environment, we can allow players to correlate and to form coalitions; in such an environment, the right solution concept is *coalition-proof correlated equilibrium*. One can very well characterise and thereby analyse the set of (coalition-proof) correlated equilibria for the models presented here. We briefly present these game theoretic concepts here. Finally, we discuss the theory of social situations (Greenberg, 1990). It will be really interesting to apply these powerful techniques to understand and analyse environmental games such as the two models presented in the first part of this paper.

The rest of the paper is organised as follows. In the following section, we recall some well known and popular solution concepts of cooperative and non-cooperative game theory. Sections 3 and 4 describe and analyse the two game theoretic models respectively. In Section 5, we discuss some new game theoretic solution concepts.

2. SOLUTION CONCEPTS

2.1. Non-cooperative Games: Nash, Strong Nash and Coalition-proof Nash Equilibria

Consider any strategic form game G with finite set of players N, finite sets of pure strategies $S_1,...,$ S_n, and utility functions $u_1,..., u_n$, $u_i : S\ (= \prod_{i \in N} S_i) \rightarrow \mathbb{R}$, for all i.

$s^* \in S$ is a *strong Nash equilibrium* (Aumann, 1959) if and only if for all $J \subseteq N$ and for all $s_J \in \prod_{joJ} S_j$, there exists a player $i \in J$ such that $u_i\ (s^*) \geq u_i\ (s_J, s^*_{-J})$

A *reduced* game (Peleg and Tijs, 1996) with respect to $s \in S$ and a coalition $J \subset N$, $G^{J,\,s}$, denotes the game induced on the coalition J by actions s_{-J}, i.e., $G^{J,\,s} \equiv [\ J, \{S_i\}_{ioJ}, \{u\mathrm{N}_i\}_{ioJ}\ ]$, where, $u\mathrm{N}_i : \prod_{ioJ} S_i \rightarrow \mathbb{R}$ is given by, $u\mathrm{N}_i(t_J) = u_i(t_J, s_{-J})$ for all $i \in J$ and $t_J \in \prod_{ioJ} S_i$.

In a single player game, $s^* \in S$ is a *coalition-proof Nash equilibrium* (Bernheim, Peleg and Whinston, 1987) if and only if s^* maximizes $u_1(s)$. Now, for $n > 1$, assume that coalition-proof Nash equilibrium has been defined for games with fewer than n players. Then,

(i) $s^* \in S$ is *self-enforcing* if for all coalitions $J \subset N$, s^*_J is a coalition-proof Nash equilibrium in the reduced game $G^{J,\,s*}$.

(ii) $s^* \in S$ is coalition-proof Nash equilibrium if it is self-enforcing and Pareto undominated among the self-enforcing strategy vectors.

For the special case of two-person games, it follows immediately from the above definition that the set of coalition-proof Nash equilibria is exactly the set of Nash equilibria which are not Pareto dominated by any other Nash equilibrium.

2.2. Coalitional Form Games : Core

A coalitional form game is characterised by the set of players, N, hence, the set of all possible coalitions, 2^N, and a *value* (*characteristic*) *function*, $v : 2^N \to \mathbb{R}$, assigning a value to each of the possible coalitions. We assume that the value or worth of any coalition S, $v(S)$, is transferable and thereby can be shared among the members. A game in coalitional form can therefore be denoted by $< N, v >$.

A solution is simply an *allocation*, a split of the total value (of the grand coalition) among all the agents.

An allocation x *dominates* another allocation y through a coalition S ($x \succ_S y$) if $x_i > y_i$ for all $i \in S$ and $\sum_i x_i \leq v(S)$. A coalition S *blocks* y if there exists x such that $x \succ_S y$.

Core is the set of un-dominated allocations (the set of allocations that no coalition can block). Core is a set which can be characterized easily by the following result.

$x \in$ core if and only if $\sum_i x_i = v(N)$ and $\sum_i x_i \geq v(S)$, for all S.

3. A SIMPLE ENVIRONMENTAL GAME

In this section we present a simple game theoretic model based on the work by Endres and Finus (1998). Readers are encouraged to see the references cited in their paper as well.

Consider two neighbouring countries who are emitting a pollutant. The pollutant obviously affects the global atmosphere. Suppose now the countries are thinking of reducing the level of emission. A reduction in the emission level would reduce the global pollution and hence both countries would benefit. However, such an abatement also involves a cost. Thus reducing emission reduces the environmental damage cost but increases the abatement cost. So the countries have to consider the total cost of their emissions. Let E_1 and E_2 be the level of emissions of the two countries respectively. Let $E = E_1 + E_2$. Let the total cost of country 1 be

$$TC_1 = \frac{c_1}{2}E^2 + \frac{b}{2}(\frac{a}{b} - E_1)^2$$

where a, b and c_1 are constants.

The first term represents the environmental damage caused by the total pollutant while the second term represents the abatement cost. The level a/b can be interpreted as the emission level where the country does not abate at all, in which case the abatement cost is zero. Let us therefore say that the decision country 1 has to make is to choose an emission level between 0 and a/b.

The cost function is so chosen that the environment cost increases with the aggregate emissions; moreover, it increases at an increasing rate. Similarly, reducing emissions from a/b increases the abatement cost at an increasing rate.

Country 2 faces a similar situation. Suppose the total cost of country 2 is given by

$$TC_2 = \frac{c_2}{2}E^2 + \frac{b}{4}(\frac{a}{b} - E_2)^2$$

where a, b are as in the cost function of country 1 and c_2 is another constant. Two countries are therefore different in terms of the parameters in the cost function. Country 2 has a different parameter c_2 in the environmental damage cost; it also exhibits half of the country 1's abatement cost.

From the total cost functions, we can find the marginal environmental damage cost, *MDC* and the marginal abatement cost (due to emission reduction), *MAC*. They are given by

$MDC_1 = c_1E$; $MDC_2 = c_2E$; $MAC_1 = a - bE_1$; $MAC_2 = ½(a - bE_2)$.

From a social point of view, if we have to choose E_1 and E_2, we should minimise the total cost of the society, namely, $TC = TC_1 + TC_2 = DC_1 + AC_1 + DC_2 + AC_2$

The first order conditions of the above problem are given by

$MAC_1 = MDC_1 + MDC_2$

$MAC_2 = MDC_1 + MDC_2$

We therefore obtain the socially optimal level of E_1 and E_2.

$$E_1^S = \frac{a(b + c_1 + c_2)}{b(b + 3c_1 + 3c_2)}$$

and

$$E_2^S = \frac{a(b - c_1 - c_2)}{b(b + 3c_1 + 3c_2)}$$

The total emission is given by

$$E^S = E_1^S + E_2^S = \frac{2a}{b + 3c_1 + 3c_2}$$

To guarantee $E_2^S \geq 0$, let us impose a non-negativity constraint on the parameters, namely, $b \geq c_1 + c_2$. We can obviously model this situation as a non-cooperative game. The two countries are the two players in the game. The pure strategies are the emission levels. The pure strategy sets are $S_1 = S_2 = [0, a/b]$. Given a strategy profile (E_1, E_2), the resulting payoffs are (negative of) the total costs. We can easily find the Nash equilibrium of the game. First, given other country's strategy, we can find the best response of one country. From the first order conditions ($MAC_i = MDC_i$), the best responses are

$$BR_1(E_2) = \frac{a - c_1 E_2}{b + c_1}$$

and

$$BR_2(E_1) = \frac{a - 2c_2 E_1}{b + c_2}$$

Hence the Nash equilibrium emission levels are

$$E_1^N = \frac{a(b - c_1 + 2c_2)}{b(b + c_1 + 2c_2)}$$

and

$$E_2^N = \frac{a(b + c_1 - 2c_2)}{b(b + c_1 + 2c_2)}$$

and the total equilibrium emission level is

$$E^N = E_1^N + E_2^N = \frac{2a}{b + c_1 + 2c_2}$$

Again, to ensure positive emission rates, we require two constraints, namely, $b \geq c_1 - 2c_2$ and $b \geq 2c_2 - c_1$. The first one is weaker than the constraint we already have imposed and hence can be dropped.

It is clear from the above that the total Nash equilibrium emission is higher than the total socially optimal level. The countries therefore can negotiate to reach an agreement. Finus and Rundshagen (1998a) have considered three types of agreements: a socially optimal solution, a uniform emission tax (a tax equally applied in both countries) and a uniform emission reduction quota (an equal percentage emission reduction from a base year). They have checked stability for these agreements according to the concept of renegotiation-proofness (Farrell and Maskin, 1989). The stability requirements depend crucially on the parameters of the two countries and the type of agreement.

Finus and Rundshagen (1998b) have considered a similar model with N countries and analysed the coalition-formation process. In this process three issues are decided: abatement target, instrumental choice and coalitional size. Within a dynamic framework stability of a grand coalition and of sub-coalitions have been analysed, applying the concept of coalition-proofness (Bernheim et al, 1987).

4. AN ECONOMY WITH MULTILATERAL ENVIRONMENTAL EXTERNALITIES

This section is based solely on Chander and Tulkens (1997). Consider an economy with n agents (countries) who are denoted by i, $i \in N = \{1, ..., n\}$. There are three types of commodities in this economy.

i) a private good, whose quantities are denoted by $x \geq 0$ if they are consumed and by $y \geq 0$ if they are produced.

ii) pollutant discharges, the quantities of which are denoted by $p \geq 0$.

iii) ambient pollutant quantities denoted by $z \leq 0$.

Agent i is characterised by a utility function, $u_i(x_i, z)$. Let us assume for all i, $u_i(x_i, z) = x_i + v_i(z)$ where $v_i(z)$ is concave, differentiable and let $dv_i / dz \equiv B_i(z) > 0$ for all $z \leq 0$. Agent i is also associated with a technology, described by the production function $y_i = g_i(p_i)$ where g_i is strictly concave and differentiable. Let us also assume that there exists $p_i^0 > 0$ such that $dy_i / dp_i \equiv > 0$ if $p_i < p_i^0$, $= 0$ if $p_i \geq p_i^0$ and $= \infty$ if $p_i = 0$. Inputs are not explicitly mentioned above and are subsumed in the function g_i. A feasible state of the economy is a vector (x, p, z) such that $\sum_{i\,o\,N} x_i \leq \sum_{i\,o\,N} g_i(p)$ and $z = - \sum_{i\,o\,N} p_i$. A feasible state (x, p, z) is Pareto efficient if there exists no other feasible state (xN, pN, zN) for which $u_i(x_iN, zN) \geq u_i(x_i, z)$ for all $i \in N$ with strict inequality for at least one i.

It is easy to characterise an efficient state. From the first order conditions, an efficient state is characterised by the system of inequalities $\sum_{j\,o\,N} B_j(z) = (_i(p_i)$, $i = 1, ..., n$.

Under the assumptions of the model, several efficient states exist and in any efficient state $p_i > 0$ for all i. Moreover, one can prove that in all Pareto efficient states, the vector of emission levels (p_1, ...,

p_n) is the same (Proposition 1, Chander and Tulkens, 1997).

Let us now model the above economy as a non-cooperative game. Agents (countries) are the players in the game. A strategy of player i is to choose choose x_i and p_i. Let the (pure) strategy set of country i be $T_i = \{(x_i, p_i): 0 \# p_i \# p^0_i , 0 \# x_i \# g_i (p^0 i)\}$.

Let us also consider a coalition, S, of several countries. Suppose a coalition S can jointly choose their strategies. In that case, the space of joint strategies of players in S would be

$$T(s) = \{(x_i , p_i)_{i,s} : 0 \# p_i \# p^0_i \text{ œ } i \in S \text{ and } 0 \leq \sum_{i \in s} x_i \leq \sum_{i \in s} g_i(p_i^0) \}$$

Clearly, $T(S)$ e $\prod_{i\, o\, S} T_i$. Any strategy profile $((x_1 , p_1) , ..., (x_n , p_n))$ induces a feasible state (x, p, z) of the economy if $z = - \sum p_i$. Thus we can define the payoff of country i corresponding to a strategy profile $((x_1 , p_1) , ..., (x_n , p_n))$ as $u_i (x_i, z) = x_i + v_i(z)$ where $z = - \sum p_j$. The non-cooperative game is therefore well-defined.

A strategy profile $((x_1^{\,*}, p_1^{\,*}) , ..., (x_n^{\,*}, p_n^{\,*}))$ is a Nash equilibrium of the above game if for all $i \in N$, $(x_i^{\,*}, p_i^{\,*})$ maximises $x_i + v_i(z)$ subject to $x_i \# g_i (pi)$ and $p_i + z = - \sum_{j..i} p^*_j$. One can characterise the Nash equilibrium by the first order conditions of the players' maximisation problems which yield the following system of equalities $\prod_i (z^*) = \%_i (p^*_i); i = 1, .. , n.$

The above system differs from the system of equalities which characterises an efficient state of the economy. The Nash equilibrium therefore is not efficient. One can however prove that the Nash equilibrium of the above game exists and is unique (Proposition 2, Chander and Tulkens, 1997).

We can apply the concepts of strong Nash equilibrium and coalition-proof Nash equilibrium. Unfortunately however there does not exist a strong Nash equilibrium of the above game as was reserved earlier, the Nash equilibrium can not induce a Pareto efficient state. The concept of coalition-proof Nash does not add much to our study either. A coalition-proof Nash equilibrium is also a Nash equilibrium and in the above game the Nash equilibrium is unique; therefore, any coalition-proof Nash equilibrium - if exists at all - can not be different from the Nash equilibrium characterised above.

We can also associate a cooperative game with our economy. We have to associate, with every coalition *S*, the number *v(S)*, the value of the coalition *S*. Obviously, the value or the worth of a coalition is the highest aggregate payoff $\sum_{i \in S} u_i$ that the members of the coalition can achieve using some strategy. The pair [*N*, *v(.)*] consisting of the set of players, *N*, and the characteristic function *v(.)*defines a cooperative game with transferable utilities associated with the economy. The characteristic function is given by $v(S) = Max_{\{(xi, pi)i, s\}} \quad \sum_{i \in S} x_i + v_i(z)$

Notice that the variable *z* involves choices made by all the players including those who are not members of *S*. Thus the worth of a coalition is not only a function of actions taken by its members but also of actions of players outside the coalition. Hence one has to explicitly specify the actions of the players outside the coalition. In the notion of α-characteristic function and α-core, it is assumed that the players outside a coalition would adopt strategies which are the least favourable to the coalition. Chander and Tulkens (1995, 1997) proposed a new concept in which the players outside a coalition adopt individually best response strategies and thereby characterised the γ-core of such a game. Tulkens (1998) studied the two extreme approaches regarding the feasibility and as a consequence the likelihood of cooperation among countries in the above framework.

5. NEW DIRECTIONS

There exists a vast number of papers where environmental problems have been modelled as games and game theoretic solution concepts have been applied to solve such problems. To mention a few, Hoel (1992) presented a dynamic game of CO_2 emissions and analysed the open-loop and Markov perfect (Fudenberg and Tirole, 1991) equilibria. Ulph (1992) examined the question of the choice of environmental policy instruments in a model of strategic international trade between countries and compared the Nash and subgame perfect equilibria. Guth and Pethig (1992) analysed a signalling game

(Cho and Kreps, 1987) between a polluting firm that can save abatement costs by illegal waste emissions and a monitoring controller. When deciding on whether to dispose of its waste legally or illegally the firm does not know whether the controller is sufficiently qualified to detect the firm's illegal releases. The firm has the option of undertaking a small-scale deliberate "exploratory pollution accident" to get a hint about the controller's qualification before deciding on how to dispose of its waste. The controller may or may not respond to that "accident" by a thorough investigation thus revealing his type to the firm. Guth and Pethig (1992) formulated such a sequential game of asymmetric information as a signalling game and analysed the equilibria. Jeppesen and Andersen (1998) applied Rabin's (1993) idea of fairness into environmental games introduced by Barrett (1992, 1994). Their paper turned the focus away from the rather simple Nash equilibria to the more realistic fairness equilibria where countries are rewarding the "good ones" and punishing the "bad ones". Hurley and Shogren (1998) examined the theory and behaviour of an environmental conflict with one-sided asymmetric information. They explored the nature of the equilibrium in a conflict between a Northern and a Southern country battling over the preservation or development of a Southern resource.

The purpose of this section however is to mention a few recent developments in the game theory literature and to discuss a few new solution concepts which might be used to analyse the environmental problems already discussed. Let us first recall these concepts for the sake of completeness.

5.1. Correlation

In any non-cooperative game, if the players can *correlate* their strategies, a new self-enforcing agreement emerges, namely, *correlated equilibrium* (Aumann, 1974, 1987). The Nash equilibrium payoffs can be improved upon by correlating the strategies in all games other than *strategically zero-sum* games (Moulin and Vial, 1978). A *correlation device* consists of finite message sets, one for each player, and a probability distribution over the product of these message sets. Given any normal form game, an extended game induced by a correlation device is a game where first, according to the probability distribution, the device chooses a message profile and tells each player privately his message and then the players play the given game; thus a strategy of a player in the extended game is a map from the set of messages to the set of actions. A *correlated equilibrium* is a pair consisting of a correlation device and a strategy profile such that the strategy profile forms a Nash equilibrium in the extended game induced by the device.

Formally, for a game with complete information, a *correlation device* (mechanism), d, consists of finite message sets $M_1,..., M_n$, and a probability distribution p over $M = \prod_{i \in N} M_i$. Given a game G and a device d, an extended game G_d is a game where the device d first chooses a message profile $m = (m_1,..., m_n)$ according to p, tells each player i, privately and confidentially only his component m_i; then the players play the game G after getting the recommended message. One can restrict to *direct* correlation devices only (Myerson, 1982). A correlation device is *direct* (*canonical*) if $M_i = S_i$ for all i. Thus, a direct correlation device is just a distribution p over S satisfying $p(s) \geq 0$ for all $s \in S$ and $\sum_{s \in S} p(s) = 1$.

A direct correlation device can thus be thought of as a *mediator* who tries to help the players to coordinate their actions. He recommends, the recommendations being determined according to a commonly known distribution, each player separately an action; each player is then free to choose any action. A strategy of player i, δ_i, in this extended game is thus a map from S_i to S_i and the payoff is given by $u_i^*(\delta_1,...., \delta_n) = \sum_s p(s)u_i(\delta_1(s_1),...., \delta_n(s_n))$. The obedient strategy for player i, δ_i^*, is the identity map, i.e, $\delta_i^*(s_i) = s_i$ for all $s_i \in S_i$. A direct correlation device is a *correlated equilibrium* if the obedient strategy profile δ^* is a Nash equilibrium of the extended game. for all $i \in N$, $\sum_s p(s)u_i(s_i, s_{-i}) \geq \sum_s p(s)u_i(\delta_i(s_i), s_{-i})$ for all $\delta_i : S_i \rightarrow S_i$.

Other ways of correlation are also conceivable. For example, Moulin and Vial (1978) proposed a correlation scheme in which the players decide either to be told the outcome of the lottery and commit himself to play the pure strategy selected by the lottery or not to be told the outcome and play some other (mixed) strategy. Gerard-Varet and Moulin (1978) used this notion to study correlation in duopoly markets.

5.2. Coalition-Proof Correlated Equilibrium

Recently, several studies (Moulin, 1982; Einy and Peleg, 1995; Milgrom and Roberts, 1996; Moreno and Wooders, 1996; Ray, 1996, 1998) have been made to understand a non-cooperative situation in which the players can form coalitions as well as correlate their strategies. Ray (1996) has defined the notion of *coalition-proof correlated equilibrium* as a coalition-proof Nash equilibrium of the game extended by a correlation device. This notion assumes quite a natural way of defining such a concept.

A *Coalition-proof correlated equilibrium* of the game G is a pair $(d, (\phi_i)_{i \in N})$, where the strategy (pure or behavioural) vector $(\phi_1, \ldots, \phi_n)$ is a coalition-proof Nash equilibrium of the extended game G_d. A *direct coalition-proof correlated equilibrium* of the game G is a coalition-proof correlated equilibrium where $M_i = S_i$, and ϕ_i is the identity map, for all i.

5.3. Theory of Social Situations

In the theory of social situations (Greenberg, 1990), the notion of a *situation* provides a complete description of a game or any social environment. In any social environment (a game), we have several positions and the description (rules) of the environment (the game) tells us how these positions can be induced. Thus, a situation consists of a set of possible *positions* and an *inducement correspondence*.

A position specifically tells us the current state, i.e., the set of individuals (players) involved, the set of all possible outcomes and the preferences (utility functions) of the individuals over the set of outcomes.

A *position*[1], G, is a triple, $G = (N(G), X(G), \{f_i(G)\}_{i \in N(G)})$, where, $N(G)$ is the set of players, $X(G)$ is the set of all feasible outcomes and $f_i(G)$ is the utility function of player i in position G over the outcomes, that is, $f_i(G): X(G) \rightarrow \mathbb{R}$, for all i.

[1]In this subsection, we use all the notations from Greenberg (1990) and thus in this paper, some of the symbols, such as *G*, are used for different objects. Hope the readers will not find it confusing. Also, following Greenberg, all the inclusions in this subsection are weak.

From any given position of a social environment or a game (formally, a situation), an individual can induce a set of positions. The *inducement correspondence* precisely describes this set. Formally, the set of positions that individual i can induce from a position G when a particular outcome $x \in X(G)$ is proposed is given by, $\gamma(\{i\}|G, x)$. The same reasoning can be extended to a group of individuals. Given any position G, and a proposed outcome $x \in X(G)$, let $\gamma(J|G, x)$ denote the set of positions that a coalition J, $J \subset N(G)$ can induce from G when x is proposed. We are now ready to formally define a situation.

A *situation* is a pair (γ, Γ), where Γ is a set of positions and the mapping γ, is called the *inducement correspondence*, satisfies the condition that for all $G \in \Gamma$, $J \subset N(G)$ and $x \in X(G)$, $\gamma(J|G, x) \subset \Gamma$. (i.e., Γ is closed under γ.)

The only requirement imposed on the inducement correspondence, γ, is that the set of players in each position that a coalition J can induce includes, but need not coincide with, the players in J. Formally, we assume, in any situation (γ, Γ), for all $G \in \Gamma$, $J \subset N(G)$ and $x \in X(G)$, if a position $H \in \gamma(J|G, x)$, then $J \subset N(H)$.

A situation, (γ, Γ), just describes a social environment. It does not however, suggest anything about the expected outcomes for different positions in a situation. A *solution* for a position G, denoted by $\mathcal{F}(G)$, is a subset of the set of feasible outcomes, $X(G)$. As we are interested in the outcomes for all the positions in a situation, we define the following notion.

Let (γ, Γ) be a situation, where Γ is a collection of positions. A mapping σ that assigns to each position $G \in \Gamma$ a solution, $\sigma(G) \subset X(G)$, is called a *standard of behaviour* (*SB*) for Γ.

Following von Neumann and Morgenstern (1947), we will call an *SB* stable if it satisfies both the *internal* and *external* stability concepts. We shall say that the *SB* σ is *internally stable* for (γ, Γ) if for all $G \in \Gamma$, $x \in \mathcal{F}(G)$ implies that there exists no coalition $J \subset N(G)$ and position $H \in \gamma(J|G, x)$, such that J benefits by rejecting x and inducing H realizing that the solution to H is given by $\mathcal{F}(H)$. We shall say that the *SB* σ is *externally stable* for (γ, Γ) if for all $G \in \Gamma$, $x \in X(G)\backslash\mathcal{F}(G)$ implies that there exists a coalition $J \subset N(G)$ and a position $H \in \gamma(J|G, x)$, such that J benefits by rejecting x and inducing H realizing that the solution to H is given by $F(H)$.

We shall say that the *SB* σ is *optimistic internally stable* for (γ, Γ) if for all $G \in \Gamma$, $x \in F(G)$ implies that there do not exist a coalition $J \subset N(G)$, a position $H \in \gamma(J|G, x)$, and an outcome $y \in F(H)$ such that for all $i \in J$, $f_i(H)(y) > f_i(G)(x)$. We shall say that the *SB* σ is *optimistic externally stable* for (γ, Γ) if for all $G \in \Gamma$, $x \in X(G) \backslash F(G)$ implies that there exist a coalition $J \subset N(G)$, a position $H \in \gamma(J|G, x)$, and an outcome $y \in F(H)$ such that $f_i(H)(y) > f_i(G)(x)$ for all $i \in J$.

Combining the two stability conditions, we have, if σ is both optimistic internally and externally stable for (γ, Γ) then σ is an *optimistic stable standard of behaviour* (*OSSB*) for (γ, Γ).

The notion of *ODOM* as defined below provides an alternative and useful way to define *OSSB*. $ODOM(\sigma, G) = \{x \in X(G) | \exists J \subset N(G), H \in \gamma(J|G, x)$, and $y \in F(H)$ such that for all $i \in J$, $f_i(H)(y) > f_i(G)(x)\}$. Obviously, σ is an *OSSB* if and only if, for all $G \in \Gamma$, $F(G) = X(G) \backslash ODOM(F, G)$.

One should definitely use the above concepts to analyse environmental problems. For example, one should try to find the set of correlated equilibria and the coalition-proof correlated equilibria for the games described in Sections 3 and 4. Similarly, like Gerard-Varet and Moulin (1978), one could use the concepts of Moulin and Vial (1978) to the problems discussed here. The economy in section 4 can be modelled as a social situation and therefore one should try to find the *OSSB* for that situation. It will be really interesting to see how these new concepts can shed new lights to our understanding of the environmental problems.

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